JMAP
REGENTS BY DATE

NY Algebra II Regents Exam Questions from Spring 2015 to August 2018 Sorted by Date

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1 If $a$, $b$, and $c$ are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x + b)\left(x^2 - 2cx + c^2\right)$?

2 Which equation represents a parabola with a focus of $(0,4)$ and a directrix of $y = 2$?

3 If the terminal side of angle $\theta$, in standard position, passes through point $(-4,3)$, what is the numerical value of $\sin \theta$?

4 A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the $t$ represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

5 Use the properties of rational exponents to determine the value of $y$ for the equation:

\[ \frac{3}{\sqrt[3]{x^8}} = x^y, \quad x > 1 \]
6 Write \((5 + 2yi)(4 - 3i) - (5 - 2yi)(4 - 3i)\) in \(a + bi\) form, where \(y\) is a real number.

7 Use an appropriate procedure to show that \(x - 4\) is a factor of the function \(f(x) = 2x^3 - 5x^2 - 11x - 4\). Explain your answer.

8 Solve algebraically for all values of \(x\):
\[
\sqrt{x - 5} + x = 7
\]

9 Monthly mortgage payments can be found using the formula below:
\[
M = \frac{P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n}{\left( 1 + \frac{r}{12} \right)^n - 1}
\]

\[
M = \text{monthly payment} \\
P = \text{amount borrowed} \\
r = \text{annual interest rate} \\
n = \text{number of monthly payments}
\]

The Banks family would like to borrow $120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the fewest number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than $720.

10 Solve the following system of equations algebraically for all values of \(x, y,\) and \(z\):
\[
x + 3y + 5z = 45
\]
\[
6x - 3y + 2z = -10
\]
\[
-2x + 3y + 8z = 72
\]

11 Write an explicit formula for \(a_n\), the \(n\)th term of the recursively defined sequence below.
\[
a_1 = x + 1
\]
\[
a_n = x(a_{n-1})
\]
For what values of \(x\) would \(a_n = 0\) when \(n > 1\)?

12 Stephen’s Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products \(A, B\), and the new product. Nine out of fifty participants preferred Stephen’s new cola to products \(A\) and \(B\). The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen’s new product, each of sample size 50, simulated 100 times.

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.
13 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on both issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

14 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form $f(t) = A \cos(Bt)$, where $A$ and $B$ are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in $t$ hours since 8:30 a.m. On the grid below, graph one cycle of this function.

People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.
15 What is the solution set of the equation \( \frac{3x + 25}{x + 7} - 5 = \frac{3}{x} \)?

1) \( \left\{ \frac{3}{2}, 7 \right\} \)
2) \( \left\{ \frac{7}{2}, -3 \right\} \)
3) \( \left\{ -\frac{3}{2}, 7 \right\} \)
4) \( \left\{ -\frac{7}{2}, -3 \right\} \)

16 Functions \( f, g, \) and \( h \) are given below.

\[
\begin{align*}
\text{ } & f(x) = \sin(2x) \\
\text{ } & g(x) = f(x) + 1 \\
\text{ } & h(x) = \text{ } \\
\end{align*}
\]

Which statement is true about functions \( f, g, \) and \( h \)?

1) \( f(x) \) and \( g(x) \) are odd, \( h(x) \) is even.
2) \( f(x) \) and \( g(x) \) are even, \( h(x) \) is odd.
3) \( f(x) \) is odd, \( g(x) \) is neither, \( h(x) \) is even.
4) \( f(x) \) is even, \( g(x) \) is neither, \( h(x) \) is odd.

17 The expression \( \frac{6x^3 + 17x^2 + 10x + 2}{2x + 3} \) equals

1) \( 3x^2 + 4x - 1 + \frac{5}{2x + 3} \)
2) \( 6x^2 + 8x - 2 + \frac{5}{2x + 3} \)
3) \( 6x^2 - x + 13 - \frac{37}{2x + 3} \)
4) \( 3x^2 + 13x + \frac{49}{2} + \frac{151}{2x + 3} \)

18 The solutions to the equation \( -\frac{1}{2}x^2 = -6x + 20 \) are

1) \( -6 \pm 2i \)
2) \( -6 \pm 2\sqrt{19} \)
3) \( 6 \pm 2i \)
4) \( 6 \pm 2\sqrt{19} \)

19 What is the completely factored form of \( k^4 - 4k^3 + 8k^2 - 32k + 12k^2 - 48 \)?

1) \( (k - 2)(k - 2)(k + 3)(k + 4) \)
2) \( (k - 2)(k - 2)(k + 6)(k + 2) \)
3) \( (k + 2)(k - 2)(k + 3)(k + 4) \)
4) \( (k + 2)(k - 2)(k + 6)(k + 2) \)

20 Which statement is incorrect for the graph of the function \( y = -3 \cos \left[ \frac{\pi}{3} (x - 4) \right] + 7 \)?

1) The period is 6.
2) The amplitude is 3.
3) The range is \([4,10]\).
4) The midline is \( y = -4 \).

21 Algebraically determine the values of \( x \) that satisfy the system of equations below.

\[
\begin{align*}
\text{ } & y = -2x + 1 \\
\text{ } & y = -2x^2 + 3x + 1 \\
\end{align*}
\]
22 The results of a poll of 200 students are shown in the table below:

<table>
<thead>
<tr>
<th></th>
<th>Preferred Music Style</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Techno</td>
</tr>
<tr>
<td>Female</td>
<td>54</td>
</tr>
<tr>
<td>Male</td>
<td>36</td>
</tr>
</tbody>
</table>

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

23 For the function $f(x) = (x - 3)^3 + 1$, find $f^{-1}(x)$.

24 Given: $h(x) = \frac{2}{9} x^3 + \frac{8}{9} x^2 - \frac{16}{13} x + 2$

$k(x) = -|0.7x| + 5$

State the solutions to the equation $h(x) = k(x)$, rounded to the nearest hundredth.

25 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

26 Rewrite the expression

\[(4x^2 + 5x)^2 - 5(4x^2 + 5x) - 6\]

as a product of four linear factors.

27 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton’s Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

$T_a$ = the temperature surrounding the object

$T_0$ = the initial temperature of the object

$t$ = the time in hours

$T$ = the temperature of the object after $t$ hours

$k$ = decay constant

The turkey reaches the temperature of approximately 100°F after 2 hours. Find the value of $k$, to the nearest thousandth, and write an equation to determine the temperature of the turkey after $t$ hours. Determine the Fahrenheit temperature of the turkey, to the nearest degree, at 3 p.m.
28 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups’ final grades is shown below:

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{x}$</td>
<td>80.16</td>
<td>83.8</td>
</tr>
<tr>
<td>$S_x$</td>
<td>6.9</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students’ final grades were rerandomized 500 times. The results are shown below.

Using the simulation to determine if there is a significant difference in the final grades. Explain your answer.

29 Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and $z(-5) = 0$, algebraically determine all the zeros of $z(x)$.

30 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the nearest ten thousandth, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

31 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the nearest tenth. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.
0616AII Common Core State Standards

1 When $b > 0$ and $d$ is a positive integer, the expression $(3b)^d$ is equivalent to

1) $\frac{1}{\left(\frac{\sqrt[3]{3b}}{2}\right)^2}$
2) $(\sqrt[3]{3b})^d$
3) $\frac{1}{\sqrt[3]{3b^d}}$
4) $(\frac{\sqrt[3]{3b}}{2})^2$

2 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, $T$, are left in the semester?

1) $\frac{255 + 93T}{3T} = 90$
2) $\frac{255 + 90T}{3T} = 93$
3) $\frac{255 + 93T}{T+3} = 90$
4) $\frac{255 + 90T}{T+3} = 93$

3 Given $i$ is the imaginary unit, $(2 - yi)^2$ in simplest form is

1) $y^2 - 4yi + 4$
2) $-y^2 - 4yi + 4$
3) $-y^2 + 4$
4) $y^2 + 4$

4 Which graph has the following characteristics?

- three real zeros
- as $x \to -\infty$, $f(x) \to -\infty$
- as $x \to \infty$, $f(x) \to \infty$

5 The solution set for the equation $\sqrt{56-x} = x$ is

1) {-8,7}
2) {-7,8}
3) {7}
4) { }
6 The zeros for \( f(x) = x^4 - 4x^3 - 9x^2 + 36x \) are
1) \( \{0, \pm 3, 4\} \)
2) \( \{0, 3, 4\} \)
3) \( \{0, \pm 3, -4\} \)
4) \( \{0, 3, -4\} \)

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Given the results of her coin flips and of her computer simulation, which statement is most accurate?
1) 73 of the computer's next 100 coin flips will be heads.
2) 50 of her next 100 coin flips will be heads.
3) Her coin is not fair.
4) Her coin is fair.

8 If \( g(c) = 1 - c^2 \) and \( m(c) = c + 1 \), then which statement is not true?
1) \( g(c) \cdot m(c) = 1 + c - c^2 - c^3 \)
2) \( g(c) + m(c) = 2 + c - c^2 \)
3) \( m(c) - g(c) = c + c^2 \)
4) \( \frac{m(c)}{g(c)} = \frac{-1}{1 - c} \)

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the nearest whole percent, is
1) 6
2) 48
3) 68
4) 95

10 The formula below can be used to model which scenario?
\[
\begin{align*}
a_1 &= 3000 \\
a_n &= 0.80a_{n-1}
\end{align*}
\]
1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of $3000, and each year it grows by 80%.
4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are
1) independent
2) dependent
3) mutually exclusive
4) complements

12 A solution of the equation \( 2x^2 + 3x + 2 = 0 \) is
1) \( -\frac{3}{4} + \frac{i}{4}\sqrt{7} \)
2) \( -\frac{3}{4} + \frac{1}{4}i \)
3) \( -\frac{3}{4} + \frac{1}{4}\sqrt{7} \)
4) \( \frac{1}{2} \)
13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, \( H \), in feet, above the ground of one of the six-person cars can be modeled by

\[
H(t) = 70 \sin \left(\frac{2\pi}{7} (t - 1.75)\right) + 80,
\]

where \( t \) is time, in minutes. Using \( H(t) \) for one full rotation, this car's minimum height, in feet, is

1) 150
2) 70
3) 10
4) 0

14 The expression \( \frac{4x^2 + 5x + 10}{2x + 3} \) is equivalent to

1) \( 2x^2 + 3x - 7 + \frac{31}{2x + 3} \)
2) \( 2x^2 - 3x + 7 - \frac{11}{2x + 3} \)
3) \( 2x^2 + 2.5x + 5 + \frac{15}{2x + 3} \)
4) \( 2x^2 - 2.5x - 5 - \frac{20}{2x + 3} \)

15 Which function represents exponential decay?

1) \( y = 2^{0.3t} \)
2) \( y = 1.2^{3t} \)
3) \( y = \left(\frac{1}{2}\right)^x \)
4) \( y = 5^{-x} \)

16 Given \( f^{-1}(x) = \frac{3}{4}x + 2 \), which equation represents \( f(x) \)?

1) \( f(x) = \frac{4}{3}x - \frac{8}{3} \)
2) \( f(x) = -\frac{4}{3}x + \frac{8}{3} \)
3) \( f(x) = \frac{3}{4}x - 2 \)
4) \( f(x) = -\frac{3}{4}x + 2 \)

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, \( \theta \), intercepts the circle in Quadrant II at point \( C \). The \( y \)-coordinate of point \( C \) is 8. What is the value of \( \cos \theta \)?

1) \( \frac{3}{5} \)
2) \( \frac{3}{4} \)
3) \( \frac{3}{5} \)
4) \( \frac{4}{5} \)

18 Which statement about the graph of \( c(x) = \log_6 x \) is false?

1) The asymptote has equation \( y = 0 \).
2) The graph has no \( y \)-intercept.
3) The domain is the set of positive reals.
4) The range is the set of all real numbers.

19 The equation \( 4x^2 - 24x + 4y^2 + 72y = 76 \) is equivalent to

1) \( 4(x - 3)^2 + 4(y + 9)^2 = 76 \)
2) \( 4(x - 3)^2 + 4(y + 9)^2 = 121 \)
3) \( 4(x - 3)^2 + 4(y + 9)^2 = 166 \)
4) \( 4(x - 3)^2 + 4(y + 9)^2 = 436 \)
20 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.

Which statement about this function is incorrect?
1) The degree of the polynomial is even.
2) There is a positive leading coefficient.
3) At two pH values, there is a relative maximum value.
4) There are two intervals where the function is decreasing.

21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let \( m \) represent months.]
1) \( (1.0525)^m \)
2) \( (1.0525)^{\frac{m}{12}} \)
3) \( (1.00427)^m \)
4) \( (1.00427)^{\frac{m}{12}} \)

22 Which value, to the nearest tenth, is not a solution of \( p(x) = q(x) \) if \( p(x) = x^3 + 3x^2 - 3x - 1 \) and \( q(x) = 3x + 8 \)?
1) -3.9
2) -1.1
3) 2.1
4) 4.7

23 The population of Jamesburg for the years 2010-2013, respectively, was reported as follows: 250,000 250,937 251,878 252,822 How can this sequence be recursively modeled?
1) \( j_n = 250,000(1.00375)^{n-1} \)
2) \( j_n = 250,000 + 937(n-1) \)
3) \( j_1 = 250,000 \)
4) \( j_1 = 250,000 \quad j_n = 1.00375j_{n-1} \)

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Which equation best represents the value of the voltage as it flows through the electric wires, where \( t \) is time in seconds?
1) \( V = 120 \sin(t) \)
2) \( V = 120 \sin(60t) \)
3) \( V = 120 \sin(60\pi t) \)
4) \( V = 120 \sin(120\pi t) \)

25 Solve for \( x \): \( \frac{1}{x} - \frac{1}{3} = -\frac{1}{3x} \)

26 Describe how a controlled experiment can be created to examine the effect of ingredient \( X \) in a toothpaste.

27 Determine if \( x - 5 \) is a factor of \( 2x^3 - 4x^2 - 7x - 10 \). Explain your answer.
28. On the axes below, graph one cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).

29. A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

30. The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

31. Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

32. A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the nearest percent.

33. Solve the system of equations shown below algebraically.

\[
\begin{align*}
(x - 3)^2 + (y + 2)^2 &= 16 \\
2x + 2y &= 10
\end{align*}
\]

34. Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, $S_n$, for Alexa's total earnings over $n$ years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the nearest cent.

35. Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the nearest hundredth.

