TWELFTH YEAR MATHEMATICS
12A (Advanced Algebra)

Monday, January 27, 1964—1:15 to 4:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answers on the line to the right.

1. Express the number $\frac{2 + i\sqrt{3}}{5 - i\sqrt{3}}$ as an equivalent fraction with a real denominator. 1.....

2. The graph defined by $y = 2x^2 + bx + c$ passes through the points (1,5) and (2,6). Find the value of b. 2.....

3. Express in the form $a + bi$ the product $2(\cos 119° + i \sin 119°)(\cos 121° + i \sin 121°)$. 3.....

4. Solve for x: $16^{3x} = 8^{x+4}$ 4.....

5. Write the coordinates of the minimum point of the parabola $y = x^2 - 6x + 13$. 5.....

6. The sides of a triangle are segments of the lines whose equations are $x = 0$, $y = 0$ and $y = -x + 2$. Find the area of the triangle. 6.....

7. The line $y = 4x - 4$ is tangent to $y = x^2$. Find the coordinates of the point of tangency. 7.....

8. A and B can do a job in c days. If A does half as much work per day as B, express in terms of c the number of days it would take A alone to do the job. 8.....

9. Solve for x: $7 - 2x < 5(x - 7)$ 9.....

10. If the expression $x^5 + 2x^2 - k$, in which k is a constant, is divided by $x + 2$, the remainder is zero. Find the value of k. 10.....

11. If 3 is one root of the equation $x^3 - 3x^2 + 9x - 27 = 0$, find the other two roots. 11.....

12. If $\log_a 2 = b$ and $\log_a 3 = c$, express $\log_a 24$ in terms of b and c. 12.....

13. Write an equation of the line perpendicular to $2x - 3y = 1$ and passing through (2,7). 13.....

14. Find the value of x for which $\log (x - 2) + \log (x + 2) = \log (x^2 - 2x + 3)$. 14.....

15. Find to the *nearest tenth* the value of $\log_2 310$. 15.....

16. Write the eighth term only in the expansion of $\left(1 - \frac{1}{x}\right)^{10}$. 16.....

17. A quantity z varies directly with the product of x and y^2. If $z = 3$ when $x = 100$ and $y = 3$, find the positive value of y when $z = 50$ and $x = 6$. 17.....

18. How many odd numbers x such that $100 < x < 1{,}000$ can be formed from the digits 1, 2, 5, 8, 9, if repetition of digits is *not* permitted? 18.....

19. Find the minimum value of $x^3 - 12x + 3$ for $x > 0$. 19.....

Directions (20-24): Indicate the correct completion for *each* of the following by writing the number 1, 2, 3 or 4 in the space provided at the right.

20. The sum of $-3 - 2i$ and $+5 - 3i$ is a complex number which, when represented graphically, lies in quadrant (1) I (2) II (3) III (4) IV 20.....

21. The 12th term of an arithmetic progression is $2x$ and the 17th term is $7x$. The first term is (1) x (2) $-9x$ (3) $-13x$ (4) $-14x$ 21.....

22. The velocity v in feet per second of a moving body is given by the equation $v = 5t^3 - 9t$ where t is the time in seconds. The acceleration, measured in feet per second per second is a (1) constant (2) linear function of t (3) quadratic function of t (4) cubic function of t 22.....

23. A root of $x^3 = 1$ is (1) $\dfrac{-1 + i\sqrt{3}}{2}$ (2) $\dfrac{1 - i\sqrt{3}}{2}$ (3) $\dfrac{1 + \sqrt{3}}{2}$ (4) $\dfrac{-1 - \sqrt{3}}{2}$ 23.....

24. Factors of the expression $a^2 + b^2$ are (1) $(a + b)$ $(a + b)$ (2) $(a + bi)$ $(a - bi)$ (3) $(a + bi)$ $(a + bi)$ (4) $(a + b)$ $(a - b)$ 24.....

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the line at the right.*

25. If $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$, find the value of $f(\frac{1}{2}) - g(2.25)$. 25.....

26. A circle whose center is at $(-2,2)$ passes through the point $(-6,-1)$. Write an equation of this circle. 26.....

27. Mary is one of 10 students of whom two are to be chosen at random for a committee. What is the probability that Mary will be chosen for the committee? 27.....

