

The University of the State of New York

293D HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 24, 1945 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find, correct to the *nearest tenth*, the real root of the equation $x^3 + x^2 - 3x - 10 = 0$ [10]

22 The roots of the equation $x^3 + 3x^2 - 6x + k = 0$ are in arithmetic progression. Represent the roots by $r - d$, r and $r + d$.

a Using the coefficient of the second term of the equation, find r . [3]

b Using the factor theorem, find k . [4]

c Find the other two roots of the equation. [3]

23 a Using the same set of axes, draw the graphs of $y^2 = 4(x - 2)$ and $x^2 + y^2 = 16$. [6, 2]

b From the graphs made in answer to a find, correct to the *nearest tenth*, the real values of x and y that satisfy both equations. [2]

24 a Using the formula $A = P(1 + r)^n$ find, correct to the *nearest dollar*, the sum of money that will amount to \$4500 in 12 years at 3% interest compounded annually. [5]

b Find the value of x correct to the *nearest tenth*: $5^x = 39.5$ [5]

25 The safe load S of a horizontal beam standing on edge varies jointly as the width w and the square of the depth d and inversely as the length l , that is, $S = \frac{kwd^2}{l}$ where k is the constant of variation depending on the material of which the beam is made. If a beam whose dimensions are $w = 4$ in., $d = 6$ in. and $l = 20$ ft will safely support a load of 1200 lb., find the safe load of a beam of the same material whose dimensions are $w = 6$ in., $d = 8$ in. and $l = 16$ ft. [10]

26 The parabola $(x - h)^2 = 6(y - k)$ passes through the points (8, 5) and (2, -1). Find h and k . [10]

27 Given the equation $ax^3 + bx^2 + cx + d = 0$ in which the coefficients are rational numbers and a is not equal to zero; tell whether *each* of the following statements is *always true*, *sometimes true* or *never true*:

a The product of the roots of the equation is $-\frac{d}{a}$. [2]

b The three roots of the equation are negative. [2]

c If r is a factor of d and s is a factor of a , then $\frac{r}{s}$ is a root of the equation. [2]

d If two roots of the equation are rational, the third root is irrational. [2]

e If $p + \sqrt{-q}$, in which p and q are positive integers, is a root of the equation, then $p - \sqrt{-q}$ is also a root. [2]

*28 a Express $2(\cos 30^\circ + i \sin 30^\circ)$ in the form $a + bi$. [4]

b Express $1 + i$ in polar form. [4]

c Find the modulus of $2 + i\sqrt{3}$. [2]

*29 A rectangular field is to contain 10 acres. [1 acre = 160 sq. rd]

a If y equals the perimeter of the field and x equals its length, express y as a function of x . [2]

b Find the first derivative of y with respect to x . [4]

c Find the dimensions of the field if the perimeter is to be a minimum. [4]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Find the product of $4 + 3i$ and $4 - 3i$ 1.....
- 2 The complex number $-1 + i\sqrt{3}$ is represented graphically by the point P . How many degrees are there in the obtuse angle formed by the line OP , O being the origin, and the axis of real numbers? 2.....
- 3 Write the equation of the circle whose center is the origin and which passes through the point $(3, 4)$. 3.....
- 4 Find the slope of the straight line which passes through the points $(2, 3)$ and $(8, 5)$. 4.....
- 5 The sum of an infinite geometric progression is 1.25 and the ratio is 0.2; find the first term. 5.....
- 6 Find the third term of the expansion of $(a - \frac{1}{b})^8$ 6.....
- 7 Solve the formula $A = \frac{rs}{r+s}$ for r . 7.....
- 8 Find the rational root of the equation $x^3 - 4x^2 + 2x + 4 = 0$ 8.....
- 9 How many negative roots has the equation $x^5 + x^2 + 12 = 0$? 9.....
- 10 In how many points does the graph of $y = x$ intersect the graph of $y = x^3 - 4x$? 10.....
- 11 Solve for x : $\log \sqrt[3]{x} = .1400$ 11.....
- 12 Express $\log_b a$ as the quotient of two logarithms. 12.....
- 13 Transform the equation $x^3 + \frac{1}{2}x^2 - 3x - \frac{1}{8} = 0$ into an equation whose roots are the roots of the given equation each multiplied by 2. 13.....
- 14 How many permutations are there of 8 different things taken 3 at a time? 14.....
- 15 If from a penny, a nickel, a dime, a quarter and a half-dollar two coins are chosen at random, what is the probability that the combined values of the two coins will exceed 50 cents? 15.....
- Directions (questions 16-19) — Indicate whether each statement is true or false by writing the word *true* or the word *false* on the line at the right.
- 16 If $px^2 + q$ is divided by $x - r$, where p and q are positive integers and r is real, the remainder is always positive. 16.....
- 17 As x increases from 0, the function $x^2 + x^{-1}$ decreases and then increases. 17.....

18 The graph of the equation $y = x^2 + px + q$ is symmetric with respect to the y axis if p is zero.

19 The roots of the equation $x^2 + 2kx + 1 = 0$ are imaginary for all values of k which are greater than -1 but less than $+1$.

20 On the diagram below, represent the numbers 2 and $3 - 2i$ and find their sum graphically.

18.....

19.....