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% − 50% split. Explain what statistical evidence supports this concern.
36 Which function shown below has a greater average rate of change on the interval \([-2,4]\)? Justify your answer.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0.3125</td>
</tr>
<tr>
<td>-3</td>
<td>0.625</td>
</tr>
<tr>
<td>-2</td>
<td>1.25</td>
</tr>
<tr>
<td>-1</td>
<td>2.5</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
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<tr>
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</tr>
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<td>5</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>320</td>
</tr>
</tbody>
</table>

\[g(x) = 4x^3 - 5x^2 + 3\]

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function

\[N(t) = N_0(e^{-rt}),\]

where \(N(t)\) is the amount left in the body, \(N_0\) is the initial dosage, \(r\) is the decay rate, and \(t\) is time in hours. Patient \(A\), \(A(t)\), is given 800 milligrams of a drug with a decay rate of 0.347. Patient \(B\), \(B(t)\), is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, \(A(t)\) and \(B(t)\), to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

To the nearest hour, \(t\), when does the amount of the given drug remaining in patient \(B\) begin to exceed the amount of the given drug remaining in patient \(A\)? The doctor will allow patient \(A\) to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the nearest tenth of an hour, how long patient \(A\) will have to wait to take another 800 milligram dose of the drug.
1 Which equation has $1 - i$ as a solution?
1) $x^2 + 2x - 2 = 0$
2) $x^2 + 2x + 2 = 0$
3) $x^2 - 2x - 2 = 0$
4) $x^2 - 2x + 2 = 0$

2 Which statement(s) about statistical studies is true?
I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don’t like math.
1) I, only
2) II, only
3) I and III
4) III and IV

3 To the nearest tenth, the value of $x$ that satisfies $2^x = -2x + 11$ is
1) 2.5
2) 2.6
3) 5.8
4) 5.9

4 The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?
1) 0.3803
2) 0.4612
3) 0.8415
4) 0.9612

5 Which factorization is incorrect?
1) $4k^2 - 49 = (2k + 7)(2k - 7)$
2) $a^3 - 8b^3 = (a - 2b)(a^2 + 2ab + 4b^2)$
3) $m^3 + 3m^2 - 4m + 12 = (m - 2)^2 (m + 3)$
4) $t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t + 1)(t + 2)(t + 3)$

6 Sally’s high school is planning their spring musical. The revenue, $R$, generated can be determined by the function $R(t) = -33t^2 + 360t$, where $t$ represents the price of a ticket. The production cost, $C$, of the musical is represented by the function $C(t) = 700 + 5t$. What is the highest ticket price, to the nearest dollar, they can charge in order to not lose money on the event?
1) $t = 3$
2) $t = 5$
3) $t = 8$
4) $t = 11$
7 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Text Messages per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0–10</td>
</tr>
<tr>
<td>15–18</td>
<td>4</td>
</tr>
<tr>
<td>19–22</td>
<td>6</td>
</tr>
<tr>
<td>23–60</td>
<td>25</td>
</tr>
</tbody>
</table>

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

1) \( \frac{157}{229} \)
2) \( \frac{157}{312} \)
3) \( \frac{157}{384} \)
4) \( \frac{157}{456} \)

8 A recursive formula for the sequence 18, 9, 4.5, \ldots is

1) \( g_1 = 18 \)
\( g_n = \frac{1}{2} g_{n-1} \)
2) \( g_1 = 18 \)
\( g_n = 18 \left( \frac{1}{2} \right)^{n-1} \)
3) \( g_1 = 18 \)
\( g_n = 2g_{n-1} \)
4) \( g_1 = 18(2)^{n-1} \)

9 Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1) \( \sum_{n=1}^{6} 8(1.10)^{n-1} \)
2) \( \sum_{n=1}^{6} 8(1.10)^n \)
3) \( 8 - 8(1.10)^6 \)
4) \( \frac{8 - 8(0.10)^n}{1.10} \)

10 A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave decreasing, only?

1) \((0, 200)\)
2) \((100, 300)\)
3) \((200, 400)\)
4) \((300, 400)\)

11 The expression \( \frac{x^3 + 2x^2 + x + 6}{x + 2} \) is equivalent to

1) \( x^2 + 3 \)
2) \( x^2 + 1 + \frac{4}{x + 2} \)
3) \( 2x^2 + x + 6 \)
4) \( 2x^2 + 1 + \frac{4}{x + 2} \)
12 A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the diagram below.

Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to
1) 0.01
2) 0.03
3) 0.06
4) 0.12

13 An equation to represent the value of a car after \( t \) months of ownership is \( v = 32,000(0.81)^{\frac{t}{12}} \). Which statement is not correct?
1) The car lost approximately 19% of its value each month.
2) The car maintained approximately 98% of its value each month.
3) The value of the car when it was purchased was $32,000.
4) The value of the car 1 year after it was purchased was $25,920.

14 Which equation represents an odd function?
1) \( y = \sin x \)
2) \( y = \cos x \)
3) \( y = (x + 1)^3 \)
4) \( y = e^{5x} \)

15 The completely factored form of \( 2d^4 + 6d^3 - 18d^2 - 54d \) is
1) \( 2d(d^2 - 9)(d + 3) \)
2) \( 2d(d^2 + 9)(d + 3) \)
3) \( 2d(d + 3)^2(d - 3) \)
4) \( 2d(d - 3)^2(d + 3) \)
16. Which diagram shows an angle rotation of 1 radian on the unit circle?

1) 

2) 

3) 

4) 

17. The focal length, $F$, of a camera’s lens is related to the distance of the object from the lens, $J$, and the distance to the image area in the camera, $W$, by the formula below.

$$\frac{1}{J} + \frac{1}{W} = \frac{1}{F}$$

When this equation is solved for $J$ in terms of $F$ and $W$, $J$ equals

1) $\frac{F - W}{FW}$
2) $\frac{FW}{F - W}$
3) $\frac{FW}{W - F}$
4) $\frac{1}{F} - \frac{1}{W}$

18. The sequence $a_1 = 6, a_n = 3a_{n-1}$ can also be written as

1) $a_n = 6 \cdot 3^n$
2) $a_n = 6 \cdot 3^{n+1}$
3) $a_n = 2 \cdot 3^n$
4) $a_n = 2 \cdot 3^{n+1}$
19 Which equation represents the set of points equidistant from line \( \ell \) and point \( R \) shown on the graph below?

1) \( y = -\frac{1}{8} (x + 2)^2 + 1 \)
2) \( y = -\frac{1}{8} (x + 2)^2 - 1 \)
3) \( y = -\frac{1}{8} (x - 2)^2 + 1 \)
4) \( y = -\frac{1}{8} (x - 2)^2 - 1 \)

20 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I \( (m + p)^2 = m^2 + 2mp + p^2 \)
II \( (x + y)^3 = x^3 + 3xy + y^3 \)
III \( (a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2 \)

1) I, only
2) I and II
3) II and III
4) I and III

21 The graph of \( p(x) \) is shown below.

What is the remainder when \( p(x) \) is divided by \( x + 4 \)?

1) \( x - 4 \)
2) \( -4 \)
3) \( 0 \)
4) \( 4 \)

22 A payday loan company makes loans between $100 and $1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a $300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

1) \( 300 \cdot (1.30)^{\frac{365}{14}} \)
2) \( 300 \cdot (1.30)^{\frac{365}{365}} \)
3) \( 300 \cdot (0.30)^{\frac{365}{365}} \)
4) \( 300 \cdot (1.30)^{\frac{365}{14}} \)
23 Which value is not contained in the solution of the system shown below?
\[ a + 5b - c = -20 \]
\[ 4a - 5b + 4c = 19 \]
\[ -a - 5b - 5c = 2 \]
1) −2
2) 2
3) 3
4) −3

24 In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State \( t \) years after 2010?
1) \( P_t = 19,378,000(1.5)^t \)
2) \( P_0 = 19,378,000 \)
\[ P_t = 19,378,000 + 1.015P_{t-1} \]
3) \( P_t = 19,378,000(1.015)^{t-1} \)
4) \( P_0 = 19,378,000 \)
\[ P_t = 1.015P_{t-1} \]

25 The volume of air in a person’s lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

26 Explain how \( \left( \frac{1}{3} \right)^2 \) can be written as the equivalent radical expression \( \sqrt[3]{9} \).

27 Simplify \( xi(i - 7i)^2 \), where \( i \) is the imaginary unit.

28 Using the identity \( \sin^2 \theta + \cos^2 \theta = 1 \), find the value of \( \tan \theta \), to the nearest hundredth, if \( \cos \theta = -0.7 \) and \( \theta \) is in Quadrant II.

29 Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth’s wait time unusual? Justify your answer.

30 The \( x \)-value of which function’s \( x \)-intercept is larger, \( f \) or \( h \)? Justify your answer.

\[ f(x) = \log(x - 4) \]

<table>
<thead>
<tr>
<th>x</th>
<th>h(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>−2</td>
</tr>
</tbody>
</table>
31 The distance needed to stop a car after applying the brakes varies directly with the square of the car’s speed. The table below shows stopping distances for various speeds.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>6.25</td>
<td>25</td>
<td>56.25</td>
<td>100</td>
<td>156.25</td>
<td>225</td>
<td>306.25</td>
</tr>
</tbody>
</table>

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

32 Given events $A$ and $B$, such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether $A$ and $B$ are independent or dependent.

33 Find algebraically the zeros for $p(x) = x^3 + x^2 - 4x - 4$. On the set of axes below, graph $y = p(x)$.

34 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the nearest day, the amount of time needed before the amount of I–131 in the patient’s body is approximately 7 milligrams.

35 Solve the equation $\sqrt{2x - 7} + x = 5$ algebraically, and justify the solution set.
Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

<table>
<thead>
<tr>
<th>Group 1 (seconds)</th>
<th>Group 2 (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.4</td>
<td>23.3</td>
</tr>
<tr>
<td>18.1</td>
<td>18.8</td>
</tr>
<tr>
<td>18.2</td>
<td>22.1</td>
</tr>
<tr>
<td>19.6</td>
<td>12.7</td>
</tr>
<tr>
<td>18.6</td>
<td>16.9</td>
</tr>
<tr>
<td>16.2</td>
<td>24.4</td>
</tr>
<tr>
<td>16.1</td>
<td>21.2</td>
</tr>
<tr>
<td>15.3</td>
<td>21.2</td>
</tr>
<tr>
<td>17.8</td>
<td>16.3</td>
</tr>
<tr>
<td>19.7</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Mean = 17.7  Mean = 19.1

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva’s hypothesis may be incorrect. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.

Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.
Seth’s parents gave him $5000 to invest for his 16th birthday. He is considering two investment options. Option A will pay him 4.5% interest compounded annually. Option B will pay him 4.6% compounded quarterly. Write a function of option A and option B that calculates the value of each account after \( n \) years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option B will earn than option A to the nearest cent. Algebraically determine, to the nearest tenth of a year, how long it would take for option B to double Seth’s initial investment.
1 Relative to the graph of \( y = 3 \sin x \), what is the shift of the graph of \( y = 3 \sin \left(x + \frac{\pi}{3}\right)\)?

1) \( \frac{\pi}{3} \) right  
2) \( \frac{\pi}{3} \) left  
3) \( \frac{\pi}{3} \) up  
4) \( \frac{\pi}{3} \) down

2 A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If \( t \) represents the time, in weeks, and \( P(t) \) is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?

1) 56  
2) 152  
3) 3688  
4) 81,920

3 Factored completely, \( m^5 + m^3 - 6m \) is equivalent to

1) \( (m + 3)(m - 2) \)  
2) \( (m^2 + 3m)(m^2 - 2) \)  
3) \( m(m^4 + m^2 - 6) \)  
4) \( m(m^2 + 3)(m^2 - 2) \)

4 If \( \sin^2(32^\circ) + \cos^2(M) = 1 \), then \( M \) equals

1) \( 32^\circ \)  
2) \( 58^\circ \)  
3) \( 68^\circ \)  
4) \( 72^\circ \)

5 What is the solution to the system of equations \( y = 3x - 2 \) and \( y = g(x) \) where \( g(x) \) is defined by the function below?

6 Which statement about statistical analysis is false?

1) Experiments can suggest patterns and relationships in data.  
2) Experiments can determine cause and effect relationships.  
3) Observational studies can determine cause and effect relationships.  
4) Observational studies can suggest patterns and relationships in data.
7 The expression \( \left( \frac{m^2}{m^{\frac{1}{3}}} \right)^{\frac{1}{2}} \) is equivalent to

1) \(-\sqrt[6]{m^5}\)
2) \(\frac{1}{\sqrt[6]{m^5}}\)
3) \(-m^{\frac{5}{3}}\sqrt{m}\)
4) \(\frac{1}{m^{\frac{5}{3}}\sqrt{m}}\)

8 What is the inverse of the function \( y = \log_3 x \)?

1) \( y = x^3 \)
2) \( y = \log_3 3 \)
3) \( y = 3^x \)
4) \( x = 3^y \)

9 Gabriel performed an experiment to see if planting 13 tomato plants in black plastic mulch leads to larger tomatoes than if 13 plants are planted without mulch. He observed that the average weight of the tomatoes from tomato plants grown in black plastic mulch was 5 ounces greater than those from the plants planted without mulch. To determine if the observed difference is statistically significant, he rerandomized the tomato groups 100 times to study these random differences in the mean weights. The output of his simulation is summarized in the dotplot below.

Given these results, what is an appropriate inference that can be drawn?

1) There was no effect observed between the two groups.
2) There was an effect observed that could be due to the random assignment of plants to the groups.
3) There is strong evidence to support the hypothesis that tomatoes from plants planted in black plastic mulch are larger than those planted without mulch.
4) There is strong evidence to support the hypothesis that tomatoes from plants planted without mulch are larger than those planted in black plastic mulch.

10 If \( p(x) = ab^x \) and \( r(x) = cd^x \), then \( p(x) \cdot r(x) \) equals

1) \( ac(b + d)^x \)
2) \( ac(b + d)^{2x} \)
3) \( ac(bd)^x \)
4) \( ac(bd)^{x^2} \)
11 The solution to the equation $18x^2 - 24x + 87 = 0$ is

1) $\frac{2}{3} \pm 6i\sqrt{158}$
2) $\frac{2}{3} \pm \frac{1}{6} i\sqrt{158}$
3) $\frac{2}{3} \pm 6i\sqrt{158}$
4) $\frac{2}{3} \pm \frac{1}{6} i\sqrt{158}$

12 When \( g(x) = \frac{2}{x+2} \) and \( h(x) = \log(x+1) + 3 \) are graphed on the same set of axes, which coordinates best approximate their point of intersection?

