

ADVANCED ALGEBRA

Wednesday, January 25, 1956—9.15 a.m. to 12.15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. If $f(x) = \frac{2x}{3x-h}$, find $f(h)$. 1.....
2. Write an equation of the line passing through the point (2,0) and parallel to the line $2x - y + 3 = 0$. 2.....
3. Write in simplest form the *fifth* term in the expansion of $(x - \sqrt{y})^6$. 3.....
4. Find the remainder when $x^{16} + 5$ is divided by $x + 1$. 4.....
5. Express in simplest form the product of $3 + 6i$ and $2 + i$. 5.....
6. Express the repeating decimal 0.0909 . . . as a common fraction. 6.....
7. A graph whose equation may be written in the form $y = x^2 + px + q$ has an axis of symmetry whose equation is $x = -2$. Find the value of p . 7.....
8. In the equation $2x^2 - 20x + 40 + k = 0$, find the value of k so that the roots shall be equal. 8.....
9. If two roots of the equation $x^3 + ax^2 + bx + c = 0$ (in which a , b and c represent integers) are 1 and $2 - 3i$, find the value of a . 9.....
10. If the graphs of $y^2 - x^2 = 4$ and $x = 2$ are drawn on the same set of axes, how many points do the graphs have in common? 10.....
11. How many different three-digit numbers greater than 300 can be made using the digits 1, 2, 3 and 4 if no digits are repeated? 11.....
12. Solve the equation $\frac{K + y}{K - y} = K$ for y in terms of K . 12.....

Directions (13-17): Indicate the correct completion for *each* of the following by writing on the line at the right the letter a , b or c .

13. The fraction $\frac{\log x^n}{\log y^n}$ is equal to (a) $\frac{\log nx}{\log ny}$ (b) $\frac{\log x}{\log y}$
 (c) $\log \left(\frac{x}{y} \right)^n$ 13.....

14. If r varies directly as s and inversely as t , an equation representing this relationship, using k as the constant of variation, is
 (a) $rt = ks$ (b) $r = kst$ (c) $rs = kt$ 14.....
15. If the probability that an event will happen is $\frac{1}{x}$, then
 the probability that the event will not happen is (a) $\frac{1-x}{x}$
 (b) $\frac{x-1}{x}$ (c) $\frac{1+x}{x}$ 15.....
16. When n is not equal to 1, ${}_n C_n$ is (a) less than ${}_n C_{n-1}$
 (b) equal to ${}_n C_{n-1}$ (c) greater than ${}_n C_{n-1}$ 16.....
17. The equation $\sqrt{x+6} = -x$ has (a) no roots (b) one root
 (c) two roots 17.....
18. Transform the equation $x^3 - 2x^2 - x + 1 = 0$, whose roots are a , b and c , into an equation whose roots are $3a$, $3b$ and $3c$. 18.....
19. Transform the equation $x^3 - 2x^2 - x + 1 = 0$ into an equation whose roots are those of the original equation increased by 1. 19.....
20. How many imaginary roots has the equation
 $x^5 + x^3 + 1 = 0$? 20.....

Part II

Answer five questions from this part. Show all work.

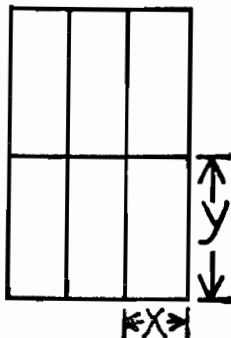
21. Solve the equation $2x^4 - 5x^3 + 4x^2 - 5x + 2 = 0$. [10]
22. Find to the nearest tenth the larger of the two positive roots of
 $x^3 + 3x^2 - 13x + 1 = 0$. [10]
23. a. State and prove the Remainder Theorem. [1, 5]
 b. State and prove the Factor Theorem. [1, 3]
24. a. If $7.6^x = 2.1$, find the value of x to the nearest hundredth. [4]
 b. If $y = 0.49^{-1.9}$, find the value of y to the nearest tenth. [6]
25. a. Draw the graph of $y = x^3 - 3x^2$ for the interval from $x = -1$ to $x = 3$. [4]
 b. On the same set of axes used in part a, draw the graph of
 $y = -x^2 + 4x - 4$ for the interval from $x = 0$ to $x = 4$. [4]
 c. The axis of symmetry of the graph of $y = -x^2 + 4x - 4$ intersects the graph of $y = x^3 - 3x^2$ at the point P . Using the graphs drawn in parts a and b, find the coordinates of P . [2]
26. In a geometric progression of four terms, the third term exceeds the first by 42 and the fourth term exceeds the second by 56. Find the four terms of the progression. [10]

27. A gardener can cultivate his large vegetable garden in 1 hour and 30 minutes less time with a power cultivator than with a hand cultivator. One day his power cultivator broke down 25 minutes after he started to use it, and he required 1 hour and 50 minutes to finish cultivating his garden with the hand cultivator. How long would it have taken him to cultivate the garden using only the power cultivator? [6, 4]

- *28. a. Which of the numbers -3 , $4i$, $-3 + 4i$ has the smallest modulus? [2]
 b. Which number in part a has the smallest amplitude (angle)? [2]
 c. Express $3 - 3i$ in polar form. [3]
 d. Express $2(\cos 240^\circ + i \sin 240^\circ)$ in $a + bi$ form. [3]

*29. A total of 360 feet of fencing is to be used to construct 6 equal rectangular enclosures, each with dimensions of x and y as shown in the figure.

- a. Express y as a function of x . [2]
 b. Express the area A of one of the six enclosures in terms of x . [1]
 c. Using the *derivative*, find the value of x for which A is a maximum. [5]
 d. Find the value of y for which A is a maximum. [2]



* These questions are based upon the optional topics in the syllabus.