January 25, 1961

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

1. Find the remainder if $3x^8 + 6x^3 + 7$ is divided by $x + 1$. 1_____

2. Express the discriminant of the equation $x^2 + mx - m = 0$ as a function of m. 2_____

3. Solve for x: $4^{2x-1} = \sqrt[3]{2}$ 3_____

4. Write an equation of the circle whose center is at $(-3, 4)$ and whose radius is 5. 4_____

5. Find the value of $\sqrt[3]{1.34}$ to the *nearest hundredth*. 5_____

6. Express $0.23333\ldots$ as a common fraction. 6_____

Directions (7-10): These questions refer to $\triangle ABC$ where A, B and C are the points $(-1, 2)$, $(5, 4)$ and $(2, 7)$, respectively.

7. Find the slope of the altitude from C to AB. 7_____

8. Find the length of AC. 8_____

9. An equation of the line through C and B is $x + y = 9$. Write an equation of the family of lines parallel to CB. 9_____

10. Find the *coordinates* of the point where the line through C and B intersects the x-axis. 10_____

11. If the product of the roots of the equation $ax^2 + 4x - 9 = 0$ is 3, find the sum of the roots. 11_____

12. If $f(x) = x^{-2/3} + 8x^0 - x^{-1}$, find $f(64)$. 12_____

13. Find the maximum value of y if $y = -2x^2 + 12x - 18$. 13_____

14. Find the abscissa of the point where the graph of $y = \log_3 x$ intersects the x-axis. 14_____

15. Between what two consecutive integers does the positive root of $x^4 + 2x^2 - 6 = 0$ lie? 15_____

16. Find in *simplest form* the fourth term of the geometric progression $\sqrt[3]{2}, \sqrt{2}, \sqrt[3]{8}, \ldots$. 16_____

17. Find the value of $\log_{16} \frac{1}{8}$. 17_____

18. If $y = 2x^2 - x + 1$, find the average rate of change of y with respect to x in the interval from $x = 2$ to $x = 5$. 18_____

19. Write in *simplest form* the ninth term in the expansion of $(a^{\frac{1}{2}} + x^{\frac{1}{2}})^{10}$. 19_____

20. It took 12 men 5 hours to build an airstrip. If 4 more men had been hired to work at the same rate, how many hours would the job have taken? 20_____

21. A is a member of a group of 10 from which a committee of 3 is to be chosen at random. What is the probability that A will be a member of this committee? 21_____

22. How many odd numbers of three digits each can be formed from the digits 2, 4, 6 and 7, if repetitions are permitted? 22_____

Directions (23-25): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

23. Which of the following is a rational integral function of x?

(1) $2x^6 + 3$ (2) $3x^{\frac{1}{2}} - 1$ (3) $4 - \frac{1}{x^2}$ (4) $17\sqrt[3]{x} + 1$ 23_____

24. If x is divided by y, the quotient is 3 and the remainder is 2. This relation may be expressed by (1) $\frac{x}{y} = 3\frac{2}{y}$

(2) $\frac{x}{y} = 3y + 2$ (3) $\frac{x}{y} = 3y + \frac{2}{y}$ (4) $\frac{x}{y} = 3 + \frac{2}{y}$ 24_____

25. If the first term of an arithmetic progression is a and the common difference is $2a$, the sum of the first n terms is

(1) n^2 (2) $2an^2$ (3) an^2 (4) a^2n^2 25_____

Part II

*Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

26. What is the rational root of the equation $5x^3 - x^2 + 5x - 1 = 0$? $(5x-1)(x-1)(x+i)$ 26_____

27. Given: A parabola whose equation is $y = 2x^2 - 3x - 2$. Find the slope of the line tangent to the curve at the point (2, 0). 27_____

