Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. No credit will be allowed unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient.

1 Solve \(2\sqrt{x^2 - 2x - 3 + x^2} - 2x = 6\)

2 Prove that the number of combinations of \(n\) things taken \(r\) at a time is equal to the number of combinations of \(n\) things taken \(n - r\) at a time.

If the number of permutations of \(n\) things taken 3 at a time is equal to 10 times the number of permutations of \(n\) things taken 2 at a time, what is the value of \(n\)?

3 Represent graphically the complex numbers 
\(-3 + 2\sqrt{-1}\) and \(1 - 4\sqrt{-1}\);
then represent their sum and their difference.

4 Prove that if a determinant has two rows identical, its value is zero.

Evaluate the déterminant
\[
\begin{vmatrix}
1 & 1 & 1 & 1 \\
1 & 1+x & 1 & 1 \\
1 & 1 & 1+x & 1 \\
1 & 1 & 1 & 1+x
\end{vmatrix}
\]

5 Find by means of determinants the value of \(y\) from the following equations:
\[3x^2 - 7y + 5z = 9\]
\[x + 8y - 3z = 4\]
\[5x - 4z - 2 = 0\]

6 Prove that if \(a + b\sqrt{-1}\) is a root of an equation with real coefficients \(a - b\sqrt{-1}\) is also a root.

7 Transform \(x^3 - \frac{x^2}{2} + \frac{15x}{36} - \frac{13}{108} = 0\) into an equation whose coefficients are integers, the coefficient of the first term being 1. Find a rational root of the resulting equation. What is the commensurable root of the original equation?

8 Apply Descartes' rule of signs to determine the nature of the roots of the equation \(x^3 + 3x - 5 = 0\)

Graph the function \(y = x^3 + 3x - 5\). What information concerning the roots of the equation is afforded by the graph?

9 One root of the equation \(x^4 - 4x^3 + 5x^2 + 2x + 52 = 0\) is \(3 - 2\sqrt{-1}\); find the other roots.

10 In the equation \(2x^3 + 3x - 90 = 0\), find by Horner's method, to two decimal places, a positive root lying between 3 and 4.