The University of the State of New York

280th High School Examination

ADVANCED ALGEBRA

Wednesday, January 22, 1941 — 9.15 a. m. to 12.15 p. m., only

Instructions

*Do not open this sheet until the signal is given.*

**Part I**

*This part is to be done first and the maximum time allowed for it is one and one half hours.* Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

**Part II**

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Name of school ................................................ Name of pupil ........................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 If every term of an arithmetic progression is increased by a constant $k$, is the resulting series an arithmetic progression? [Answer yes or no.] 1.

2 Write the $fourth$ term of the expansion $(x + \frac{1}{x^n})^4$ 2.

3 Is the product of two conjugate complex numbers always a real number? [Answer yes or no.] 3.

4 If $f(x) = 3x^n + 2x^2 + x$, what is the value of $f(0)$? 4.

5 Write the equation of the straight line which is parallel to the graph of $4x + 3y = 12$ and which passes through the origin. 5.

6 In how many points does the graph of $y^2 = 4x$ intersect the graph of $y = 2x$? 6.

Questions 7-10 refer to the equation $2x^4 - 3x^2 - 6 = 0$

7 What is the number of negative roots? 7.

8 What is the number of complex (imaginary) roots? 8.

9 What is the sum of the roots? 9.

10 What is the product of the roots? 10.

11 Write as an equation the following statement: $x$ varies inversely as the square of $y$. 11.

12 Write the equation of the lowest possible degree with real coefficients which has for two of its roots 3 and $1 + i$ 12.

13 Write the equation whose roots are one half the roots of the equation $x^4 - 4x^2 + 32 = 0$ 13.

14 Write the equation whose roots are less by two than the roots of $x^4 - 8x^3 + 24x^2 - 32x + 20 = 0$ 14.

15 Find the real root of the equation $x^{\frac{3}{2}} + x^{\frac{1}{2}} = \frac{9}{8}$ 15.

16 Solve $x^3 = .3588$ for $x$ correct to the nearest hundredth. 16.

17 In how many ways can six different books be arranged on a shelf? 17.

18 Given five coins: a cent, a nickel, a dime, a quarter and a half dollar. How many different sums consisting of two coins can be formed? 18.

19 If three coins are tossed, what is the probability of obtaining three "heads"? 19.

20 If $r^2 = s^2$, then, when $x$ is not equal to 0, (a) $log r = 2 log s$; (b) $log r = \frac{log s}{2}$; (c) $log r = (log s)^2$. Which is correct, (a), (b) or (c)? 20.

[3]
Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Solve the equation $4x^4 + 9x^2 - 11x + 3 = 0$ \[10\]

22 Find, correct to the nearest tenth, the real root of the equation $x^3 + x - 5 = 0$ \[10\]

23 Given the following simultaneous equations:
   \[x^2 + y^2 = 18\]
   \[y^2 = 3x\]
   a Solve for $x$ and $y$ algebraically. \[4\]
   b Solve for $x$ and $y$ graphically. \[5\]
   c Explain the difference in the number of solutions. \[1\]

24 Given the formula $V = \frac{\pi N r^2}{270}$
   If $V = 12.40$ and $N = 47$, find $r$ correct to the nearest tenth. \[\pi = 3.14\] \[10\]

25 If the equation $x^3 + ax^2 + bx + c = 0$ has one root which is the negative of another, show that $ab = c$. \[10\]

26 The square of the logarithm of a number exceeds the logarithm of the number by 2. Find the number. \[10\]

27 A grocer has a barrel full of vinegar. He draws out 4 gallons and fills the barrel with water. He then draws out 8 gallons. There are now 21 gallons of the original vinegar left in the barrel. What is the capacity of the barrel? \[7, 3\]

*28 The frequency distribution of point scores made by a class of 51 pupils is given below.

<table>
<thead>
<tr>
<th>Scores</th>
<th>95</th>
<th>90</th>
<th>85</th>
<th>80</th>
<th>75</th>
<th>70</th>
<th>65</th>
<th>60</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pupils</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>13</td>
<td>17</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

   a Compute the mean score. \[4\]
   b Compute the median score. \[2\]
   c Draw the graph of the frequency distribution. \[4\]

*29 The velocity $v$, in feet per second, of a moving body is given by the equation $v = 48t^2 - 2t^3$, where $t$ is the time in seconds.
   a What is the maximum velocity attained by the body? \[5\]
   b What is the maximum acceleration attained? \[5\]

* This question is based on one of the optional topics in the syllabus.