ADVANCED ALGEBRA

Monday, January 21, 1907 — 9.15 a. m. to 12.15 p. m., only

Answer eight questions. Give each step of solution. Reduce fractions to lowest terms. Express final result in its simplest form and mark it Ans. Each complete answer will receive 12½ credits. Papers entitled to 75 or more credits will be accepted if written by students in class A; those entitled to 60 or more credits will be accepted if written by students in class B.

1 Solve as a quadratic \( \frac{x^2-1}{2x} + \frac{2x}{x^2-1} = \frac{25}{12} \)

2 Form the equation whose roots are \(-a, \frac{a + \sqrt{b}}{4}\) and \(\frac{a - \sqrt{b}}{4}\)

Perform operations necessary to remove fractions and radicals.

3 How many odd numbers of five different figures each can be formed from the digits 3, 4, 5, 6, 7?

4 Find the sum to infinity of \(-\frac{3}{4}, \frac{3}{10}, -\frac{3}{5} \ldots \)

5 Find the value of \(x\) in each of the following: \(\log_{\frac{1}{5}} (\frac{1}{3}) = x; \quad (\frac{1}{4})^{x+2} = 2^{\frac{7}{8}}\)

6 Find to three places of decimals the fifth root of 35, by the application of the binomial theorem.

7 Find the successive derivatives with respect to \(x\) of \(x^6 - 3x^4 - 12x^2 + 17\)

8 Transform \(x^3 - 9x^2 + 10x - 3 = 0\) into another equation whose second term shall be wanting.

9 Revert to four terms \(y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \)

10 Find the scale of relation and the sum of the following recurring series of the second order: \(5 + 7x + 11x^2 + 19x^3 + \ldots \)

11 Without expanding the determinants, prove that

\[
\begin{vmatrix}
  a & b & c \\
  m & r & s \\
  x & y & z
\end{vmatrix} = \begin{vmatrix}
  r & b & y \\
  m & a & x \\
  x & y & z
\end{vmatrix} = \begin{vmatrix}
  m & r & s \\
  s & c & z \\
  a & b & c
\end{vmatrix}
\]

12 Determine by the graphic method the approximate values of the roots of

\[
\begin{cases}
  x^2 + y^2 = 25 \\
  xy = 12
\end{cases}
\]