28. The graphs of $y = \log_{10} x$ and $x + y = 1$ are drawn on the same set of axes. What are the coordinates of the point(s) of intersection? 28.....

29. Express $8i^4 - 3i^3 + 3i^2 + 6i + 4$ in the form $a + bi$. 29.....

30. Using the formula $F = e^{rt}$, find F to the *nearest tenth* when $e = 2.72$, $r = 0.01$ and $t = 10$. 30.....

31. One root of $f(x) = 0$ is $3 - i$ where $f(x)$ is a polynomial with real coefficients. If the degree of $f(x)$ is n, express in terms of n the largest possible number of real roots of $f(x) = 0$. 31.....

32. The sum of an infinite geometric progression is $\frac{3\sqrt{2}}{5}$. If the common ratio is $\frac{-2}{3}$, find the first term. 32.....

33. Express the repeating decimal 2.171717 . . ., in which the digits 1 and 7 are repeated endlessly as indicated, in the form $\frac{p}{q}$ where p and q are integers. 33.....

34. A law of gravitational attraction between two masses, m_1 and m_2, states that force (F) varies directly as the product of the masses and inversely as the square of the distance (d) between them. Using k as the constant of variation, write this law as an equation in F, m_1, m_2, d and k. 34.....

35. Three coins are tossed simultaneously. What is the probability that they will show two heads and a tail? 35.....

36. Find the total number of lines determined by 50 points, no three of which are collinear. 36.....

37. If the average rate of change of $f(x)$ over the interval $x = 4$ to $x = 8$ is 12, and $f(4) = 17$, find the value of $f(8)$. 37.....

38. A polynomial equation $f(x) = 0$ has only one negative real root. Express to the *nearest tenth* an approximation of that root, given the following information: $f(-1) > 0$, $f(-2) < 0$, $f(-1.1) > 0$, $f(-1.2) < 0$, $f(-1.15) > 0$. 38.....

*39. The equation of a certain line can be written as

$$\begin{vmatrix} x & y & 1 \\ 0 & 1 & 1 \\ -2 & 6 & 1 \end{vmatrix} = 0.$$

Write the same equation in the form $ax + by + c = 0$. 39.....

*40. Transform $r = 4 \cos \theta$ from polar to rectangular coordinates. 40.....

41. The inequality $x^2 - x + 1 < 0$ is true for (1) all real values of x, (2) some but not all real values of x, (3) no real values of x. Answer (1), (2) or (3). 41.....

Directions (42-48): For each *of those chosen,* write in the space provided the *number* preceding the expression that best completes the statement.

42. When graphed on the same set of axes, the total number of points of intersection of the graphs of $y = \frac{12}{x}$ and $x^2 + y^2 = 1$ is (1) 0 (2) 2 (3) 3 (4) 4 42.....

43. The points (1,1) and (—4,—14) determine a straight line. Point P whose abscissa is 3 lies on this line. The ordinate of point P is (1) $\frac{5}{3}$ (2) 7 (3) 3 (4) 11 43.....

44. If in the equation $y = 3^x$ the variable x is decreased by 3, then y is (1)decreased by 3 (2)divided by 3 (3)decreased by 27 (4)divided by 27 44.....

45. Which of the following is a negative irrational number in the interval $-3 < x < -2$? (1) $\sqrt{-2.5}$ (2) $-\sqrt{5}$ (3) $-\sqrt{2.5}$ (4) $-\sqrt{\frac{25}{4}}$ 45.....

46. Given the equation $ax^2 + bx + c = 0$ in which a, b and c are positive integers and b is the mean proportional between a and c, then the roots of the equation are (1)real, rational, equal (2)real, rational, unequal (3)real, irrational, unequal (4)imaginary 46.....

47. If the roots of $x^2 - 3x + 1 = 0$ are m and n, then $3(m + n) + mn$ is (1) 8 (2) —8 (3) 10 (4) —10 47.....

48. The numbers p, q, r and k are real. One root of $x^4 + px^3 + qx^2 + rx + k = 0$ is $2 - i$. Two of the roots are equal integers. The coefficient p (1)must be an even integer (2)must be an odd integer (3)may be either an even or an odd integer (4)need not be an integer 48.....