1) \((-0.9, 1.8)\)
2) \((-0.9, 1.9)\)
3) \((1.4, 3.3)\)
4) \((1.4, 3.4)\)

13 The price of a postage stamp in the years since the end of World War I is shown in the scatterplot below.

![Price of a Postage Stamp Since End of World War I](scatterplot.png)

The equation that best models the price, in cents, of a postage stamp based on these data is

1) \( y = 0.59x - 14.82 \)
2) \( y = 1.04(1.43)^x \)
3) \( y = 1.43(1.04)^x \)
4) \( y = 24\sin(14x) + 25 \)

14 The eighth and tenth terms of a sequence are 64 and 100. If the sequence is either arithmetic or geometric, the ninth term can \textit{not} be

1) \(-82\)
2) \(-80\)
3) 80
4) 82
15 The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity $I_0$ to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, $I$, and the decibel rating, $d$, of this sound is found using $d = 10 \log \frac{I}{I_0}$. The threshold sound audible to the average person is $1.0 \times 10^{-12}$ W/m² (watts per square meter). Consider the following sound level classifications:

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Decibels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderate</td>
<td>45-69 dB</td>
</tr>
<tr>
<td>Loud</td>
<td>70-89 dB</td>
</tr>
<tr>
<td>Very loud</td>
<td>90-109 dB</td>
</tr>
<tr>
<td>Deafening</td>
<td>&gt;110 dB</td>
</tr>
</tbody>
</table>

How would a sound with intensity $6.3 \times 10^{-3}$ W/m² be classified?
1) moderate  
2) loud  
3) very loud  
4) deafening

16 Pedro and Bobby each own an ant farm. Pedro starts with 100 ants and says his farm is growing exponentially at a rate of 15% per month. Bobby starts with 350 ants and says his farm is steadily decreasing by 5 ants per month. Assuming both boys are accurate in describing the population of their ant farms, after how many months will they both have approximately the same number of ants?
1) 7  
2) 8  
3) 13  
4) 36

17 What is the solution, if any, of the equation $\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}$?
1) −1  
2) −5  
3) all real numbers  
4) no real solution

18 In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects 95% of the Critical Reading scores?
1) $430 \pm 115$  
2) $430 \pm 230$  
3) $496 \pm 115$  
4) $496 \pm 230$

19 Which statement regarding the graphs of the functions below is untrue?

\[ f(x) = 3 \sin 2x, \text{ from } -\pi < x < \pi \]
\[ g(x) = (x - 0.5)(x + 4)(x - 2) \]
\[ h(x) = \log_2 x \]
\[ j(x) = -|4x - 2| + 3 \]

1) $f(x)$ and $j(x)$ have a maximum $y$-value of 3.  
2) $f(x)$, $h(x)$, and $j(x)$ have one $y$-intercept.  
3) $g(x)$ and $j(x)$ have the same end behavior as $x \to -\infty$.  
4) $g(x)$, $h(x)$, and $j(x)$ have rational zeros.

20 When $g(x)$ is divided by $x + 4$, the remainder is 0. Given $g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8$, which conclusion about $g(x)$ is true?
1) $g(4) = 0$  
2) $g(-4) = 0$  
3) $x - 4$ is a factor of $g(x)$.  
4) No conclusion can be made regarding $g(x)$.
21 Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of $B$ dollars after $m$ months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after $m$ months.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.00</td>
</tr>
<tr>
<td>10</td>
<td>1172.00</td>
</tr>
<tr>
<td>19</td>
<td>1352.00</td>
</tr>
<tr>
<td>36</td>
<td>1770.80</td>
</tr>
<tr>
<td>60</td>
<td>2591.90</td>
</tr>
<tr>
<td>69</td>
<td>2990.00</td>
</tr>
<tr>
<td>72</td>
<td>3135.80</td>
</tr>
<tr>
<td>73</td>
<td>3186.00</td>
</tr>
</tbody>
</table>

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

1) month 10 to month 60  
2) month 19 to month 69  
3) month 36 to month 72  
4) month 60 to month 73

22 Which graph represents a cosine function with no horizontal shift, an amplitude of 2, and a period of $\frac{2\pi}{3}$?

1)  
2)  
3)  
4)  

23 According to a pricing website, Indroid phones lose 58% of their cash value over 1.5 years. Which expression can be used to estimate the value of a $300 Indroid phone in 1.5 years?

1) $300e^{-0.87}$  
2) $300e^{-0.63}$  
3) $300e^{-0.58}$  
4) $300e^{-0.42}$

24 A cardboard box manufacturing company is building boxes with length represented by $x + 1$, width by $5 - x$, and height by $x - 1$. The volume of the box is modeled by the function below.

Over which interval is the volume of the box changing at the fastest average rate?

1) $[1, 2]$  
2) $[1, 3.5]$  
3) $[1, 5]$  
4) $[0, 3.5]$

25 Express $(1 - i)^3$ in $a + bi$ form.
26 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario. State one conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

27 Using the unit circle below, explain why \( \csc \theta = \frac{1}{y} \).

28 The function \( M(t) \) represents the mass of radium over time, \( t \), in years.

\[
M(t) = 100e^{\frac{\ln \frac{1}{2}}{1590} t}
\]

Determine if the function \( M(t) \) represents growth or decay. Explain your reasoning.

29 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

30 Given the equal terms \( \frac{1}{2}\sqrt[3]{x^5} \) and \( \frac{5}{y^6} \), determine and state \( y \), in terms of \( x \).

31 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

<table>
<thead>
<tr>
<th></th>
<th>Comedy Series</th>
<th>Drama Series</th>
<th>Reality Series</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>95</td>
<td>65</td>
<td>70</td>
<td>230</td>
</tr>
<tr>
<td>Females</td>
<td>80</td>
<td>70</td>
<td>110</td>
<td>260</td>
</tr>
<tr>
<td>Total</td>
<td>175</td>
<td>135</td>
<td>180</td>
<td>490</td>
</tr>
</tbody>
</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.
32 Given \( f(x) = 3x^2 + 7x - 20 \) and \( g(x) = x - 2 \), state the quotient and remainder of \( \frac{f(x)}{g(x)} \), in the form \( q(x) + \frac{r(x)}{g(x)} \).

33 Algebraically determine the values of \( h \) and \( k \) to correctly complete the identity stated below.

\[
2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k
\]

34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer. Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose. Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

35 The guidance department has reported that of the senior class, 2.3% are members of key club, \( K \), 8.6% are enrolled in AP Physics, \( P \), and 1.9% are in both. Determine the probability of \( P \) given \( K \), to the nearest tenth of a percent. The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.
36 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

\( P_n \) = present amount borrowed

\( n \) = number of monthly pay periods

\( PMT \) = monthly payment

\( i \) = interest rate per month

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

37 The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation

\[ s = \sqrt{3 - 2t + 6} \]

where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \). How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.
1. The graph of the function \( p(x) \) is sketched below.

Which equation could represent \( p(x) \)?

1) \( p(x) = (x^2 - 9)(x - 2) \)
2) \( p(x) = x^3 - 2x^2 + 9x + 18 \)
3) \( p(x) = (x^2 + 9)(x - 2) \)
4) \( p(x) = x^3 + 2x^2 - 9x - 18 \)

2. What is the solution to \( 8(2^{x+3}) = 48 \)?

1) \( x = \frac{\ln 6}{\ln 2} - 3 \)
2) \( x = 0 \)
3) \( x = \frac{\ln 48}{\ln 16} - 3 \)
4) \( x = \ln 4 - 3 \)

3. Cheap and Fast gas station is conducting a consumer satisfaction survey. Which method of collecting data would most likely lead to a biased sample?

1) interviewing every 5th customer to come into the station
2) interviewing customers chosen at random by a computer at the checkout
3) interviewing customers who call an 800 number posted on the customers' receipts
4) interviewing every customer who comes into the station on a day of the week chosen at random out of a hat

4. The expression \( 6x^3(-4xi + 5) \) is equivalent to

1) \( 2x - 5i \)
2) \( -24x^2 - 30xi \)
3) \( -24x^2 + 30x - i \)
4) \( 26x - 24x^2i - 5i \)

5. If \( f(x) = 3|x| - 1 \) and \( g(x) = 0.03x^3 - x + 1 \), an approximate solution for the equation \( f(x) = g(x) \) is

1) \( 1.96 \)
2) \( 11.29 \)
3) \( (-0.99, 1.96) \)
4) \( (11.29, 32.87) \)

6. Given the parent function \( p(x) = \cos x \), which phrase best describes the transformation used to obtain the graph of \( g(x) = \cos(x + a) - b \), if \( a \) and \( b \) are positive constants?

1) right \( a \) units, up \( b \) units
2) right \( a \) units, down \( b \) units
3) left \( a \) units, up \( b \) units
4) left \( a \) units, down \( b \) units

7. The solution to the equation \( 4x^2 + 98 = 0 \) is

1) \( \pm 7 \)
2) \( \pm 7i \)
3) \( \pm \frac{7\sqrt{2}}{2} \)
4) \( \pm \frac{7i\sqrt{2}}{2} \)
8 Which equation is represented by the graph shown below?

![Graph Image]

1) \( y = \frac{1}{2} \cos 2x \)
2) \( y = \cos x \)
3) \( y = \frac{1}{2} \cos x \)
4) \( y = 2 \cos \frac{1}{2} x \)

9 A manufacturing company has developed a cost model, \( C(x) = 0.15x^3 + 0.01x^2 + 2x + 120 \), where \( x \) is the number of items sold, in thousands. The sales price can be modeled by \( S(x) = 30 - 0.01x \). Therefore, revenue is modeled by \( R(x) = x \cdot S(x) \). The company's profit, \( P(x) = R(x) - C(x) \), could be modeled by

1) \( 0.15x^3 + 0.02x^2 - 28x + 120 \)
2) \( -0.15x^3 - 0.02x^2 + 28x - 120 \)
3) \( -0.15x^3 + 0.01x^2 - 2.01x - 120 \)
4) \( -0.15x^3 + 32x + 120 \)

10 A game spinner is divided into 6 equally sized regions, as shown in the diagram below.

![Game Spinner Image]

For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times, assuming the probability of winning each spin is \( \frac{1}{6} \). The output of the simulation is shown in the diagram below.

![Simulation Output Image]

Which explanation is appropriate for Miles and his dad to make?
1) The spinner was likely unfair, since the number 6 failed to occur in about 20% of the simulations.
2) The spinner was likely unfair, since the spinner should have landed on the number 6 by the sixth spin.
3) The spinner was likely not unfair, since the number 6 failed to occur in about 20% of the simulations.
4) The spinner was likely not unfair, since in the output the player wins once or twice in the majority of the simulations.

11 Which binomial is a factor of \( x^4 - 4x^2 - 4x + 8 \)?

1) \( x - 2 \)
2) \( x + 2 \)
3) \( x - 4 \)
4) \( x + 4 \)
12 Given that $\sin^2 \theta + \cos^2 \theta = 1$ and $\sin \theta = -\frac{2}{5}$, what is a possible value of $\cos \theta$?

1) $\frac{5 + \sqrt{2}}{5}$
2) $\frac{\sqrt{23}}{5}$
3) $\frac{3\sqrt{3}}{5}$
4) $\frac{\sqrt{35}}{5}$

13 A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P = 714(0.75)^d$, where $P$ is the population, in thousands, $d$ decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after $y$ years. Suzanne's model is best represented by

1) $P = 714(0.6500)^y$
2) $P = 714(0.8500)^y$
3) $P = 714(0.9716)^y$
4) $P = 714(0.9750)^y$

14 The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

I: dependent
II: independent
III: mutually exclusive

1) I, only
2) II, only
3) I and III
4) II and III

15 Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914 \sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238 \sin(0.525x - 2.148) + 86.729$. Which statement can *not* be concluded based on the average monthly temperature models $x$ months after starting data collection?

1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.
2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.
3) The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.
4) The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree.

16 For $x \neq 0$, which expressions are equivalent to one divided by the sixth root of $x$?

I. $\frac{\sqrt[6]{x}}{3x}$
II. $\frac{1}{x^{\frac{1}{6}}}$
III. $\frac{1}{x^{\frac{1}{3}}}$

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

17 A parabola has its focus at (1,2) and its directrix is $y = -2$. The equation of this parabola could be

1) $y = 8(x + 1)^2$
2) $y = \frac{1}{8} (x + 1)^2$
3) $y = 8(x - 1)^2$
4) $y = \frac{1}{8} (x - 1)^2$
18 The function \( p(t) = 110e^{0.03922t} \) models the population of a city, in millions, \( t \) years after 2010. As of today, consider the following two statements:

I. The current population is 110 million.
II. The population increases continuously by approximately 3.9% per year.
This model supports

1) I, only
2) II, only
3) both I and II
4) neither I nor II

19 To solve \( 2x^2 - 2 - 11x = \frac{8}{x^2 - 2x} \), Ren multiplied both sides by the least common denominator. Which statement is true?

1) 2 is an extraneous solution.
2) \( \frac{7}{2} \) is an extraneous solution.
3) 0 and 2 are extraneous solutions.
4) This equation does not contain any extraneous solutions.

20 Given \( f(9) = -2 \), which function can be used to generate the sequence \(-8, -7.25, -6.5, -5.75, \ldots \)?

1) \( f(n) = -8 + 0.75n \)
2) \( f(n) = -8 - 0.75(n - 1) \)
3) \( f(n) = -8.75 + 0.75n \)
4) \( f(n) = -0.75 + 8(n - 1) \)

21 The function \( f(x) = 2^{-0.25x} \cdot \sin \left( \frac{\pi}{2} x \right) \) represents a damped sound wave function. What is the average rate of change for this function on the interval \([-7, 7]\), to the nearest hundredth?

1) \(-3.66\)
2) \(-0.30\)
3) \(-0.26\)
4) \(3.36\)

22 Mallory wants to buy a new window air conditioning unit. The cost for the unit is $329.99. If she plans to run the unit three months out of the year for an annual operating cost of $108.78, which function models the cost per year over the lifetime of the unit, \( C(n) \), in terms of the number of years, \( n \), that she owns the air conditioner?

1) \( C(n) = 329.99 + 108.78n \)
2) \( C(n) = 329.99 + 326.34n \)
3) \( C(n) = \frac{329.99 + 108.78n}{n} \)
4) \( C(n) = \frac{329.99 + 326.34n}{n} \)

23 The expression \( \frac{-3x^2 - 5x + 2}{x^3 + 2x^2} \) can be rewritten as

1) \( \frac{-3x - 3}{x^2 + 2x} \)
2) \( \frac{-3x - 1}{x^2} \)
3) \(-3x^{-1} + 1\)
4) \(-3x^{-1} + x^{-2}\)

24 Jasmine decides to put $100 in a savings account each month. The account pays 3% annual interest, compounded monthly. How much money, \( S \), will Jasmine have after one year?

