7 Transform the equation \( x^3 - \frac{1}{2}x^2 + \frac{1}{3}x + \frac{1}{3} = 0 \) into an equation of the form \( x^3 + px + q = 0 \), where \( p \) and \( q \) are integers. [12\text{\frac{1}{2}}]

8 The sum of an infinite number of terms of a decreasing geometric progression is \( \frac{3}{5} \) and the sum of their squares is also \( \frac{3}{5} \); write the first three terms of the progression. [12\text{\frac{1}{2}}]

9 In the following set of equations determine the values of \( k \) so that two values of \( x \) in the solution shall be equal, that is, the graph of the second equation shall be tangent to the graph of the first equation: [The drawing of the two graphs is not required.]

\[
\begin{align*}
y &= x^3 - 3x^2 - 8x + 1 \\
y &= x + k
\end{align*}
\]

[12\text{\frac{1}{2}}]

10 The edges of two hollow cubes differed by 10 inches. When a certain quantity of water was poured into the larger cube there remained 1578 cubic inches of space not filled with water. When the second cube was filled from this same quantity of water there were 142 cubic inches of water left. Find the dimensions of each cube. [7, 5\text{\frac{1}{2}}]

11 Solve the following set of equations:

\[
\begin{align*}
x^3 + y^3 &= 468 \\
x^2y + xy^2 &= 420
\end{align*}
\]

[12\text{\frac{1}{2}}]

12 A rectangular tin box with an open top and square base is to have a total outside area of 48 square feet.

\( a \) Express the volume \( y \) in terms of a base edge \( x \). [5]

\( b \) Plot the graph of the relation expressed in answer to \( a \) for values of \( x \) from 0 to 7 inclusive. [5]

\( c \) From the graph determine the length of a base edge that will make the greatest volume. [2\text{\frac{1}{2}}]