New York State Education Department
208th High School Examination
ADVANCED ALGEBRA

Monday, January 20, 1913—9.15 a.m. to 12.15 p.m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

1 For what values of $k$ are the roots of the equation
   \[ x^2 + k(x + 1) + 3 = 0 \]
   (a) complex, (b) real, (c) equal?

2 Two bodies, A and B, are moving at a constant rate and in the same direction around the circumference of a circle whose length is 20; A makes one circuit in two seconds less time than B and at the end of one minute A has made one circuit more than B. At what rates are A and B moving?

3 Deduce the formula for the number of permutations of $n$ things taken all at a time, when $p$ are of one kind, $q$ of another kind and $r$ of a third kind.

4 Prove the relation \( (1 - i)^3 = \frac{-4i}{1 + i} \) \[ i = \sqrt{-1} \]

5 Solve the following for $y$, using determinants of the 4th order:
   \[
   \begin{align*}
   x + y + z + w &= 0 \\
   x + 2y + 3z + 4w &= 0 \\
   x + 3y + 6z + 10w &= 0 \\
   x + 4y + 10z + 20w &= -1
   \end{align*}
   \]

6 Solve \( x^4 - 22x^2 + 12x + 48 = 0 \)

7 Write, with integral coefficients, the equation of lowest degree two of whose roots are \( \sqrt{2} \) and \( -6\sqrt{-1} \).

8 Assuming that every rational integral equation in one variable has one root, prove that it has no more than $n$ roots.

9 State Descartes' rule of signs. By means of this rule show that the equation \( x^4 + ax + b = 0 \) can not have four real roots unless $a$ and $b$ both equal zero.

10 Plot the equation \( x^3 + 7x^2 - 11x - 2 = y \). Compute by Horner's method, to two places of decimals, the positive root of the left member of this equation set equal to zero.