The University of the State of New York

305TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 26, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find to the *nearest tenth* the real root of the equation $x^3 + 2x - 20 = 0$ [10]

22 Solve the equation $3x^4 - x^3 - 7x^2 - 5x - 2 = 0$ [10]

23 *a* Draw the graph of $y = 2^{x-1}$ from $x = 0$ to $x = 4$ [4]

b On the same axes as used in answer to *a*, draw the graph of $y = 4x - x^2$ from $x = 0$ to $x = 4$ [4]

c From the graphs made in answer to *a* and *b*, estimate, to *tenths*, the roots of the equation $4x - x^2 = 2^{x-1}$ [2]

24 The formula $N = 19.9\, e^{.25t}$ gives the number (N) of bacteria per unit volume in a certain culture at various times (t hours). For this culture, find to the *nearest hour* the time it would take for N to become 115. [Use $e = 2.718$] [10]

25 Find the *seventh* term of an arithmetic progression whose first, second and fifth terms are in geometric progression and whose first term is 2. [10]

26 A plumber can do a certain piece of work in a hours and his helper can do the same work in b hours. The plumber works alone for h hours, is then joined by his helper, and together they finish the work. How long does the helper work? [10]

27 *a* Prove: ${}_nC_r = {}_nC_{n-r}$ [6]

b Prove: $\log_b a = \dfrac{\log a}{\log b}$ [4]

*28 *a* Express $-1 + i\sqrt{3}$ in polar form. [3]

b Express $6(\cos 210° + i \sin 210°)$ in the form $a + bi$. [3]

c Express *one* of the imaginary roots of $x^3 - 1 = 0$ in polar form. [3]

d If $a + bi$ is multiplied by i, by how many degrees is the amplitude increased? [1]

*29 Given the curve $y = x^2 - 6x + 4$

a Find the slope of the line tangent to the curve at any point P. [4]

b Find the slope when P is the point $(2, -4)$. [2]

c Find the equation of the line perpendicular to the tangent to the curve $y = x^2 - 6x + 4$ at the point $(2, -4)$. [4]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.. Name of school..

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express $\dfrac{1}{1-2i}$ as a fraction with a real denominator. 1...............

2 Express the repeating decimal 0.535353... as a common fraction in lowest terms. 2...............

3 If the graph of $y = ax^2 + bx + c$ has no point in common with the x-axis, is the discriminant of the equation $ax^2 + bx + c = 0$ (*a*) positive, (*b*) zero or (*c*) negative? 3...............

4 Write in simplest form the *third* term in the expansion of $(x^2 - \frac{1}{2})^8$ 4...............

5 Using k as the constant of variation, write the equation representing the relationship: F varies directly as the square of v and inversely as r. 5...............

6 Write the equation of a straight line passing through the point $(-2, 3)$ and parallel to the line $y = 2x - 3$ 6...............

7 If the graphs of $x^2 + y^2 = 9$ and $y = x^2$ are plotted on the same axes, how many points do the graphs have in common? 7...............

8 Write the equation of third degree with integral coefficients, two of whose roots are 2 and $\sqrt{-3}$ 8...............

9 How many imaginary roots has the equation $x^5 + 3x - 5 = 0$? 9...............

10 Transform the equation $x^3 - 5x^2 + 2x - 1 = 0$ into an equation whose roots are those of the original equation each diminished by 1. 10...............

11 Transform the equation $x^3 - 8x^2 + 24x - 80 = 0$ into an equation whose roots are those of the original equation each divided by 2. 11...............

12 Find the remainder when $x^{15} + 1$ is divided by $x - 1$ 12...............

13 If $f(x) = 3x^{-1} + 2x^{\frac{1}{2}} + 3x^0$, find $f(9)$ 13...............

14 The executive board of a certain society consists of 8 members. How many committees of 4 can be chosen from this board if a certain member of the board is to serve on every committee? 14...............

15 How many *odd* numbers of 3 digits each can be written with the digits 0, 1, 2, 3, 4 if repetitions are allowed? 15...............

16 A bag contains 5 white balls and 5 red balls. If 3 balls are drawn at random from this bag, what is the probability that all 3 will be white? 16...............

17 Solve for x: $2^{x+1} = 8^x$ 17...............

18 If $\log x^3 = 6$, find x. 18...............

19 The diagonal of a rectangle is d and one of its sides is s. Express the area of the rectangle as a function of d and s. 19...............

20 If $y = x^2 - 2x$, express x as a function of y. 20...............