January 25, 1967

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answers in the spaces provided.

1. If the probability that a basketball player will succeed in making a certain shot is 0.4, what is the probability that he will not succeed?

2. Solve for $x$: $4^x = 2^{x+4}$

3. Express $\frac{1}{2 + i\sqrt{3}}$ as an equivalent fraction with a real denominator.

4. Solve for $y$: $\sqrt{3y} - 8 = y - 6$

5. If $(a + bi) + (2 - i) = 3 + i$, find the value of $b$.

6. The height(s) above the ground, in feet, of a ball thrown vertically upward is given by the equation $s = 44 + 80t - 16t^2$, where $t$ is measured in seconds. Find the velocity of the ball in feet per second when $t = 1$.

7. How many distinct five-letter arrangements can be made from the letters of the word "ADDED"?

8. Find the rational root of the equation $2x^3 - x^2 - x - 3 = 0$.

9. A root of $x^3 + x^2 + x - 7 = 0$ lies between 1.4 and 1.5. Find this root to the nearest tenth.

10. Find the values of $x$ which satisfy the inequality $3 + x \leq 4x - 5$.

11. Find the remainder, independent of $x$, when $ax^2 + bx + c$ is divided by $x - r$.

12. For what value of $k$ is 2 a root of the equation $2x^4 - 6x^3 + 4kx^2 + 13 = 0$?

13. $F$ varies directly as $m$ and inversely as $d^2$. If $F = 96$ when $m = 4$ and $d = 6$, find $F$ when $m = 6$ and $d = 8$.

14. How many committees each consisting of 3 boys and 2 girls can be chosen from a group of 8 boys and 5 girls?

15. Find the numerical value of $\log_{10} 64$.

16. If the graphs of the equations $4y - 2x = 7$ and $ax + 5y = 10$ are perpendicular to each other, find the numerical value of $a$.

17. Solve for $x$ in terms of $y$: $y = \frac{2x + 7}{x - 3}$.
18. The cost of a telephone call is $a$ cents for the first three minutes and $b$ cents per minute for each minute thereafter. If $n$ is an integer greater than 3, write an expression for the cost of a call for $n$ minutes.

19. The 5th term of an arithmetic progression is $s$ and the 15th term is $t$. Find the common difference in terms of $s$ and $t$.

Directions (20-24): Indicate the correct completion for each of the following by writing the number 1, 2, 3, or 4 in the space provided.

20. One of the equations for the family of lines passing through the point whose coordinates are $(0, -3)$ is (1) $y = -3x + b$ (2) $y = b$ (3) $y = mx - 3$ (4) $y = mx$

21. The sum of the roots of the equation $x^3 - 3x^2 + 2x - 1 = 0$ exceeds the product of the roots by (1) 1 (2) 2 (3) 3 (4) 4

22. The graph of the equation $3x^2 + 12x - 20y + 42 = 0$ is (1) a parabola (2) an ellipse (3) a circle (4) a hyperbola

23. The coordinates of the point of inflection of $y = \frac{x^3}{6} + \frac{x^2}{2} + 2x - 1$ are (1) $\left( -1, -\frac{3}{2} \right)$ (2) $\left( -1, -\frac{8}{3} \right)$ (3) $\left( 1, \frac{5}{3} \right)$ (4) $\left( 1, \frac{7}{2} \right)$

24. The numerical value of $10^{10} \cdot 3$ is (1) $\frac{10}{3}$ (2) $\frac{3}{10}$ (3) $3$ (4) $\frac{1}{3}$

Part II

Answer sixteen questions from this part, 25-48. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers in the space provided.

25. Given $i$ is the imaginary unit, write $(-i)^{56}$ in simplest form.

26. Determine all the values of $x$ for which the inequality $x^2 - x - 6 < 0$ is true.

*27. The area of a triangle, expressed in the form $\frac{1}{2}$

is 14. Find the value of $x$.

28. Four points $A, B, C,$ and $D,$ which represent complex numbers plotted in the complex plane, are the vertices of parallelogram $ABCD$. If $A$ represents $0 + 0i$, $B$ represents $5 + i$, and $D$ represents $2 + 7i$, what complex number does $C$ represent?
29. Multiply $2(\cos 115^\circ + i \sin 115^\circ)$ by $6(\cos 245^\circ + i \sin 245^\circ)$ and express the result in the rectangular form, $a + bi$.  

