The University of the State of New York

317TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 21, 1953 — 9.15 a. m. to 12.15 p. m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find to the *nearest tenth* the positive root of the equation $x^3 + 3x - 5 = 0$. [10]

22 Solve the equation $2x^4 + 5x^3 + 3x^2 + x - 2 = 0$. [10]

23 *a* Using the formula $A = P(1 + r)^n$, find A to the *nearest dollar* when $P = \$625$, $r = 3\%$ and $n = 17$. [5]

b Solve the equation $5^x = 12$ for x to the *nearest tenth.* [5]

24 *a* On the same set of axes draw the graphs of $(x - 1)^2 + y^2 = 9$ and $y = x^2 - 3x$ [4, 4]

b From the graphs made in answer to *a,* estimate to the *nearest tenth* the values of x and y common to both equations. [2]

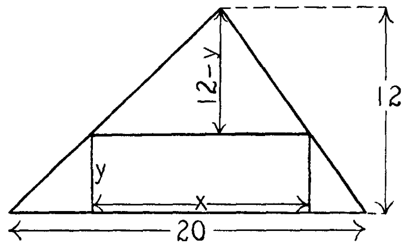
25 Given the equation $ax^2 + bx + c = 0$

(1) *Derive* the formula for the roots of this equation in terms of *a, b* and *c*. [6]

(2) Prove that the sum of the roots of this equation is equal to $-\dfrac{b}{a}$ and that the product is equal to $\dfrac{c}{a}$. [1, 3]

26 Two points, A and B, are 45 miles apart. A cabin cruiser leaves A for B at the same time that another cabin cruiser leaves B for A, going over the same route. The cruisers travel at uniform rates and meet at the end of 2 hours. Upon reaching its destination, each cruiser starts its return trip, without delay, and they meet for the second time 15 miles from B. Find the rate of each cruiser. [10]

27 A rectangle whose dimensions are x and y is inscribed in a triangle as shown in the drawing. The base of the triangle is 20 and its altitude is 12.



a Express y in terms of x. [Suggestion: Use similar triangles.] [3]

b Express the area of the rectangle in terms of x. [2]

c Find the value of x for which the area of the rectangle is a maximum. [5]

*28 *a* Show that the lines whose equations are $ax + by = c$ and $bx - ay = d$ are perpendicular. [5]

b Find the equation of the line through the point (1,4) and perpendicular to the line joining the points (—2,5) and (4,3). [5]

*29 *a* Express $-3 + 3i$ in polar form. [2]

b Express $2\sqrt{2}(\cos 315° + i \sin 315°)$ in the form $a + bi$. [2]

c Express the imaginary roots of $x^3 - 1 = 0$ in polar form. [2]

d Show that one of the roots given in answer to *c* is equal to the square of the other. [4]

* This question is based upon one of the optional topics in the syllabus.

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Fill in the following lines:

Name of pupil...Name of school...

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

1 Express in the form $a + bi$ the product of $(3 + i\sqrt{2})$ and $(2 - i\sqrt{2})$. 1................

2 What is the value of the x-intercept of the line $3x - y - 9 = 0$? 2................

3 Write the equation of the line whose slope is 4 and which passes through the point $(-2,3)$. 3................

4 Write in simplest form the *fourth term* in the expansion of $(a - b^2)^7$. 4................

5 Solve for x: $3^{x+2} = 27^x$ 5................

6 W varies directly as r^2 and inversely as s. If r is multiplied by 2, by what number must s be multiplied in order that W remain unchanged? 6................

7 Determine the value of k for which the graph of the function $y = 2x^2 - 4x + k$ will be tangent to the x-axis. 7................

8 How many imaginary roots has the equation $x^6 + 4x^4 - x^2 - 8 = 0$? 8................

9 Three of the roots of an equation with rational coefficients are 3, $2 + i$ and $1 + \sqrt{5}$. What is the lowest possible degree of the equation? 9................

10 Transform the equation $x^3 + 6x^2 - 4x - 9 = 0$ into an equation whose roots are the roots of the given equation each increased by 1. 10................

11 Transform the equation $x^3 - 3x + 2 = 0$ into an equation whose roots are those of the original equation each multiplied by 3. 11................

12 Find the value of k for which $x - 1$ is a factor of $x^3 + 2x^2 - kx - 1$ 12................

13 If $f(x) = x^3 - 5x + 6$, find $f(2)$. 13................

14 Find the common ratio of the geometric progression log 3, log 9, log 81, ... 14................

15 Find the sum of the infinite geometric progression 1, 0.8, 0.64, ... 15................

16 How many distinct permutations can be made from the letters of the word *running* if they are all used every time? 16................

17 How many different amounts of money can be formed from a nickel, a dime, a quarter and a half dollar, using any number of these four coins at a time? 17................

18 A, B, C, D and E are members of a panel. If three members are chosen by lot from that panel, what is the probability that A will be among those chosen? 18................

19 Find the equation of the axis of symmetry of the graph of the function $x^2 - 2x + 3 = y$. 19................

20 What is the maximum number of points in which two hyperbolas can intersect? 20................