1) \( S = 100(1.03)^{12} \)
2) \( S = \frac{100 - 100(1.0025)^{12}}{1 - 1.0025} \)
3) \( S = 100(1.0025)^{12} \)
4) \( S = \frac{100 - 100(1.03)^{12}}{1 - 1.03} \)

25 Given \( r(x) = x^3 - 4x^2 + 4x - 6 \), find the value of \( r(2) \). What does your answer tell you about \( x - 2 \) as a factor of \( r(x) \)? Explain.
26 The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the nearest integer, weighed less than 8.25 pounds.

27 Over the set of integers, factor the expression $4x^3 - x^2 + 16x - 4$ completely.

28 The graph below represents the height above the ground, $h$, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, $t$, in seconds.

Identify the period of the graph and describe what the period represents in this context.

29 Graph $y = 400(.85)^x - 6$ on the set of axes below.

30 Solve algebraically for all values of $x$:

\[
\sqrt{x} - 4 + x = 6
\]

31 Write $\sqrt[3]{x} \cdot \sqrt{x}$ as a single term with a rational exponent.
Data collected about jogging from students with two older siblings are shown in the table below.

<table>
<thead>
<tr>
<th>Student Does Not Jog</th>
<th>Neither Sibling Jogs</th>
<th>One Sibling Jogs</th>
<th>Both Siblings Jog</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Jogs</td>
<td>1168</td>
<td>1823</td>
<td>1380</td>
</tr>
<tr>
<td></td>
<td>188</td>
<td>416</td>
<td>400</td>
</tr>
</tbody>
</table>

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.

Solve the following system of equations algebraically for all values of $x$, $y$, and $z$:

\[
\begin{align*}
x + y + z &= 1 \\
2x + 4y + 6z &= 2 \\
-x + 3y - 5z &= 11
\end{align*}
\]

Jim is looking to buy a vacation home for $172,600 near his favorite southern beach. The formula to compute a mortgage payment, $M$, is

\[
M = P \cdot \frac{r(1+r)^N}{(1+r)^N - 1}
\]

where $P$ is the principal amount of the loan, $r$ is the monthly interest rate, and $N$ is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With no down payment, determine Jim's mortgage payment, rounded to the nearest dollar. Algebraically determine and state the down payment, rounded to the nearest dollar, that Jim needs to make in order for his mortgage payment to be $1100.

Graph $y = \log_2(x + 3) - 5$ on the set of axes below. Use an appropriate scale to include both intercepts.

Describe the behavior of the given function as $x$ approaches -3 and as $x$ approaches positive infinity.
36 Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if 50% or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that 50% of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40, simulated 100 times.

![Graph showing proportions of customers preferring the new procedure](image)

Assume the set of data is approximately normal and the dealership wants to be 95% confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the nearest hundredth. Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is 32.5%. The dealership decides not to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.

37 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form

\[ A = A_0 \left( \frac{1}{2} \right)^{\frac{t}{h}} \]

that models this situation, where \( h \) is the constant representing the number of hours in the half-life, \( A_0 \) is the initial mass, and \( A \) is the mass \( t \) hours after 3 p.m. Using this equation, solve for \( h \), to the nearest ten thousandth. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the nearest tenth of an hour.
0817AII Common Core State Standards

1 The function \( f(x) = \frac{x - 3}{x^2 + 2x - 8} \) is undefined when \( x \) equals
   1) 2 or -4
   2) 4 or -2
   3) 3, only
   4) 2, only

2 Which expression is equivalent to \((3k - 2i)^2\), where \( i \) is the imaginary unit?
   1) \(9k^2 - 4\)
   2) \(9k^2 + 4\)
   3) \(9k^2 - 2ik - 4\)
   4) \(9k^2 - 12i + 4\)

3 The roots of the equation \(x^2 + 2x + 5 = 0\) are
   1) -3 and 1
   2) -1, only
   3) \(-1 + 2i\) and \(-1 - 2i\)
   4) \(-1 + 4i\) and \(-1 - 4i\)

4 The solution set for the equation \(\sqrt{x + 14} - \sqrt{2x + 5} = 1\) is
   1) \{6\}
   2) \{6\}
   3) \{18\}
   4) \{2,22\}

5 As \( x \) increases from 0 to \( \frac{\pi}{2} \), the graph of the equation \( y = 2 \tan x \) will
   1) increase from 0 to 2
   2) decrease from 0 to -2
   3) increase without limit
   4) decrease without limit
7 Which diagram represents an angle, \( \alpha \), measuring \( \frac{13\pi}{20} \) radians drawn in standard position, and its reference angle, \( \theta \)?

1)  

2)  

3)  

4)  

8 What are the zeros of \( P(m) = (m^2 - 4)(m^2 + 1) \)?

1) 2 and \(-2\), only

2) 2, \(-2\), and \(-4\)

3) \(-4\), \(i\), and \(-i\)

4) 2, \(-2\), \(i\), and \(-i\)

9 The value of a new car depreciates over time. Greg purchased a new car in June 2011. The value, \( V \), of his car after \( t \) years can be modeled by the equation \( \log_{0.8} \left( \frac{V}{17000} \right) = t \). What is the average decreasing rate of change per year of the value of the car from June 2012 to June 2014, to the nearest ten dollars per year?

1) 1960

2) 2180

3) 2450

4) 2770

10 Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, \( A \), of Iridium-192 present after \( t \) days would be \( A = 100 \left( \frac{1}{2} \right)^{\frac{t}{73.83}} \). Which equation approximates the amount of Iridium-192 present after \( t \) days?

1) \( A = 100 \left( \frac{73.83}{2} \right)^t \)

2) \( A = 100 \left( \frac{1}{147.66} \right)^t \)

3) \( A = 100(0.990656)^t \)

4) \( A = 100(0.116381)^t \)

11 The distribution of the diameters of ball bearings made under a given manufacturing process is normally distributed with a mean of 4 cm and a standard deviation of 0.2 cm. What proportion of the ball bearings will have a diameter less than 3.7 cm?

1) 0.0668

2) 0.4332

3) 0.8664

4) 0.9500
12 A polynomial equation of degree three, \( p(x) \), is used to model the volume of a rectangular box. The graph of \( p(x) \) has \( x \) intercepts at \(-2, 10, \) and \( 14 \). Which statements regarding \( p(x) \) could be true?
A. The equation of \( p(x) = (x - 2)(x + 10)(x + 14) \).
B. The equation of \( p(x) = -(x + 2)(x - 10)(x - 14) \).
C. The maximum volume occurs when \( x = 10 \).
D. The maximum volume of the box is approximately 56.
1) \( A \) and \( C \)
2) \( A \) and \( D \)
3) \( B \) and \( C \)
4) \( B \) and \( D \)

13 Which expression is equivalent to \( \frac{4x^3 + 9x - 5}{2x - 1} \), where \( x \neq \frac{1}{2} \)?
1) \( 2x^2 + x + 5 \)
2) \( 2x^2 + \frac{11}{2} + \frac{1}{2(2x - 1)} \)
3) \( 2x^2 - x + 5 \)
4) \( 2x^2 - x + 4 + \frac{1}{2x - 1} \)

14 The inverse of the function \( f(x) = \frac{x + 1}{x - 2} \) is
1) \( f^{-1}(x) = \frac{x + 1}{x + 2} \)
2) \( f^{-1}(x) = \frac{2x + 1}{x - 1} \)
3) \( f^{-1}(x) = \frac{x + 1}{x - 2} \)
4) \( f^{-1}(x) = \frac{x - 1}{x + 1} \)

15 Which expression has been rewritten correctly to form a true statement?
1) \( (x + 2)^2 + 2(x + 2) - 8 = (x + 6)x \)
2) \( x^4 + 4x^2 + 9x^2y^2 - 36y^2 = (x + 3y)^2(x - 2)^2 \)
3) \( x^3 + 3x^2 - 4xy^2 - 12y^2 = (x - 2y)(x + 3)^2 \)
4) \( (x^2 - 4)^2 - 5(x^2 - 4) - 6 = (x^2 - 7)(x^2 - 6) \)

16 A study conducted in 2004 in New York City found that 212 out of 1334 participants had hypertension. Kim ran a simulation of 100 studies based on these data. The output of the simulation is shown in the diagram below.

At a 95% confidence level, the proportion of New York City residents with hypertension and the margin of error are closest to
1) proportion \( \approx .16 \); margin of error \( \approx .01 \)
2) proportion \( \approx .16 \); margin of error \( \approx .02 \)
3) proportion \( \approx .01 \); margin of error \( \approx .16 \)
4) proportion \( \approx .02 \); margin of error \( \approx .16 \)
17 Which scenario is best described as an observational study?
1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.
3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.

18 Which sinusoid has the greatest amplitude?

![Graph of sine functions]

1) $y = 3 \sin(\theta - 3) + 5$
2) $y = 3 \sin(\theta - 3) + 5$
3) $y = -5 \sin(\theta - 1) - 3$

19 Consider the system shown below.

$2x - y = 4$

$(x + 3)^2 + y^2 = 8$

The two solutions of the system can be described as
1) both imaginary
2) both irrational
3) both rational
4) one rational and one irrational

20 Which binomial is not a factor of the expression $x^3 - 11x^2 + 16x + 84$?

1) $x + 2$
2) $x + 4$
3) $x - 6$
4) $x - 7$

21 A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

1) 29
2) 58
3) 120
4) 149

22 A public opinion poll was conducted on behalf of Mayor Ortega's reelection campaign shortly before the election. 264 out of 550 likely voters said they would vote for Mayor Ortega; the rest said they would vote for his opponent. Which statement is least appropriate to make, according to the results of the poll?

1) There is a 48% chance that Mayor Ortega will win the election.
2) The point estimate ($\hat{p}$) of voters who will vote for Mayor Ortega is 48%.
3) It is most likely that between 44% and 52% of voters will vote for Mayor Ortega.
4) Due to the margin of error, an inference cannot be made regarding whether Mayor Ortega or his opponent is most likely to win the election.
23 What does \( \left( \frac{-54x^9}{y^4} \right)^{\frac{2}{3}} \) equal?

1) \( \frac{9x^{6\sqrt{4}}}{y^{2\sqrt{y}}} \)

2) \( \frac{9x^{6\sqrt{4}}}{y^{2\sqrt{y}}} \)

3) \( \frac{9x^{6\sqrt{4}}}{y^{3\sqrt{y}}} \)

4) \( \frac{9x^{6\sqrt{4}}}{y^{2\sqrt{y}}} \)

24 The Rickertts decided to set up an account for their daughter to pay for her college education. The day their daughter was born, they deposited $1000 in an account that pays 1.8% compounded annually. Beginning with her first birthday, they deposit an additional $750 into the account on each of her birthdays. Which expression correctly represents the amount of money in the account \( n \) years after their daughter was born?

1) \( a_n = 1000(1.018)^n + 750 \)

2) \( a_n = 1000(1.018)^n + 750n \)

3) \( a_0 = 1000 \)

4) \( a_0 = 1000 \)

25 \( a_n = a_{n-1}(1.018) + 750 \)

26 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

27 Verify the following Pythagorean identity for all values of \( x \) and \( y \):

\[ (x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2 \]

28 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said "What are the odds I got all of that kind?" Mrs. Jones replied, "simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual." Explain how this simulation could be used to solve the problem.

29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79, .... Write a recursive formula for Candy's sequence. Determine the eighth term in Candy's sequence.

30 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was $1.25 an hour and in 2015, it was $8.75. Algebraically determine the rate of growth to the nearest percent.

31 Algebraically determine whether the function \( f(x) = x^4 - 3x^2 - 4 \) is odd, even, or neither.
32 On the axes below, sketch a possible function \( p(x) = (x - a)(x - b)(x + c) \), where \( a, b, \) and \( c \) are positive, \( a > b \), and \( p(x) \) has a positive \( y \)-intercept of \( d \). Label all intercepts.

33 Solve for all values of \( p \):
\[
\frac{3p}{p - 5} - \frac{2}{p + 3} = \frac{p}{p + 3}
\]

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed $2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed $6.25 to replace his library card and pay the fine for the overdue book. Suppose the total amount Simon owes when the book is \( n \) days late can be determined by an arithmetic sequence. Determine a formula for \( a_n \), the \( n \)th term of this sequence. Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

35 a) On the axes below, sketch \textit{at least one} cycle of a sine curve with an amplitude of 2, a midline at \( y = -\frac{3}{2} \), and a period of \( 2\pi \).

b) Explain any differences between a sketch of
\[
y = 2 \sin \left( x - \frac{\pi}{3} \right) - \frac{3}{2}
\]
and the sketch from part a.
36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

<table>
<thead>
<tr>
<th>Hours (x)</th>
<th>Average Number of Spores (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>260</td>
</tr>
<tr>
<td>4</td>
<td>1130</td>
</tr>
<tr>
<td>6</td>
<td>16,380</td>
</tr>
</tbody>
</table>

Using these data, write an exponential regression equation, rounding all values to the nearest thousandth. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the nearest quarter hour, that the meat can be kept at room temperature safely.

37 The value of a certain small passenger car based on its use in years is modeled by \( V(t) = 28482.698(0.684)^t \), where \( V(t) \) is the value in dollars and \( t \) is the time in years. Zach had to take out a loan to purchase the small passenger car. The function \( Z(t) = 22151.327(0.778)^t \), where \( Z(t) \) is measured in dollars, and \( t \) is the time in years, models the unpaid amount of Zach's loan over time. Graph \( V(t) \) and \( Z(t) \) over the interval \( 0 \leq t \leq 5 \), on the set of axes below.

State when \( V(t) = Z(t) \), to the nearest hundredth, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a $3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the nearest year, how long will it take Zach to cancel this policy? Justify your answer.
1 The operator of the local mall wants to find out how many of the mall's employees make purchases in the food court when they are working. She hopes to use these data to increase the rent and attract new food vendors. In total, there are 1023 employees who work at the mall. The best method to obtain a random sample of the employees would be to survey
1) all 170 employees at each of the larger stores
2) 50% of the 90 employees of the food court
3) every employee
4) every 30th employee entering each mall entrance for one week

2 What is the solution set for $x$ in the equation below?
\[
\sqrt{x+1} - 1 = x
\]
1) \{1\}
2) \{0\}
3) \{-1,0\}
4) \{0,1\}

3 For the system shown below, what is the value of $z$?
\[
y = -2x + 14 \\
3x - 4z = 2 \\
3x - y = 16
\]
1) 5
2) 2
3) 6
4) 4

4 The hours of daylight, $y$, in Utica in days, $x$, from January 1, 2013 can be modeled by the equation $y = 3.06 \sin(0.017x - 1.40) + 12.23$. How many hours of daylight, to the nearest tenth, does this model predict for February 14, 2013?
1) 9.4
2) 10.4
3) 12.1
4) 12.2

5 A certain pain reliever is taken in 220 mg dosages and has a half-life of 12 hours. The function $A = 220 \left(\frac{1}{2}\right)^{\frac{t}{12}}$ can be used to model this situation, where $A$ is the amount of pain reliever in milligrams remaining in the body after $t$ hours. According to this function, which statement is true?
1) Every hour, the amount of pain reliever remaining is cut in half.
2) In 12 hours, there is no pain reliever remaining in the body.
3) In 24 hours, there is no pain reliever remaining in the body.
4) In 12 hours, 110 mg of pain reliever is remaining.

