

January 25, 1962

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Write your answer on the line at the right.

- Given $f(x) = x^2 - 2x$, express $f \left\{ \frac{1}{2a} \right\}$ as a single fraction. 1 _____
- Express $0.297297 \dots$ in the form $\frac{a}{b}$ where a and b are integers. 2 _____
- Write an equation of the straight line perpendicular to the line $3x + 2y = 5$ and passing through the point $(-1, 2)$. 3 _____
- Find the length of the radius of the circle $x^2 + y^2 - 8x + 14y = 35$. 4 _____
- How many three-digit numbers greater than 100 may be formed using the digits 0, 1, 3, 5 if repetitions of digits are *not* allowed? 5 _____
- An equation with real coefficients has $1 - i$, 3 and 4 among its roots. What is the lowest possible degree of the equation? 6 _____
- Solve for x : $8^{2x-3} = 16^{x-2}$ 7 _____
- If $f(x) = -x^2 + x + c$, for what value of c will $f(x) = 0$ have equal roots? 8 _____
- Given: a varies directly as b and inversely as the square of c . If $a = 12$ when $b = 3$ and $c = 2$, find a when $b = \frac{1}{4}$ and $c = \frac{1}{3}$. 9 _____
- How many straight lines are determined by a set of nine points if no three points of the set are collinear? 10 _____
- Express $\frac{5 + i\sqrt{2}}{3 - i\sqrt{2}}$ as an equivalent fraction with a real denominator. 11 _____

Directions (12-16): Write on the line at the right of *each* of the following the number preceding the expression that best completes the statement.

- When drawn on the same set of axes, the graphs of $x^2 - 3y^2 = 4$ and $y^2 = x$ intersect in _____
 (1) only one point
 (2) only two points (3) no points (4) four points 12 _____
- If $x = \frac{y+a}{y-a}$, then $y =$ _____ (1) $\frac{x-1}{a-ax}$
 (2) $\frac{a-ax}{x+1}$ (3) $\frac{a+ax}{x-1}$ (4) $\frac{a+ax}{x+1}$ 13 _____

14. Given: $\log_{10}x = M$. If b is a positive real number $\neq 1$, then M is equal to
- (1) $\frac{\log_b x}{\log_b 10}$ (2) $(\log_b x)(\log_b 10)$
- (3) $\log_b x - \log_b 10$ (4) $\frac{\log_b 10}{\log_b x}$ 14 _____
15. If $f(x) = \frac{x - 2}{x + 5}$, then the value of x for which $f(x)$ is undefined is
- (1) -2 (2) 2 (3) 5 (4) -5 15 _____
16. The graph of $2^x = y$ has a point on the
- (1) positive portion of the x -axis (2) negative portion of the x -axis
- (3) positive portion of the y -axis (4) negative portion of the y -axis 16 _____
17. Express $x^3 + 2x^2 - 5x - 6$ as an indicated product of three linear factors. 17 _____
18. Solve for x to the nearest integer: $(1.5)^x = 25$ 18 _____
19. From a deck of fifty-two cards containing four aces, one card is drawn at random. What is the probability that the card selected is *not* an ace? 19 _____
20. Given: $\log k = 2 \log a - \frac{1}{3} \log b$. Express k in terms of a and b . 20 _____
21. If x and y are real numbers and if $2x + 8i = 3 + iy$, find the numerical value of x . 21 _____
22. Find the modulus of the complex number $-2 + 3i$. 22 _____
23. Express $6i^4 - 3i^3$ in the form $a + bi$. 23 _____
24. When $x^3 - 2x^2 + cx + 9$ is divided by $x - 2$, the remainder is 3. Find the value of c . 24 _____

Part II

Answer sixteen questions from this part, 25-48. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

25. Express one of the imaginary roots of $x^6 - 1 = 0$ in polar form. 25 _____
26. Find the coordinates of the point of inflection of the graph of $f(x) = x^3 - 6x^2$. 26 _____

27. The first three terms of a geometric progression are x , y and z , in that order. Express x in terms of y and z . 27. _____

28. Divide $12(\cos 110^\circ + i \sin 110^\circ)$ by $3(\cos 50^\circ + i \sin 50^\circ)$. 28. _____

29. A merchant bought a articles for c cents each. If each article had cost d cents less, how many articles could he have bought for the same amount of money? 29. _____

30. Write in simplest form the *fourth* term of the expansion of

$$\left\{ x + \frac{1}{x^2} \right\}^9$$

30. _____

31. If the roots of the equation $x^3 + px^2 + qx + r = 0$ are 2, 3 and 4, find the value of r . 31. _____

32. For what value of k will the graph of the equation $y - 2 = 3(x - k)$ pass through the point $(4, 2)$? 32. _____

33. Transform $x^2 + y^2 + 2x - y = 1$ into an equation in polar coordinates. 33. _____

*34. If $\begin{vmatrix} x & 0 & 1 \\ 1 & 2 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$, find the value of x . 34. _____

35. A positive root of $x^3 - 9x - 9 = 0$ lies between 3 and 4. Find this root to the *nearest integer*. 35. _____

36. Find an equation of the tangent to the curve $y = x^2$ at the point $(2, 4)$. 36. _____

37. Given $y = x^2 + 3x$. Find the average rate of change of y as x increases from 1 to 3. 37. _____

38. If two balls are drawn at random from a bag containing 4 red balls and 4 white balls, what is the probability that both are red? 38. _____

39. A motorist travels from town A to town B at the uniform rate of x miles per hour and returns over the same route at the uniform rate of y miles per hour. Express in terms of x and y the average speed at which the motorist has made the entire trip. 39. _____

40. Solve the inequality: $5x + 2 \geq 3(x - 1)$ 40. _____

*41. Write in determinant form the equation of the straight line that passes through the points $(2, -3)$ and $(5, 4)$. 41. _____

Directions (42-43): For each of the following *which you select*, indicate whether the statement is true for (1) all real values of a , b and c (2) some, but not all, real values of a , b and c (3) no real value of a , b and c by writing on the line at the right the number 1, 2 or 3.

42. If $a < b$ and $b = \frac{2}{3}c$, then $a < c$. 42. _____

43. If $a < (b + c)$, then $-a < -(b + c)$. 43. _____

Directions (44-48): For each of the following *which you select*, write the *number* preceding the expression that best completes the statement or answers the question.

44. All lines of the family $y = 3x + k$ have the (1) same x -intercept (2) same slope (3) same y -intercept (4) common point (2, 3) 44. _____

45. What is the value of the abscissa of the minimum point of the graph of $f(x) = ax^2 + bx + c$? [Assume $a > 0$.]
 (1) 1 (2) 0 (3) $-\frac{b}{2a}$ (4) $\sqrt{b^2 - 4ac}$ 45. _____

46. Which is *not* a possible root of $4x^3 - px^2 + qx - 6 = 0$?
 (1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{3}{2}$ (4) 4 46. _____

47. The equation $\sqrt{2x + 3} + x = 0$ has (1) no real root (2) only one positive root (3) only one negative root (4) one positive and one negative root 47. _____

*48. If r and θ denote polar coordinates, then the graph of $r = \cos \theta$ is a (1) straight line perpendicular to the polar axis (2) straight line parallel to the polar axis (3) circle whose center is at the pole (4) circle whose center is not at the pole 48. _____