30. If $y = x^2 + 3x - 2$, find the average rate of change of $y$ with respect to $x$ as $x$ increases from $x = 1$ to $x = 4$. 

31. What is the abscissa of the point on the graph of $y = x^2 + 5x + 4$ where the slope of the tangent equals 9? 

32. The arithmetic mean between two numbers is $-6$ and their positive geometric mean is $4\sqrt{2}$. Find the two numbers. 

33. What is the sum of the seven numerical coefficients in the expansion of $(a + b)^6$? 

34. Express the repeating decimal $0.4333\ldots$, in which the digit 3 is repeated endlessly as indicated, in the form $\frac{a}{b}$ where $a$ and $b$ are integers. 

35. If $f(x) = x^2 + 2x - 3$, write $f(a - 3)$ as an expression free of parentheses. 

36. Change $x^2 + y^2 = 2x + 15$ from rectangular coordinates to polar coordinates. 

37. If $\log_{10} e = 0.4343$, find $\log_{10} 100$ to the nearest tenth. 

38. What is the abscissa of the point at which the graph of $y = -2x^2 + x + 3$ is intersected by its axis of symmetry? 

39. Write an equation of the line whose $x$-intercept and $y$-intercept are each twice the corresponding intercepts of the graph of the equation $5x - 2y = 10$. 

40. The endpoints of a diameter of a circle are $(6, 0)$ and $(0, 8)$. Write the equation of this circle in the form $(x - h)^2 + (y - k)^2 = r^2$. 

41. The equation $x^3 + 6x^2 + 13x + 10 = 0$ has a root $x = -2$. Express one of the remaining roots in the form $a + bi$. 

*36. Change $x^2 + y^2 = 2x + 15$ from rectangular coordinates to polar coordinates.

Directions (42-48): For each of those chosen, write in the space provided the number preceding the expression that best completes each statement or answers each question.

42. A rectangle is twice as long as it is wide. Its diagonal is $d$ inches long and its area contains $A$ square inches. The area $A$ expressed in terms of $d$ is 

\[
\frac{2d^2}{3} \quad (2) \quad \frac{3d^2}{2} \quad (3) \quad \frac{2d^2}{5} \quad (4) \quad \frac{5d^2}{2}
\]

43. The value or values of $x$ for which the expression $\frac{x - 1}{2x(x + 1)}$ is undefined would be (1) 1 only (2) $-1$ only (3) 0 only (4) 0 and $-1$  43____
44. Which point does not lie on the graph of \( y = \log x \)?
   (1) \( \left( \frac{1}{2}, -\frac{1}{2} \right) \)  
   (2) \( (1, 0) \)  
   (3) \( (-4, -1) \)  
   (4) \( (2, \frac{1}{2}) \)  

45. In how many points do the graphs of \( xy = 6 \) and \( (x + 3)^2 + (y - 3)^2 = 4 \) intersect?
   (1) 1  
   (2) 2  
   (3) 0  
   (4) 4

46. The roots of the equation \( x^2 - kx + k = 0 \) are real and unequal if
   (1) \( k < 0 \) or \( k > 4 \)  
   (2) \( 0 \leq k < 4 \)  
   (3) \( k = 0 \)  
   (4) \( k = 4 \)

47. It requires 8 hours for machine \( A \) to do a certain job alone, and it requires 15 hours for machine \( B \) to do the same job alone. If two machines of exactly the same type as \( A \) and three machines of exactly the same type as \( B \) work on this job together, in how many hours will they complete it? If \( x \) represents the time required for these 5 machines to do the job together, then a correct equation for the solution of this problem is

\[
\frac{x}{16} + \frac{x}{45} = 1
\]

\[
\frac{1}{4} + \frac{1}{5} = \frac{5}{x}
\]

\[
\frac{x}{8} + \frac{x}{15} = 1
\]

\[
\frac{x}{4} + \frac{x}{5} = 1
\]

48. A complex root of \( x^5 + 32 = 0 \) is
   (1) \( 2(\cos 72^\circ + i \sin 72^\circ) \)  
   (2) \( 2(\cos 36^\circ + i \sin 36^\circ) \)  
   (3) \( -2(\cos 108^\circ + i \sin 108^\circ) \)  
   (4) \( -2(\cos 36^\circ + i \sin 36^\circ) \)