6 The expression $(x+a)(x+b)$ can not be written as
1) $a(x+b) + x(x+b)$
2) $x^2 + abx + ab$
3) $x^2 + (a+b)x + ab$
4) $x(x+a) + b(x+a)$
7 There are 440 students at Thomas Paine High School enrolled in U.S. History. On the April report card, the students' grades are approximately normally distributed with a mean of 79 and a standard deviation of 7. Students who earn a grade less than or equal to 64.9 must attend summer school. The number of students who must attend summer school for U.S. History is closest to
1) 3
2) 5
3) 10
4) 22

8 For a given time, \(x\), in seconds, an electric current, \(y\), can be represented by \(y = 2.5 \left( 1 - 2.7^{-10x} \right) \).
Which equation is not equivalent?
1) \(y = 2.5 - 2.5 \left( 2.7^{-10x} \right)\)
2) \(y = 2.5 - 2.5 \left( 2.7^2 \right)^{-0.5x}\)
3) \(y = 2.5 - 2.5 \left( \frac{1}{2.7^{10x}} \right)\)
4) \(y = 2.5 - 2.5 \left( 2.7^{-2} \right) \left( 2.7^{0.5x} \right)\)

9 What is the quotient when \(10x^3 - 3x^2 - 7x + 3\) is divided by \(2x - 1\)?
1) \(5x^2 + x + 3\)
2) \(5x^2 - x + 3\)
3) \(5x^2 - x - 3\)
4) \(5x^2 + x - 3\)

10 Judith puts $5000 into an investment account with interest compounded continuously. Which approximate annual rate is needed for the account to grow to $9110 after 30 years?
1) 2%
2) 2.2%
3) 0.02%
4) 0.022%

11 If \(n = \sqrt{a^5}\) and \(m = a\), where \(a > 0\), an expression for \(\frac{n}{m}\) could be
1) \(a^{\frac{5}{2}}\)
2) \(a^4\)
3) \(\sqrt[3]{a^2}\)
4) \(\sqrt[3]{a^3}\)

12 The solutions to \(x + 3 - \frac{4}{x - 1} = 5\) are
1) \(\frac{3}{2} \pm \frac{\sqrt{17}}{2}\)
2) \(\frac{3}{2} \pm \frac{\sqrt{17}}{i}\)
3) \(\frac{3}{2} \pm \frac{\sqrt{33}}{2}\)
4) \(\frac{3}{2} \pm \frac{\sqrt{33}}{2}i\)

13 If \(ae^{bt} = c\), where \(a\), \(b\), and \(c\) are positive, then \(t\) equals
1) \(\ln \left( \frac{c}{ab} \right)\)
2) \(\ln \left( \frac{cb}{a} \right)\)
3) \(\frac{\ln \left( \frac{c}{a} \right)}{b}\)
4) \(\frac{\ln \left( \frac{c}{a} \right)}{\ln b}\)
14. For which values of \( x \), rounded to the nearest hundredth, will \( \sqrt{x^2 - 9} - 3 = \log_3 x \)?

1) 2.29 and 3.63
2) 2.37 and 3.54
3) 2.84 and 3.17
4) 2.92 and 3.06

15. The terminal side of \( \theta \), an angle in standard position, intersects the unit circle at \( P \left( \frac{1}{3}, \frac{\sqrt{8}}{3} \right) \). What is the value of \( \sec \theta \)?

1) \(-3\)
2) \(\frac{3\sqrt{8}}{8}\)
3) \(\frac{1}{3}\)
4) \(\frac{\sqrt{8}}{3}\)

16. What is the equation of the directrix for the parabola \(-8(y - 3) = (x + 4)^2\)?

1) \(y = 5\)
2) \(y = 1\)
3) \(y = -2\)
4) \(y = -6\)

17. The function below models the average price of gas in a small town since January 1st.

\[ G(t) = -0.0049t^4 + 0.0923t^3 - 0.56t^2 + 1.166t + 3.23 \]

where \( 0 \leq t \leq 10 \).

If \( G(t) \) is the average price of gas in dollars and \( t \) represents the number of months since January 1st, the absolute maximum \( G(t) \) reaches over the given domain is about

1) \$1.60
2) \$3.92
3) \$4.01
4) \$7.73

18. Written in simplest form, \( \frac{c^2 - d^2}{d^2 + cd - 2c^2} \) where \( c \neq d \), is equivalent to

1) \(\frac{c + d}{d + 2c}\)
2) \(\frac{c - d}{d + 2c}\)
3) \(\frac{-c - d}{d + 2c}\)
4) \(\frac{-c + d}{d + 2c}\)

19. If \( p(x) = 2x^3 - 3x + 5 \), what is the remainder of \( p(x) \div (x - 5) \)?

1) \(-230\)
2) 0
3) 40
4) 240

20. The results of simulating tossing a coin 10 times, recording the number of heads, and repeating this 50 times are shown in the graph below.

Based on the results of the simulation, which statement is false?

1) Five heads occurred most often, which is consistent with the theoretical probability of obtaining a heads.
2) Eight heads is unusual, as it falls outside the middle 95% of the data.
3) Obtaining three heads or fewer occurred 28% of the time.
4) Seven heads is not unusual, as it falls within the middle 95% of the data.
21 What is the inverse of \( f(x) = -6(x - 2) \)?

1) \( f^{-1}(x) = -2 - \frac{x}{6} \)
2) \( f^{-1}(x) = 2 - \frac{x}{6} \)
3) \( f^{-1}(x) = \frac{1}{-6(x - 2)} \)
4) \( f^{-1}(x) = 6(x + 2) \)

22 Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

1) $11,622,614.67
2) $17,433,922.00
3) $116,226,146.80
4) $1,743,392,200.00

23 If the function \( g(x) = ab^x \) represents exponential growth, which statement about \( g(x) \) is false?

1) \( a > 0 \) and \( b > 1 \)
2) The \( y \)-intercept is \((0, a)\).
3) The asymptote is \( y = 0 \).
4) The \( x \)-intercept is \((b, 0)\).

24 At her job, Pat earns $25,000 the first year and receives a raise of $1000 each year. The explicit formula for the \( n \)th term of this sequence is \( a_n = 25,000 + (n - 1)1000 \). Which rule best represents the equivalent recursive formula?

1) \( a_1 = 24,000 + 1000n \)
2) \( a_n = 25,000 + 1000n \)
3) \( a_1 = 25,000, a_n = a_{n-1} + 1000 \)
4) \( a_1 = 25,000, a_n = a_{n-1} + 1000 \)

25 Elizabeth tried to find the product of \((2 + 4i)\) and \((3 - i)\), and her work is shown below.

\[(2 + 4i)(3 - i) = 6 - 2i + 12i - 4i^2\]
\[= 6 + 10i - 4(1)\]
\[= 6 + 10i - 4\]
\[= 2 + 10i\]

Identify the error in the process shown and determine the correct product of \((2 + 4i)\) and \((3 - i)\).

26 A runner is using a nine-week training app to prepare for a "fun run." The table below represents the amount of the program completed, \( A \), and the distance covered in a session, \( D \), in miles.

<table>
<thead>
<tr>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Based on these data, write an exponential regression equation, rounded to the nearest thousandth, to model the distance the runner is able to complete in a session as she continues through the nine-week program.
27 A formula for work problems involving two people is shown below.
\[ \frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b} \]
- \( t_1 \) = the time taken by the first person to complete the job
- \( t_2 \) = the time taken by the second person to complete the job
- \( t_b \) = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk. Determine, to the nearest tenth of an hour, how long it would take Fred and Barney working together to build a desk.

28 Completely factor the following expression:
\[ x^2 + 3xy + 3x^3 + y \]

29 Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5% per month. The fox population had an initial value of 30 and grew continuously at the rate of 3% per month. Find, to the nearest tenth of a month, how long it takes for these populations to be equal.

30 Consider the function \( h(x) = 2\sin(3x) + 1 \) and the function \( q \) represented in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( q(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-8</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Determine which function has the smaller minimum value for the domain \([-2, 2]\). Justify your answer.
31 The zeros of a quartic polynomial function \( h \) are \(-1, \pm 2, \text{ and } 3\). Sketch a graph of \( y = h(x) \) on the grid below.

32 Explain why \( \frac{81}{4} \) equals 27.

33 Given: \( f(x) = 2x^2 + x - 3 \) and \( g(x) = x - 1 \)
Express \( f(x) \cdot g(x) - [f(x) + g(x)] \) as a polynomial in standard form.

34 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is \( \frac{1}{4} \). The probability that the student chosen is a junior is \( \frac{116}{459} \). If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is \( \frac{47}{108} \), what is the exact probability that the student selected is a junior whose favorite subject is Math? Are the events "the student is a junior" and "the student's favorite subject is Math" independent of each other? Explain your answer.

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.

a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the nearest integer.
b) A study claims "50 percent of men 21 and older in the United States are married." Do your results from part a contradict this claim? Explain.
36 The graph of \( y = f(x) \) is shown below. The function has a leading coefficient of 1.

Write an equation for \( f(x) \). The function \( g \) is formed by translating function \( f \) left 2 units. Write an equation for \( g(x) \).

37 The resting blood pressure of an adult patient can be modeled by the function \( P \) below, where \( P(t) \) is the pressure in millimeters of mercury after time \( t \) in seconds.

\[
P(t) = 24 \cos(3\pi t) + 120
\]

On the set of axes below, graph \( y = P(t) \) over the domain \( 0 \leq t \leq 2 \).

Determine the period of \( P \). Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.
1 The graphs of the equations $y = x^2 + 4x - 1$ and $y + 3 = x$ are drawn on the same set of axes. One solution of this system is

1) $(-5, -2)$
2) $(-1, -4)$
3) $(1, 4)$
4) $(-2, -1)$

2 Which statement is true about the graph of $f(x) = \left( \frac{1}{8} \right)^x$?

1) The graph is always increasing.
2) The graph is always decreasing.
3) The graph passes through $(1,0)$.
4) The graph has an asymptote, $x = 0$.

3 For all values of $x$ for which the expression is defined, $\frac{x^3 + 2x^2 - 9x - 18}{x^3 - x^2 - 6x}$, in simplest form, is equivalent to

1) $3$
2) $\frac{17}{2}$
3) $\frac{x + 3}{x}$
4) $\frac{x^2 - 9}{x(x - 3)}$

4 A scatterplot showing the weight, $w$, in grams, of each crystal after growing $t$ hours is shown below.

The relationship between weight, $w$, and time, $t$, is best modeled by

1) $w = 4t + 5$
2) $w = (1.4)^t + 2$
3) $w = 5(2.1)^t$
4) $w = 8(.75)^t$

5 Where $i$ is the imaginary unit, the expression $(x + 3i)^2 - (2x - 3i)^2$ is equivalent to

1) $-3x^2$
2) $-3x^2 - 18$
3) $-3x^2 - 18xi$
4) $-3x^2 - 6xi - 18$

6 Which function is even?

1) $f(x) = \sin x$
2) $f(x) = x^2 - 4$
3) $f(x) = |x - 2| + 5$
4) $f(x) = x^4 + 3x^3 + 4$
7. The function \( N(t) = 100e^{-0.023t} \) models the number of grams in a sample of cesium-137 that remain after \( t \) years. On which interval is the sample's average rate of decay the fastest?
1) \([1,10]\)
2) \([10,20]\)
3) \([15,25]\)
4) \([1,30]\)

8. Which expression can be rewritten as \((x + 7)(x - 1)\)?
1) \((x + 3)^2 - 16\)
2) \((x + 3)^2 - 10(x + 3) - 2(x + 3) + 20\)
3) \(\frac{(x - 1)(x^2 - 6x - 7)}{(x + 1)}\)
4) \(\frac{(x + 7)(x^2 + 4x + 3)}{(x + 3)}\)

9. What is the solution set of the equation \(\frac{2}{x} - \frac{3x}{x + 3} = \frac{x}{x + 3}\)?
1) \{3\}
2) \[\frac{3}{2}\]
3) \{-2, 3\}
4) \[-1, \frac{3}{2}\]

10. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.

If the depth, \(d\), is measured in feet and time, \(t\), is measured in hours since midnight, what is an equation for the depth of the water at the marker?
1) \(d = 5 \cos \left(\frac{\pi}{6} t\right) + 9\)
2) \(d = 9 \cos \left(\frac{\pi}{6} t\right) + 5\)
3) \(d = 9 \sin \left(\frac{\pi}{6} t\right) + 5\)
4) \(d = 5 \sin \left(\frac{\pi}{6} t\right) + 9\)

11. On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?
1) 73%
2) 36%
3) 23%
4) 12%
12 If \(x - 1\) is a factor of \(x^3 - kx^2 + 2x\), what is the value of \(k\)?
1) 0
2) 2
3) 3
4) \(-3\)

13 The profit function, \(p(x)\), for a company is the cost function, \(c(x)\), subtracted from the revenue function, \(r(x)\). The profit function for the Acme Corporation is \(p(x) = -0.5x^2 + 250x - 300\) and the revenue function is \(r(x) = -0.3x^2 + 150x\). The cost function for the Acme Corporation is
1) \(c(x) = 0.2x^2 - 100x + 300\)
2) \(c(x) = 0.2x^2 + 100x + 300\)
3) \(c(x) = -0.2x^2 + 100x - 300\)
4) \(c(x) = -0.8x^2 + 400x - 300\)

14 The populations of two small towns at the beginning of 2018 and their annual population growth rate are shown in the table below.

