195th High School Examination

Advanced Algebra

Monday, June 15, 1908—9.15 a.m. to 12.15 p.m., only

Answer eight questions.

1. Find the successive derived polynomials of
   \( \frac{1}{4}x^6 + \frac{1}{4}x^4 - \frac{1}{16}x^2 + 5x^2 - 1 = 0 \)

2. Resolve \( \frac{x^2 + x + 3}{x^3 + x^2 + x + 1} \) into partial fractions.

3. Find the sum of the infinite recurring series of the second
   order \( 1 + 3x + 5x^2 + 7x^3 + 9x^4 + \ldots \)

4. Given \( \log 2 = 0.30103 \), \( \log 3 = 0.47712 \), \( \log 5 = 0.69897 \);
   find the logarithm of \( \frac{\sqrt{88}}{\sqrt{5}} \)

5. Out of nine different pairs of gloves, in how many ways can I choose a right hand glove and a left hand glove that do not form a pair? Write a full explanation.

6. Plot the graph of \( y = x^4 - 7x + 4 \) and from the graph find the approximate values of the roots of the quadratic equation formed by making \( y \) equal to zero.

7. Prove that if \( a \) is a root of an equation the equation is divisible by \( x - a \)

8. Four roots of the equation \( x^5 - 5x^4 - 55x^3 + 245x^2 + 654x - 2520 = 0 \) are 3, 5, 7, -6; find the other root.

9. Transform the equation \( x^4 + \frac{1}{8}x^3 + \frac{3}{4}x^2 + \frac{11}{8} = 0 \) into an equation having only integral coefficients, that of the first term being unity. How do the roots of the derived equation differ from the roots of the given equation?

10. Four numbers are in arithmetic progression; the sum of the first and fourth equals 18 and the sum of the squares of the second and third equals 162\( \frac{3}{4} \). Find the numbers.

11. Express graphically the sum of the complex numbers
   \( 2 + \sqrt{-9}, 5 + \sqrt{-49} \)

12. Show by Descartes's rule of signs that the equation
   \( 2x^7 - 5x^4 + 7x - 5 = 0 \) has at least four imaginary roots.