28. The roots of the equation $x^3 + px^2 + qx + r = 0$ are 2, $-i$ and i. Find the value of q. 28_____

29. If the polynomial $x^4 - 5x^3 + 5x^2 + kx - 6$ is exactly divisible by $x - 2$, what is the value of k? 29_____

30. If 3 coins are tossed, what is the probability that exactly 2 of them will come up heads? 30_____

31. Find to the *nearest tenth*: $\log_5 2$ 31_____

32. Solve for x in terms of a: $\log 3a^4 - \log 2a = \log x$ 32_____

33. If the graphs of the equations $x^2 + y^2 = 1$ and $xy = 2$ are drawn on the same set of axes, how many points will they have in common? 33_____

34. A root of $x^3 - 6x^2 + 5x - 3 = 0$ lies between 5.1 and 5.2. Find this root to the *nearest tenth*. 34._____

35. Find the abscissa of the point of inflection of the graph of $y = x^3 - 3x^2 + 7x - 2$. 35._____

*36. Write in determinant form an equation of the line which passes through the points $(2, -1)$ and $(3, 9)$. 36._____

*37. If $\frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ 0 & -2 & 1 \\ a & 5 & 1 \end{vmatrix} = 20$, find the value of a. 37._____

Directions (38-40): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

38. If $(x^2 - x - 6) < 0$, then
(1) $-3 < x < -2$ (3) $-2 < x < 3$
(2) $1 < x < 6$ (4) $-3 < x < 2$ 38._____

39. If $M^{x-1} = N^x$, then x **equals**
(1) $\dfrac{\log N}{\log M - \log N}$ (3) $\dfrac{\log M - \log N}{\log M}$
(2) $\dfrac{\log M}{\log M - \log N}$ (4) $\dfrac{\log N - \log M}{\log N}$
39._____

40. The expression $i^{88} - i^{22}$ is equal to (1) 0 (2) 2
(3) $1 - i$ (4) $1 + i$ 40._____

Part III

*Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

41. One root of the equation $2x^3 + 7x^2 + 6x = 5$ is $\frac{1}{2}$. Write a quadratic equation which can be used to find the other two roots 41._____

42. A man piles 150 posts in layers so that each layer has one less post than the layer below. If the top layer has 3 posts, find the number of layers. 42._____

43. Solve for x: $3x - (7 + 6x) \geq 17 - x$. 43._____

44. Find the coordinates of the center of the circle whose equation is $x^2 - 14x + y^2 + 6y + 54 = 0$. 44._____

45. Express the complex number $6(\cos 30^\circ + i \sin 30^\circ)$ in $a + bi$ form. 45._____

46. Find the modulus of the complex number $-1 + 2i$. 46._____

47. The acceleration due to the earth's gravitational attraction varies inversely as the square of the distance from the center of the earth. The acceleration is 32 feet per second per second at 4,000 miles from the center. What is the acceleration in feet per second per second on a satellite 8,000 miles from the center of the earth? 47_____

48. If $f(u) = 3u + 1$ and $g(v) = v^2 - 1$, find $f(3) + g(0)$. 48_____

49. How many distinct points does the graph of $y = (x - 1)^2 (x - 2)$ have in common with the x-axis? 49_____

50. In how many distinct ways may 2 red flags, 2 white flags and 3 blue flags be arranged on a pole if all 7 flags are used in each arrangement? 50_____

51. A body moves along a line in such a way that its distance S in feet from a fixed point on the line at the end of t seconds is given by the equation: $S = 3t^3 + 2t^2 + 3$. Find the acceleration in feet per second per second when $t = 2$. 51_____

52. Given the complex number $N = 8 (\cos 60^\circ + i \sin 60^\circ)$. Express in polar form that cube root of N which, when represented graphically, lies in the *second* quadrant. 52_____

53. Write an equation of the straight line tangent to the curve $y = 2x^3 + 6$ at the point $(-1, 4)$. 53_____

*54. Transform $x^2 + y^2 - 4x + 3 = 0$ into an equation in polar coordinates. 54_____

*55. Find the radius of the circle whose equation is $r = \sin \theta$. 55_____