<table>
<thead>
<tr>
<th>Town</th>
<th>Population</th>
<th>Annual Population Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jonesville</td>
<td>1240</td>
<td>6% increase</td>
</tr>
<tr>
<td>Williamstown</td>
<td>890</td>
<td>11% increase</td>
</tr>
</tbody>
</table>

Assuming the trend continues, approximately how many years after the beginning of 2018 will it take for the populations to be equal?
1) 7
2) 20
3) 68
4) 125

15 What is the inverse of \(f(x) = x^3 - 2\)?
1) \(f^{-1}(x) = \sqrt[3]{x} + 2\)
2) \(f^{-1}(x) = \pm \sqrt[3]{x} + 2\)
3) \(f^{-1}(x) = \sqrt[3]{x} + 2\)
4) \(f^{-1}(x) = \pm \sqrt[3]{x} + 2\)
16 A 4th degree polynomial has zeros \(-5, 3, i, \) and \(-i\). Which graph could represent the function defined by this polynomial?

17 The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

1) 0.2257
2) 0.2743
3) 0.7257
4) 0.7757

18 The half-life of iodine-131 is 8 days. The percent of the isotope left in the body \(d\) days after being introduced is \(I = 100 \left( \frac{1}{2} \right)^{\frac{d}{8}}\). When this equation is written in terms of the number \(e\), the base of the natural logarithm, it is equivalent to \(I = 100e^{kd}\). What is the approximate value of the constant, \(k\)?

1) \(-0.087\)
2) 0.087
3) \(-11.542\)
4) 11.542

19 The graph of \(y = \log_2 x\) is translated to the right 1 unit and down 1 unit. The coordinates of the \(x\)-intercept of the translated graph are

1) \((0,0)\)
2) \((1,0)\)
3) \((2,0)\)
4) \((3,0)\)
20 For positive values of $x$, which expression is equivalent to $\sqrt[3]{16x^2} \cdot x^{2/3} + \sqrt[3]{8x^5}$

1) $6\sqrt[3]{x^5}$
2) $6\sqrt[3]{x^3}$
3) $4\sqrt[3]{x^2} + 2\sqrt[3]{x^5}$
4) $4\sqrt[3]{x^3} + 2\sqrt[3]{x^3}$

21 Which equation represents a parabola with a focus of $(-2,5)$ and a directrix of $y = 9$?

1) $(y - 7)^2 = 8(x + 2)$
2) $(y - 7)^2 = -8(x + 2)$
3) $(x + 2)^2 = 8(y - 7)$
4) $(x + 2)^2 = -8(y - 7)$

22 Given the following polynomials

- $x = (a + b + c)^2$
- $y = a^2 + b^2 + c^2$
- $z = ab + bc + ac$

Which identity is true?

1) $x = y - z$
2) $x = y + z$
3) $x = y - 2z$
4) $x = y + 2z$

23 On average, college seniors graduating in 2012 could compute their growing student loan debt using the function $D(t) = 29,400(1.068)^t$, where $t$ is time in years. Which expression is equivalent to $29,400(1.068)^t$ and could be used by students to identify an approximate daily interest rate on their loans?

1) $29,400 \left( \frac{1.068}{365} \right)^t$
2) $29,400 \left( \frac{1.068}{365} \right)^{365t}$
3) $29,400 \left( 1 + \frac{0.068}{365} \right)^t$
4) $29,400 \left( \frac{1}{365} \right)^{365t}$

24 A manufacturing plant produces two different-sized containers of peanuts. One container weighs $x$ ounces and the other weighs $y$ pounds. If a gift set can hold one of each size container, which expression represents the number of gift sets needed to hold 124 ounces?

1) $\frac{124}{16x + y}$
2) $\frac{x + 16y}{124}$
3) $\frac{124}{x + 16y}$
4) $\frac{16x + y}{124}$
25 A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Favorite Type of Program</th>
<th>Sports</th>
<th>Reality Show</th>
<th>Comedy Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior</td>
<td>83</td>
<td>110</td>
<td>67</td>
</tr>
<tr>
<td>Freshmen</td>
<td>119</td>
<td>103</td>
<td>54</td>
</tr>
</tbody>
</table>

A student response is selected at random from the results. State the exact probability the student response is from a freshman, given the student prefers to watch reality shows on television.

26 On the grid below, graph the function $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \leq x \leq 4$.

27 Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

28 Chuck's Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.

The driver who receives the highest number of positive comments will win the recognition. Explain one statistical bias in this data collection method.

29 Determine the quotient and remainder when $(6a^3 + 11a^2 - 4a - 9)$ is divided by $(3a - 2)$. Express your answer in the form $q(a) + \frac{r(a)}{d(a)}$. 
30 The recursive formula to describe a sequence is shown below.

\[ a_1 = 3 \]

\[ a_n = 1 + 2a_{n-1} \]

State the first four terms of this sequence. Can this sequence be represented using an explicit geometric formula? Justify your answer.

31 The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is $152,500 and they will make a $15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the nearest dollar.

\[
M = P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n \left( 1 + \frac{r}{12} \right)^n - 1
\]

- \(M\) = monthly payment
- \(P\) = amount borrowed
- \(r\) = annual interest rate
- \(n\) = total number of monthly payments

32 An angle, \(\theta\), is in standard position and its terminal side passes through the point \((2, -1)\). Find the exact value of \(\sin \theta\).

33 Solve algebraically for all values of \(x\):

\[
\sqrt{6 - 2x} + x = 2(x + 15) - 9
\]
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

<table>
<thead>
<tr>
<th></th>
<th>Scented Paper</th>
<th>Unscented Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{x}$</td>
<td>23</td>
<td>18</td>
</tr>
<tr>
<td>$s_x$</td>
<td>2.898</td>
<td>2.408</td>
</tr>
</tbody>
</table>

Calculate the difference in means in the experimental test grades (scented - unscented). A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth. Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.
35 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \( C(t) \), that represents the amount of money in the account \( t \) years after the account is opened, given that no more money is deposited into or withdrawn from the account. Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

36 The height, \( h(t) \) in cm, of a piston, is given by the equation \( h(t) = 12 \cos \left( \frac{\pi}{3} t \right) + 8 \), where \( t \) represents the number of seconds since the measurements began. Determine the average rate of change, in cm/sec, of the piston's height on the interval \( 1 \leq t \leq 2 \). At what value(s) of \( t \), to the nearest tenth of a second, does \( h(t) = 0 \) in the interval \( 1 \leq t \leq 5 \)? Justify your answer.

37 Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website's popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth? Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2} x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?
1. The solution of $87e^{0.3x} = 5918$, to the nearest thousandth, is
   1) 0.583
   2) 1.945
   3) 4.220
   4) 14.066

2. A researcher randomly divides 50 bean plants into two groups. He puts one group by a window to receive natural light and the second group under artificial light. He records the growth of the plants weekly. Which data collection method is described in this situation?
   1) observational study
   2) controlled experiment
   3) survey
   4) systematic sample

3. If $f(x) = x^2 + 9$ and $g(x) = x + 3$, which operation would not result in a polynomial expression?
   1) $f(x) + g(x)$
   2) $f(x) - g(x)$
   3) $f(x) \cdot g(x)$
   4) $f(x) ÷ g(x)$

4. Consider the function $p(x) = 3x^3 + x^2 - 5x$ and the graph of $y = m(x)$ below.

Which statement is true?
   1) $p(x)$ has three real roots and $m(x)$ has two real roots.
   2) $p(x)$ has one real root and $m(x)$ has two real roots.
   3) $p(x)$ has two real roots and $m(x)$ has three real roots.
   4) $p(x)$ has three real roots and $m(x)$ has four real roots.

5. Which expression is equivalent to $\frac{2x^4 + 8x^3 - 25x^2 - 6x + 14}{x + 6}$?
   1) $2x^3 + 4x^2 + x - 12 + \frac{86}{x + 6}$
   2) $2x^3 - 4x^2 - x + 14$
   3) $2x^3 - 4x^2 - x + \frac{14}{x + 6}$
   4) $2x^3 - 4x^2 - x$
6 Given \( f(x) = \frac{1}{2}x + 8 \), which equation represents the inverse, \( g(x) \)?

1) \( g(x) = 2x - 8 \)
2) \( g(x) = 2x - 16 \)
3) \( g(x) = \frac{1}{2}x + 8 \)
4) \( g(x) = \frac{1}{2}x - 16 \)

7 The value(s) of \( x \) that satisfy \( \sqrt{x^2 - 4x - 5} = 2x - 10 \) are

1) \( \{5\} \)
2) \( \{7\} \)
3) \( \{5,7\} \)
4) \( \{3,5,7\} \)

8 Stephanie found that the number of white-winged cross bills in an area can be represented by the formula \( C = 550(1.08)^t \), where \( t \) represents the number of years since 2010. Which equation correctly represents the number of white-winged cross bills in terms of the monthly rate of population growth?

1) \( C = 550(1.00643)^t \)
2) \( C = 550(1.00643)^{12t} \)
3) \( C = 550(1.00643)^{\frac{t}{12}} \)
4) \( C = 550(1.00643)^{t+12} \)

9 The roots of the equation \( 3x^2 + 2x = -7 \) are

1) \( -2, -\frac{1}{3} \)
2) \( -\frac{7}{3}, 1 \)
3) \( \frac{1}{3} \pm \frac{2i\sqrt{5}}{3} \)
4) \( \frac{1}{3} \pm \frac{\sqrt{11}}{3} \)

10 The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of \$75,000, which model is a recursive formula representing the value of the boat \( n \) years after it was purchased?

1) \( a_n = 75,000(0.08)^n \)
2) \( a_0 = 75,000 \\
\quad a_n = (0.92)^n \)
3) \( a_n = 75,000(1.08)^n \)
4) \( a_0 = 75,000 \\
\quad a_n = 0.92(a_{n-1}) \)

11 Given \( \cos \theta = \frac{7}{25} \), where \( \theta \) is an angle in standard position terminating in quadrant IV, and \( \sin^2 \theta + \cos^2 \theta = 1 \), what is the value of \( \tan \theta \)?

1) \( -\frac{24}{25} \)
2) \( -\frac{24}{7} \)
3) \( -\frac{24}{25} \)
4) \( -\frac{24}{7} \)
12 For \( x > 0 \), which expression is equivalent to 
\[
\frac{3\sqrt{x^2} \cdot \sqrt{x^5}}{6\sqrt{x}}
\]
1) \( x \)
2) \( x^{\frac{3}{2}} \)
3) \( x^3 \)
4) \( x^{10} \)

13 Jake wants to buy a car and hopes to save at least \$5000 for a down payment. The table below summarizes the amount of money he plans to save each week.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Saved, in Dollars</td>
<td>2</td>
<td>5</td>
<td>12.5</td>
<td>31.25</td>
<td>...</td>
</tr>
</tbody>
</table>

Based on this plan, which expression should he use to determine how much he has saved in \( n \) weeks?

1) \( 2 - 2(2.5^n) \)
2) \( 2 - 2(2.5^{n-1}) \)
3) \( \frac{1 - 2.5^n}{1 - 2.5} \)
4) \( \frac{1 - 2.5^{n-1}}{1 - 2.5} \)

14 Which expression is equivalent to
\[ x^6y^4(x^4 - 16) - 9(x^4 - 16) \]
1) \( x^{10}y^4 - 16x^6y^4 - 9x^4 - 144 \)
2) \( (x^6y^4 - 9)(x + 2)^2(x - 2) \)
3) \( (x^3y^2 + 3)(x^3y^2 - 3)(x + 2)^2(x - 2)^2 \)
4) \( (x^3y^2 + 3)(x^3y^2 - 3)(x^2 + 4)(x^2 - 4) \)

15 If \( A = -3 + 5i \), \( B = 4 - 2i \), and \( C = 1 + 6i \), where \( i \) is the imaginary unit, then \( A - BC \) equals
1) \( 5 - 17i \)
2) \( 5 + 27i \)
3) \( -19 - 17i \)
4) \( -19 + 27i \)
16. Which sketch best represents the graph of \( x = 3^y \)?

1) 

2) 

3) 

4) 

17. The graph below represents national and New York State average gas prices.

If New York State's gas prices are modeled by \( G(x) \) and \( C > 0 \), which expression best approximates the national average \( x \) months from August 2014?
1) \( G(x + C) \)
2) \( G(x) + C \)
3) \( G(x - C) \)
4) \( G(x) - C \)
18 Data for the students enrolled in a local high school are shown in the Venn diagram below.

![Venn Diagram](image)

If a student from the high school is selected at random, what is the probability that the student is a sophomore given that the student is enrolled in Algebra II?

1) \( \frac{85}{210} \)
2) \( \frac{85}{295} \)
3) \( \frac{85}{405} \)
4) \( \frac{85}{1600} \)

19 If \( p(x) = 2 \ln(x) - 1 \) and \( m(x) = \ln(x + 6) \), then what is the solution for \( p(x) = m(x) \)?

1) 1.65
2) 3.14
3) 5.62
4) no solution

20 Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

1) \( y = -4 \cos \left( \frac{\pi}{4} x \right) - 3 \)
2) \( y = -4 \cos \left( \frac{\pi}{4} x \right) + 5 \)
3) \( y = -4 \cos(8x) - 3 \)
4) \( y = -4 \cos(8x) + 5 \)

21 Given \( c(m) = m^3 - 2m^2 + 4m - 8 \), the solution of \( c(m) = 0 \) is

1) \( \pm 2 \)
2) 2, only
3) \( 2i, 2 \)
4) \( \pm 2i, 2 \)

22 The height above ground for a person riding a Ferris wheel after \( t \) seconds is modeled by

\[ h(t) = 150 \sin \left( \frac{\pi}{45} t + 67.5 \right) + 160 \text{ feet} \]

How many seconds does it take to go from the bottom of the wheel to the top of the wheel?

1) 10
2) 45
3) 90
4) 150

23 The parabola described by the equation \( y = \frac{1}{12} (x - 2)^2 + 2 \) has the directrix at \( y = -1 \). The focus of the parabola is

1) \( (2, -1) \)
2) \( (2, 2) \)
3) \( (2, 3) \)
4) \( (2, 5) \)
24 A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events $D$, that a customer uses the drive-thru, and $F$, that a customer orders French fries, are independent. The following data are given in a report:

$$P(F) = 0.8$$

$$P(F \cap D) = 0.456$$

Given this information, $P(F \mid D)$ is

1) 0.344
2) 0.3648
3) 0.57
4) 0.8

25 Over the set of integers, factor the expression $x^4 - 4x^2 - 12$.

26 Express the fraction $\frac{2x^\frac{3}{2}}{\left(16x^4\right)^\frac{1}{4}}$ in simplest radical form.

27 The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function $p(t) = 2560e^{0.017185t}$, where $t$ is time in years after 1950 and $p(t)$ is the population in millions. Determine the average rate of change of $p(t)$ in millions of people per year, from $4 \leq t \leq 8$. Round your answer to the nearest hundredth.

