The University of the State of New York 219th High School Examination

## ADVANCED ALGEBRA

Monday, June 17, 1918 - 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.
The minimum time requirement is flve recitations a week in algebra for two school years.
Answer eight questions. Each answer should be reduced to its simplest form.

1 a Plot the graph of $2 x^{3}+8 x^{2}-8$
6 If it were required to find from the graph the real roots of the equation $2 x^{3}+8 x^{2}-8=0$, explain why it would be unnecessary to extend the table of values beyond $x=-4$ and $x=1$
2 Find to two decimal places the real root of the equation $x^{3}+2 x-4=0$
3 Given the equation $x^{4}-2 x^{3}+4 x-4=0$
Without solving the equation, fill out the following statements by placing the proper number in each parenthesis; justify each statement by quoting the appropriate theorem or principle:
a The equation has ( ) roots.
$b$ It has ( ) positive roots and ( ) negative roots.
c It has ( ) fractional roots.
$d$ It has at least ( ) imaginary roots.
e All the integral roots, if any, must be factors of ( ).
$f$ The product of the roots is ( ).
4 How long is the edge of a cube, if the number of square feet in its surface exceeds the number of cubic feet in its volume by 27 ? [Solve either algebraically or graphically.]

5 Given the equation $a x^{2}+b x+c=0$ What can you tell about the character of the roots if $a$ and $c$ have opposite signs? Does the sign of $b$ affect the character of the roots? What must be true if one root is to be zero?

6 Solve $x^{2}+\frac{1}{x^{2}}+x+\frac{1}{x}=4$
$7 a$ Prove the theorem: If $a$ is a root of $f(x)=0$, then $x-a$ is a factor of $f(x)$
$b$ Prove that $x-1$ is a factor of $100 x^{7}-101 x^{3}+1$
8 a Simplify $\frac{1+2 i+3 i^{2}}{1-2 i+3 i^{2}}$
Represent the result graphically.
$b$ Find the three cube roots of 8 and show graphically that their sum is 0 .
9 Given the formula

$$
\frac{w v^{2} e}{2 g}=\frac{t r}{12}
$$

Express each of the variables $w, v$ and $t$, in terms of the other variables.
10 A freight boat leaves New York, sailing for a European port at the rate of 15 knots an hour. Five hours later a transport leaves the same harbor, sailing over the same course at the rate of 20 knots an hour. The freight boat, two hours after its departure, is delayed one hour by a defect in its machinery. In how many hours will the transport overtake the freight boat? [Solve either algebraically or graphically].
11 From 16 soldiers in how many ways can a guard of 5 be chosen? In how many ways can a guard of 7 be arranged in line? In how many ways can the 16 be divided into two equal groups?
12 A signal corps has six different flags; by using one, or two, or three flags at a time, how many different signals can be formed with these flags?
13 A rectangular sheet of cardboard, $9^{\prime \prime}$ by $12^{\prime \prime}$, is to be made into an open box by cutting out a square from each corner and turning up the sides; if the volume of the box is to be 80 cubic inches, find the length of a side of each square cut out.

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## DIRECTIONS FOR RATING

The direction, "Less than $60 \%$ of the credit should be granted when an error in compatation occurs," should be followed in rating all incorrect answers to questions which fall under the topics mentioned in "Suggestions on the Rating of Regents Examination Papers in Mathematics" under "General 3."

In all problems solved with two unknowns no credit should be given for one equation correctly formed if the other is not given or is inaccurate.

Except in schools where the "committee system" is used, teachers are uqrged to mark papers cumulatively, that is, to add the credits earned by each answer to the total credits earned by preceding answers so that the mark given to the last answer is the per cent to which the paper is entitled, e. g. consecutive answers earning 5, 7, 4 etc. respectively should be marked $5,12,16$ etc. respectively.

## 1 12f credits

a Allow $8 \frac{1}{2}$ credits for correct graph.
$b$ Allow 4 credits for explanation.
$2 \quad 12 \frac{1}{2}$ credits
Allow 3 credits for finding first figure of root.
Allow 4 eredits for finding second figure of root. Allow $5 \frac{1}{2}$ credits for finding third figure of root.
$312 \frac{1}{2}$ credits
Allow $2 \frac{1}{2}$ credits for $b, 2$ credits for each of the others.

## $412 \frac{1}{2}$ credits

Allow $4 \frac{1}{2}$ credits for correct equation.
If solved algebraically, allow 8 credits for possible roots (4 each); or
If solved graphically, allow 4 credits for correct graph and allow 4 credits for reading roots correctly.

## 5 12) credits

6 12f credits
Allow 6 credits for finding first two results. Allow 61 credits for finding other two results.

## Directions for Rating-concluded

$7 \quad 12 \frac{1}{2}$ credits
a $8 \frac{1}{2}$ credits
$b 4$ credits
$812 \frac{1}{2}$ credits
a 5 credits
Allow 4 credits for correct simplification.
Allow 1 credit for representing result graphically.
b $7 \frac{1}{2}$ credits
Allow $3 \frac{1}{2}$ credits for finding the roots.
Allow 4 credits for correct graph.
$9 \quad 12 \frac{1}{2}$ credits
$10 \quad 12 \frac{1}{2}$ credits
If solved algebraically, allow $7 \frac{1}{2}$ credits for correct equation and 5 credits for correct solution.
If solved graphically, allow 8 credits for graphs ( 4 each) and allow $4 \frac{1}{2}$ credits for reading correct result.
$1112 \frac{1}{2}$ credits
a 4 credits
b 4 credits
c $4 \frac{1}{2}$ credits
Allow no partial credit on $a, b$ or $c$.
$1212 \frac{1}{2}$ credits
See "General Suggestion 3."
$13 \quad 12 \frac{1}{2}$ credits
Allow $7 \frac{1}{2}$ credits for correct equation.
Allow 5 credits for correct solution.

