ADVANCED ALGEBRA

Monday, January 16, 1922—9.15 a.m. to 12.15 p.m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1. Express both (a) and (b) in the form \( c + di \) where

\[ i = \sqrt{-1}; \quad (a) \ (3 - 2i)^2, \quad (b) \ 4 - \sqrt{-9} \]

1 \[ 1, 1 \frac{1}{2} \]

2. From the letters of the word “facetious” how many permutations of five letters each may be formed? How many words of five letters each may be formed from three of the consonants and two of the vowels if each arrangement of the letters is considered a word? \[ 3 \frac{1}{2}, 9 \]

3. One root of the equation \( x^4 - 12x^3 + 55x^2 - 114x + a = 0 \) is \( 3 - \sqrt{2} \); find the other roots and the value of \( a \). \[ 12 \frac{1}{2} \]

4. Transform the equation \( x^3 - 5x^2 - 2x + 24 = 0 \) as follows:

(a) Multiply the roots by 2. (b) Diminish the roots by 3. (c) Solve the resulting equation in (b) and from these results state the roots of the original equation. \[ 12 \frac{1}{2} \]

5. Find the positive root of \( x^3 + 3x^2 - 3x - 18 = 0 \) to the nearest hundredth. \[ 12 \frac{1}{2} \]

6. Given the equation \( x^4 - 3x^3 - 5 = 0 \); without solving, complete the following statements:

a The equation has ( ) roots. \[ 2 \frac{1}{2} \]

b It has ( ) positive roots and ( ) negative roots. \[ 2 \frac{1}{2} \]

c It has ( ) imaginary roots. \[ 2 \frac{1}{2} \]

d The sum of the roots is ( ). \[ 2 \frac{1}{2} \]

e The product of the roots is ( ). \[ 2 \frac{1}{2} \]

7. a Plot the graph of \( y = 3x^4 - 4x^3 - 12x^2 + 3 \) from \( x = -2 \) to \( x = 3 \). \[ 8 \frac{1}{2} \]

b From the graph read to the nearest tenth the roots of \( 3x^4 - 4x^3 - 12x^2 + 3 = 0 \). \[ 4 \]

8. a Derive the fundamental formula for the sum of a finite geometric series in terms of \( a, r \) and \( n \). \[ 5 \]

b Insert three geometric means between 6 and 486. \[ 7 \frac{1}{2} \]

9. The formula \( D = \sqrt[3]{\frac{W}{0.5236(A - G)}} \) gives the diameter of a spherical balloon which is to lift a given weight \( W \); by the use of logarithms find \( D \) if \( A = 0.0807, \ G = 0.0056, \ W = 1250 \). \[ 12 \frac{1}{2} \]

10. An open box is to be made from a rectangular piece of tin 20 inches long and 8 inches wide, by cutting out equal squares from the corners and turning up the sides; how large should these squares be in order that the box may contain 128 cubic inches? \[ 12 \frac{1}{2} \]

11. In a race between two crews the first crew rows at an average rate of 360 yards per minute. The second crew starts at the same time covers 75 yards the first minute and for the next four minutes increases its rate 95 yards each minute; for the remainder of the race the crew maintains the rate that it has at the end of the fifth minute. In how many minutes will the second crew overtake the first? \[ 12 \frac{1}{2} \]