28 The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

29 Algebraically solve for $x$: \[
\frac{-3}{x + 3} + \frac{1}{2} = \frac{x}{6} - \frac{1}{2}
\]

30 Graph $t(x) = 3 \sin(2x) + 2$ over the domain $[0, 2\pi]$ on the set of axes below.

31 Solve the following system of equations algebraically. $x^2 + y^2 = 400$

$$y = x - 28$$
32. Some smart-phone applications contain "in-app" purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.

Considering the middle 95% of the data, determine the margin of error, to the nearest hundredth, for the simulated results. In the given context, explain what this value represents.

33. Solve the following system of equations algebraically for all values of \(x, y,\) and \(z\):
\[
\begin{align*}
2x + 3y - 4z &= -1 \\
x - 2y + 5z &= 3 \\
-4x + y + z &= 16
\end{align*}
\]

34. Evaluate \(j(-1)\) given
\[
j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48.
\]
Explain what your answer tells you about \(x + 1\) as a factor. Algebraically find the remaining zeros of \(j(x)\).

35. Determine, to the nearest tenth of a year, how long it would take an investment to double at a 3\(\frac{3}{4}\)\% interest rate, compounded continuously.
36 To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

Classical: 74, 83, 77, 77, 84, 82, 90, 89
Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer. To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.

Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.
37 A major car company analyzes its revenue, $R(x)$, and costs $C(x)$, in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, $x$, using the given functions.

\[
R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000 \\
C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000
\]

The company's profits can be represented as the difference between its revenue and costs. Write the profit function, $P(x)$, as a polynomial in standard form. Graph $y = P(x)$ on the set of axes below over the domain $2 \leq x \leq 16$.

Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.
2015 Algebra II Common Core State Standards Sample Items
Answer Section

1 ANS: 1
The zeros of the polynomial are at \(-b\), and \(c\). The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as \(x\) goes to negative infinity, \(f(x)\) goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

PTS: 2 REF: spr1501aii NAT: F.IF.C.7 TOP: Graphing Polynomial Functions
KEY: bimodalgraph

2 ANS: 4

A parabola with a focus of \((0,4)\) and a directrix of \(y=2\) is sketched as follows: 

By inspection, it is determined that the vertex of the parabola is \((0,3)\). It is also evident that the distance, \(p\), between the vertex and the focus is 1. It is possible to use the formula \((x-h)^2 = 4p(y-k)\) to derive the equation of the parabola as follows: 

\[
x^2 = 4y - 12
\]

\[
x^2 + 12 = 4y
\]

\[
x^2 \quad + 3 = y
\]

or A point \((x,y)\) on the parabola must be the same distance from the focus as it is from the directrix. For any such point \((x,y)\), the distance to the focus is \(\sqrt{(x-0)^2 + (y-4)^2}\) and the distance to the directrix is \(y-2\). Setting this equal leads to: 

\[
x^2 + y^2 - 8y + 16 = y^2 - 4y + 4
\]

\[
x^2 + 16 = 4y + 4
\]

\[
x^2 \quad + 3 = y
\]

PTS: 2 REF: spr1502aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

3 ANS: 1
A reference triangle can be sketched using the coordinates \((-4,3)\) in the second quadrant to find the value of \(\sin \theta\).

PTS: 2 REF: spr1503aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions
KEY: extension to reals
4  ANS: 2

\[ B(t) = 750 \left( 1.16 \right)^{\frac{12t}{12}} \approx 750(1.012)^{12t} \]

\[ B(t) = 750 \left( 1 + \frac{0.16}{12} \right)^{12t} \]

is wrong, because the growth is an annual rate that is not compounded monthly.

PTS: 2  REF: spr1504a11  NAT: A.SSE.B.3  TOP: Modeling Exponential Functions

KEY: AII

5  ANS:

\[
\frac{\frac{8}{3}}{\frac{4}{3}} = x^y
\]

\[
\frac{4}{\frac{3}{3}} = x^y
\]

\[
\frac{4}{3} = y
\]

PTS: 2  REF: spr1505a11  NAT: N.RN.A.2  TOP: Radicals and Rational Exponents

KEY: numbers

6  ANS:

\[
(4 - 3i)(5 + 2yi - 5 + 2yi)
\]

\[
(4 - 3i)(4yi)
\]

\[
16yi - 12yi^2
\]

\[
12y + 16yi
\]

PTS: 2  REF: spr1506a11  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

7  ANS:

\[
f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 128 - 80 - 44 - 4 = 0
\]

Any method that demonstrates 4 is a zero of \( f(x) \) confirms that \( x - 4 \) is a factor, as suggested by the Remainder Theorem.

PTS: 2  REF: spr1507a11  NAT: A.APR.B.2  TOP: Remainder Theorem
8. ANS:

\[ \sqrt{x - 5} = -x + 7 \quad \sqrt{x - 5} = -9 + 7 = -2 \text{ is extraneous.} \]

\[ x - 5 = x^2 - 14x + 49 \]
\[ 0 = x^2 - 15x + 54 \]
\[ 0 = (x - 6)(x - 9) \]
\[ x = 6, 9 \]

PTS: 2  REF: spr1508aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: extraneous solutions

9. ANS:

\[ 720 = \frac{120000 \left( \frac{-0.048}{12} \right) \left( 1 + \frac{0.048}{12} \right)^n}{\left( 1 + \frac{0.048}{12} \right)^n - 1} \]
\[ \frac{275.2}{12} \approx 23 \text{ years} \]

\[ 720(1.004)^n - 720 = 480(1.004)^n \]
\[ 240(1.004)^n = 720 \]
\[ 1.004^n = 3 \]
\[ n \log 1.004 = \log 3 \]
\[ n \approx 275.2 \text{ months} \]

PTS: 4  REF: spr1509aii  NAT: A.CED.A.1  TOP: Exponential Growth
10 ANS:

\[
\begin{align*}
6x - 3y + 2z &= -10 \\
-2x + 3y + 8z &= 72 \\
4x + 10z &= 62 \\
4x + 4(7) &= 20 \\
x &= -8 \\
4x + 4z &= 20 \\
z &= 7
\end{align*}
\]

\[
6(-2) - 3y + 2(7) = -10
\]

\[
-3y = -12
\]

\[
y = 4
\]

PTS: 4  
REF: spr1510a1ii  
NAT: A.REI.C.6  
TOP: Solving Linear Systems  
KEY: three variables

11 ANS:

\[
a_n = x^{n-1}(x + 1) = x^{n-1} = 0 \\
x + 1 = 0
\]

\[
x = 0 \\
x = -1
\]

PTS: 4  
REF: spr1511a1ii  
NAT: F.BF.A.2  
TOP: Sequences

12 ANS:

Yes. The margin of error from this simulation indicates that 95% of the observations fall within ± 0.12 of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to be 0.06 in the dotplot, by 2, or applying the estimated standard error formula, 

\[
\sqrt{\frac{p(1-p)}{n}} \text{ or } \sqrt{\frac{(0.25)(0.75)}{50}}
\]

and multiplying by 2. The interval 0.25 ± 0.12 includes plausible values for the true proportion of people who prefer Stephen’s new product. The company has evidence that the population proportion could be at least 25%. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 (9 out of 50) or less several times, even when the population proportion is 0.25, due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

PTS: 4  
REF: spr1512a1ii  
NAT: S.IC.B.4  
TOP: Analysis of Data
This scenario can be modeled with a Venn Diagram:

Since

\[ P(S \cup I) = P(S) + P(I) - P(S \cap I) \]

If \( S \) and \( I \) are independent, then the

\[ = 0.5 + 0.7 - 0.8 \]

\[ = 0.4 \]

Product Rule must be satisfied. However, \((0.5)(0.7) \neq 0.4\). Therefore, salary and insurance have not been treated independently.

PTS: 4   REF: spr1513aii   NAT: S.CP.A.2   TOP: Probability of Compound Events

KEY: independence

The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of \(-12\) and a maximum of 12. The value of \( A \) is \(-12\) since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter \( B \). By experimentation with technology or using the relation \( P = \frac{2\pi}{B} \) (where \( P \) is the period), it is determined that \( B = \frac{2\pi}{13} \).

\[ f(t) = -12 \cos \left( \frac{2\pi}{13} t \right) \]

In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from \( t = 13 \) to \( t = 19.5 \) (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.

PTS: 6   REF: spr1514aii   NAT: F.IF.C.7   TOP: Graphing Trigonometric Functions

KEY: graph
\[ x(x+7) \left[ \frac{3x+25}{x+7} - 5 \right] = \frac{3}{x} \]

\[ x(3x+25) - 5x(x+7) = 3(x+7) \]
\[ 3x^2 + 25x - 5x^2 - 35x = 3x + 21 \]
\[ 2x^2 + 13x + 21 = 0 \]
\[ (2x+7)(x+3) = 0 \]
\[ x = -\frac{7}{2}, -3 \]

PTS: 2  REF: fall1501aii  NAT: A.REI.A.2  TOP: Solving Rationals
KEY: rational solutions

16 ANS: 3
\[ f(x) = -f(x), \text{ so } f(x) \text{ is odd. } g(-x) \neq g(x), \text{ so } g(x) \text{ is not even. } g(-x) \neq -g(x), \text{ so } g(x) \text{ is not odd. } h(-x) = h(x), \text{ so } h(x) \text{ is even.} \]

PTS: 2  REF: fall1502aii  NAT: F.BF.B.3  TOP: Even and Odd Functions

17 ANS: 1
\[ \frac{3x^2 + 4x - 1}{2x^3 + 36x^2 + 17x + 10} = \frac{8x^2 + 16x}{8x^2 + 12x} - \frac{2x+2}{2x-3} \]
\[ \frac{8x^2 + 16x}{8x^2 + 12x} - \frac{2x+2}{2x-3} \]

PTS: 2  REF: fall1503aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: division
18. \( \text{ANS: 3} \)

\[
-2 \left( \frac{1}{2}x^2 = -6x + 20 \right) \]

\[
x^2 - 12x = -40 \]

\[
x^2 - 12x + 36 = -40 + 36 \]

\[
(x - 6)^2 = -4 \]

\[
x - 6 = \pm 2i \]

\[
x = 6 \pm 2i \]

PTS: 2  REF: fall1504aii  NAT: A.REI.B.4  TOP: Solving Quadratics  
KEY: complex solutions | completing the square

19. \( \text{ANS: 4} \)

\[
k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48 \]

\[
k^2(k^2 - 4) + 8k(k^2 - 4) + 12(k^2 - 4) \]

\[
(k^2 - 4)(k^2 + 8k + 12) \]

\[
(k + 2)(k - 2)(k + 6)(k + 2) \]

PTS: 2  REF: fall1505aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials 
KEY: factoring by grouping

20. \( \text{ANS: 4} \)

As the range is \([4,10]\), the midline is \( y = \frac{4 + 10}{2} = 7 \).

PTS: 2  REF: fall1506aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions 
KEY: mixed
21 ANS:
\[-2x + 1 = -2x^2 + 3x + 1\]
\[2x^2 - 5x = 0\]
\[x(2x - 5) = 0\]
\[x = 0, \frac{5}{2}\]

PTS: 2  REF: fall1507a1i  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems  
KEY: AII

22 ANS:
Based on these data, the two events do not appear to be independent. \(P(F) = \frac{106}{200} = 0.53\), while \(P(F|T) = \frac{54}{90} = 0.6\), \(P(F|R) = \frac{25}{65} = 0.39\), and \(P(F|C) = \frac{27}{45} = 0.6\). The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.

PTS: 2  REF: fall1508a1i  NAT: S.CP.A.4  TOP: Conditional Probability

23 ANS:
\[x = (y - 3)^3 + 1\]
\[x - 1 = (y - 3)^3\]
\[\sqrt[3]{x - 1} = y - 3\]
\[\sqrt[3]{x - 1} + 3 = y\]
\[f^{-1}(x) = \sqrt[3]{x - 1} + 3\]

PTS: 2  REF: fall1509a1i  NAT: F.BF.B.4  TOP: Inverse of Functions  
KEY: other
24 ANS:

Let $x$ equal the first integer and $x + 1$ equal the next. $(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$. $2x + 1$ is an odd integer.

PTS: 2 REF: fall1510a1ii NAT: A.REI.D.11 TOP: Other Systems KEY: AII

25 ANS:

The expression is of the form $y^2 - 5y - 6$ or $(y - 6)(y + 1)$. Let $y = 4x^2 + 5x$:

$$(4x^2 + 5x - 6)(4x^2 + 5x + 1)$$

$(4x - 3)(x + 2)(4x + 1)(x + 1)$

PTS: 2 REF: fall1511a1ii NAT: A.APR.C.4 TOP: Polynomial Identities KEY: a>1

26 ANS:

$$100 = 325 + (68 - 325)e^{-2k} \quad T = 325 - 257e^{-0.066t}$$

$$-225 = -257e^{-2k} \quad T = 325 - 257e^{-0.066(7)} \approx 163$$

$$k = \frac{\ln\left(\frac{-225}{-257}\right)}{-2}$$

$k \approx 0.066$

PTS: 4 REF: fall1513a1ii NAT: F.LE.A.4 TOP: Exponential Growth
The mean difference between the students’ final grades in group 1 and group 2 is –3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription. One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students’ final grades many (500) times. If the observed difference –3.64 is the result of the assignment of students to groups alone, then a difference of –3.64 or less should be observed fairly regularly in the simulation output. However, a difference of –3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups’ mean final grades.

\[
0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 \quad z(x) = 6x^3 + 19x^2 - 52x + 15 \\
0 = -750 + 25b + 260 + 15 \\
475 = 25b \\
19 = b
\]

\[
\begin{array}{cccc}
-11 & 3 & 0 \\
6 & -30 & 15 \\
\end{array}
\]

\[
6x^2 - 11x + 3 = 0 \\
(2x-3)(3x-1) = 0
\]

\[
x = \frac{3}{2}, \frac{1}{3}, -5
\]

\[
\text{normcdf}(510, 540, 480, 24) = 0.0994 \\
z = \frac{510 - 480}{24} = 1.25 \quad 1.25 = \frac{x - 510}{20} \quad 2.5 = \frac{x - 510}{20} \quad 535-560 \\
z = \frac{540 - 480}{24} = 2.5 \quad x = 535 \quad x = 560
\]

\[
A(t) = 100(0.5)^{\frac{t}{63}}, \text{ where } t \text{ is time in years, and } A(t) \text{ is the amount of titanium-44 left after } t \text{ years.} \\
A(10) - A(0) = \frac{89.58132 - 100}{10} = -1.041868 \quad \text{The estimated mass at } t = 40 \text{ is } 100 - 40(-1.041868) \approx 58.3. \quad \text{The actual mass is } A(40) = 100(0.5)^{\frac{40}{63}} \approx 64.3976. \quad \text{The estimated mass is less than the actual mass.}
\]
0616AII Common Core State Standards
Answer Section

1. ANS: 4  
   PTS: 2  
   REF: 061601aii  
   NAT: N.RN.A.2  
   TOP: Radicals and Rational Exponents  
   KEY: variables

2. ANS: 3  
   PTS: 2  
   REF: 061602aii  
   NAT: A.CED.A.1  
   TOP: Modeling Rationals

3. ANS: 2
   
   \[(2 - yi)(2 - yi) = 4 - 4yi + y^2 i^2 = -y^2 - 4yi + 4\]

   PTS: 2  
   REF: 061603aii  
   NAT: N.CN.A.2  
   TOP: Operations with Complex Numbers

4. ANS: 3
   The graph shows three real zeros, and has end behavior matching the given end behavior.

   PTS: 2  
   REF: 061604aii  
   NAT: F.IF.C.7  
   TOP: Graphing Polynomial Functions
   KEY: bimodalgraph

5. ANS: 3

\[\sqrt{56 - x} = x\]
\[-8 \text{ is extraneous.}\]

\[56 - x = x^2\]
\[0 = x^2 + x - 56\]
\[0 = (x + 8)(x - 7)\]
\[x = 7\]

PTS: 2  
REF: 061605aii  
NAT: A.REI.A.2  
TOP: Solving Radicals
KEY: extraneous solutions
6 ANS: 1

\[ x^4 - 4x^3 - 9x^2 + 36x = 0 \]

\[ x^3(x - 4) - 9x(x - 4) = 0 \]

\[ (x^3 - 9x)(x - 4) = 0 \]

\[ x(x^2 - 9)(x - 4) = 0 \]

\[ x(x + 3)(x - 3)(x - 4) = 0 \]

\[ x = 0, \pm 3, 4 \]

PTS: 2 REF: 061606aii NAT: A.APR.B.3 TOP: Zeros of Polynomials

7 ANS: 3 PTS: 2 REF: 061607aii NAT: S.I.C.A.2 TOP: Analysis of Data

8 ANS: 4

\[ \frac{m(c)}{g(c)} = \frac{c + 1}{1 - c^2} = \frac{c + 1}{(1 + c)(1 - c)} = \frac{1}{1 - c} \]

PTS: 2 REF: 061608aii NAT: F.BF.A.1 TOP: Operations with Functions

9 ANS: 2

\[ x + 2\sigma \text{ represents approximately 48\% of the data.} \]

PTS: 2 REF: 061609aii NAT: S.I.D.A.4 TOP: Normal Distributions

10 ANS: 4

The scenario represents a decreasing geometric sequence with a common ratio of 0.80.

PTS: 2 REF: 061610aii NAT: F.BF.A.2 TOP: Sequences

11 ANS: 1

The probability of rain equals the probability of rain, given that Sean pitches.

PTS: 2 REF: 061611aii NAT: S.C.P.A.3 TOP: Conditional Probability
12 ANS: 1

\[ x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)} = \frac{-3 \pm \sqrt{-7}}{4} = \frac{-3}{4} \pm \frac{i\sqrt{7}}{4} \]

PTS: 2  REF: 061612aii  NAT: A.REI.B.4  TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

13 ANS: 3

\[ H(t) \text{ is at a minimum at } 70(-1) + 80 = 10 \]

PTS: 2  REF: 061613aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions
KEY: maximum/minimum

14 ANS: 2

\[
\frac{2x^2 - 3x + 7}{2x + 3} \div \frac{4x^3 + 0x^2 + 5x + 10}{x + 3} \\
\]

\[
\begin{align*}
4x^3 + 6x^2 & \\
-6x^2 + 5x & \\
-6x^2 - 9x & \\
14x + 10 & \\
14x + 21 & \\
-11 & 
\end{align*}
\]

PTS: 2  REF: 061614aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: division

15 ANS: 4

\[ y = 5^{-t} = \left(\frac{1}{5}\right)^t \]

PTS: 2  REF: 061615aii  NAT: F.IF.C.7  TOP: Graphing Exponential Functions
16 \text{ ANS: 2} \begin{align*}
x &= -\frac{3}{4}y + 2 \\
-4x &= 3y - 8 \\
-4x + 8 &= 3y \\
\frac{4}{3}x + \frac{8}{3} &= y
\end{align*}
\text{PTS: 2 REF: 061616aii NAT: F.BF.B.4 TOP: Inverse of Functions}
\text{KEY: linear}

17 \text{ ANS: 1}

\text{PTS: 2 REF: 061617aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions}
\text{KEY: extension to reals}

18 \text{ ANS: 1}

\text{PTS: 2 REF: 061618aii NAT: F.IF.C.7 TOP: Graphing Logarithmic Functions}

19 \text{ ANS: 4}
\begin{align*}
4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) &= 76 + 36 + 324 \\
4(x - 3)^2 + 4(y + 9)^2 &= 436
\end{align*}
\text{PTS: 2 REF: 061619aii NAT: G.GPE.A.1 TOP: Equations of Circles}
\text{KEY: completing the square}

20 \text{ ANS: 2 PTS: 2}
\text{TOP: Graphing Polynomial Functions}
\text{REF: 061620aii NAT: F.IF.B.4}
21 ANS: 3
\[ \frac{1}{1.0525^{12}} \approx 1.00427 \]

PTS: 2  REF: 061621aii  NAT: F.BF.A.1  TOP: Modeling Exponential Functions
KEY: AII

22 ANS: 4

PTS: 2  REF: 061622aii  NAT: A.REI.D.11  TOP: Other Systems
KEY: AII

23 ANS: 3  PTS: 2  REF: 061623aii  NAT: F.BF.A.2  TOP: Sequences

24 ANS: 4

period = \( \frac{2\pi}{B} \)
\[ \frac{1}{60} \cdot \frac{2\pi}{B} \]

\[ B = 120\pi \]

PTS: 2  REF: 061624aii  NAT: F.TF.B.5  TOP: Modeling Trigonometric Functions

25 ANS:
\[ \frac{1}{x} - \frac{1}{3} = -\frac{1}{3x} \]
\[ \frac{3-x}{3x} = -\frac{1}{3x} \]
\[ 3-x = -1 \]
\[ x = 4 \]

PTS: 2  REF: 061625aii  NAT: A.REI.A.2  TOP: Solving Rationals
KEY: rational solutions
26 ANS:
Randomly assign participants to two groups. One group uses the toothpaste with ingredient $X$ and the other group uses the toothpaste without ingredient $X$.

PTS: 2 REF: 061626aii NAT: S.IC.B.3 TOP: Analysis of Data

27 ANS:
\[
\begin{array}{r}
2x^2 + 6x + 23 \\
2x^3 - 4x^2 - 7x - 10
\end{array}
\]
\[
\frac{\begin{array}{r}
2x^3 - 10x^2 \\
6x^2 - 7x \\
6x^2 - 30x \\
23x - 10 \\
23x - 115 \\
105
\end{array}}{x - 5}
\]

Since there is a remainder, $x - 5$ is not a factor.

PTS: 2 REF: 061627aii NAT: A.APR.B.2 TOP: Remainder Theorem

28 ANS:

PTS: 2 REF: 061628aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions

29 ANS:
\[
P(S \cap M) = P(S) + P(M) - P(S \cup M) = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376}
\]

PTS: 2 REF: 061629aii NAT: S.CP.B.7 TOP: Theoretical Probability
30 ANS:

The vertex of the parabola is (4, −3). The x-coordinate of the focus and the vertex is the same. Since the distance from the vertex to the directrix is 3, the distance from the vertex to the focus is 3, so the y-coordinate of the focus is 0. The coordinates of the focus are (4, 0).

PTS: 2 REF: 061630aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

31 ANS:

\[
\frac{x^3 + 9}{x^3 + 8} = \frac{x^3 + 8 + 1}{x^3 + 8}
\]

\[
\frac{x^3 + 9}{x^3 + 8} = \frac{x^3 + 9}{x^3 + 8}
\]

PTS: 2 REF: 061631aii NAT: A.APR.C.4 TOP: Polynomial Identities

32 ANS:

\[ A = Pe^{rt} \]

\[ 135000 = 100000e^{5r} \]

\[ 1.35 = e^{5r} \]

\[ \ln 1.35 = \ln e^{5r} \]

\[ \ln 1.35 = 5r \]

\[ .06 \approx r \text{ or } 6\% \]

PTS: 2 REF: 061632aii NAT: F.LE.A.4 TOP: Exponential Growth
33 ANS:

\[ y = -x + 5 \quad y = -7 + 5 = -2 \]

\[ (x - 3)^2 + (-x + 5 + 2)^2 = 16 \quad y = -3 + 5 = 2 \]

\[ x^2 - 6x + 9 + x^2 - 14x + 49 = 16 \]

\[ 2x^2 - 20x + 42 = 0 \]

\[ x^2 - 10x + 21 = 0 \]

\[ (x - 7)(x - 3) = 0 \]

\[ x = 7, 3 \]


34 ANS:

\[ S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04} \quad S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04} \approx 660778.39 \]

PTS: 4  REF: 061634aii  NAT: A.SSE.B.4  TOP: Series

35 ANS:

\[ 0.602 \pm 0.066 = 0.47 - 0.73. \] Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

PTS: 4  REF: 061635aii  NAT: S.IC.B.5  TOP: Analysis of Data

36 ANS:

\[ \frac{f(4) - f(-2)}{4 - (-2)} = \frac{80 - 1.25}{6} = 13.125 \quad g(x) \text{ has a greater rate of change} \]

\[ \frac{g(4) - g(-2)}{4 - (-2)} = \frac{179 - (-49)}{6} = 38 \]

PTS: 4  REF: 061636aii  NAT: F.IF.C.9  TOP: Comparing Functions  KEY: AII
ANS:  

\[ A(t) = 800e^{-0.347t} \]

\[ B(t) = 400e^{-0.231t} \]

\[ 800e^{-0.347t} = 400e^{-0.231t} \]

\[ 0.15 = e^{-0.347t} \]

\[ \ln 2 - 0.347t = \ln e^{-0.231t} \]

\[ \ln 2 - 0.347t = -0.231t \]

\[ \ln 2 = 0.116t \]

\[ 6 \approx t \]
0816AII Common Core State Standards

Answer Section

1 ANS: 4
If $1 - i$ is one solution, the other is $1 + i$.

$\frac{x}{(1 - i)}(x - (1 + i)) = 0$

$x^2 - x - ix - x + ix + (1 - i^2) = 0$

$x^2 - 2x + 2 = 0$

PTS: 2  REF: 081601aii  NAT: A.REI.B.4  TOP: Complex Conjugate Root Theorem

2 ANS: 1
II. Ninth graders drive to school less often; III. Students know little about adults; IV. Calculus students love math!

PTS: 2  REF: 081602aii  NAT: S.IC.B.3  TOP: Analysis of Data
KEY: bias

3 ANS: 2

PTS: 2  REF: 081603aii  NAT: A.REI.D.11  TOP: Other Systems
KEY: AII

4 ANS: 3

PTS: 2  REF: 081604aii  NAT: S.ID.A.4  TOP: Normal Distributions
KEY: probability

5 ANS: 3

$(m - 2)^2(m + 3) = (m^2 - 4m + 4)(m + 3) = m^3 + 3m^2 - 4m^2 - 12m + 4m + 12 = m^3 - m^2 - 8m + 12$

PTS: 2  REF: 081605aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials
KEY: factoring by grouping
6. ANS: 3

\[-33t^2 + 360t = 700 + 5t\]

\[-33t^2 + 355t - 700 = 0\]

\[t = \frac{-355 \pm \sqrt{355^2 - 4(-33)(-700)}}{2(-33)} \approx 3.8\]

PTS: 2

REF: 081606a

NAT: A.REI.D.11

TOP: Quadratic-Linear Systems

KEY: AII

7. ANS: 1

\[\frac{157}{25 + 47 + 157}\]

PTS: 2

REF: 081607a

NAT: S.CP.A.4

TOP: Conditional Probability

8. ANS: 1

(2) is not recursive

PTS: 2

REF: 081608a

NAT: F.BF.A.2

TOP: Sequences

9. ANS: 1

PTS: 2

REF: 081609a

NAT: F.BF.B.6

TOP: Sigma Notation

KEY: represent

10. ANS: 2

PTS: 2

REF: 081610a

NAT: F.If.B.4

TOP: Graphing Trigonometric Functions

KEY: increasing/decreasing

11. ANS: 2

\[
\frac{x^2 + 0x + 1}{x + 2} \left(\frac{x^3 + 2x^2 + x + 6}{x^3 + 2x^2}\right)
\]

\[
0x^2 + x
\]

\[
0x^2 + 0x
\]

\[
x + 6
\]

\[
x + 2
\]

\[
4
\]

PTS: 2

REF: 081611a

NAT: A.APR.D.6

TOP: Rational Expressions

KEY: division

12. ANS: 2

\[
ME = \left( z \sqrt{\frac{p(1-p)}{n}} \right) = \left( 1.96 \sqrt{\frac{(0.55)(0.45)}{900}} \right) \approx 0.03
\]

PTS: 2

REF: 081612a

NAT: S.IC.B.4

TOP: Analysis of Data
13 ANS: 1
The car lost approximately 19% of its value each year.

PTS: 2 REF: 081613aii NAT: F.LE.B.5 TOP: Modeling Exponential Functions

14 ANS: 1
The graph of \( y = \sin x \) is unchanged when rotated 180º about the origin.

PTS: 2 REF: 081614aii NAT: F.BF.B.3 TOP: Even and Odd Functions

15 ANS: 3
\[ 2d(d^3 + 3d^2 - 9d - 27) \]
\[ 2d(d^2 (d + 3) - 9(d + 3)) \]
\[ 2d(d^3 - 9)(d + 3) \]
\[ 2d(d + 3)(d - 3)(d + 3) \]
\[ 2d(d + 3)^2(d - 3) \]

PTS: 2 REF: 081615aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: factoring by grouping

16 ANS: 1
PTS: 2

17 ANS: 3
\[ \frac{1}{J} = \frac{1}{F} - \frac{1}{W} \]
\[ \frac{1}{J} = \frac{W-F}{FW} \]
\[ J = \frac{FW}{W-F} \]

PTS: 2 REF: 081617aii NAT: A.REI.A.2 TOP: Solving Rationals
KEY: rational solutions

18 ANS: 3
PTS: 2

19 ANS: 4
The vertex is \((2, -1)\) and \(p = 2\).
\[ y = -\frac{1}{4(2)} (x - 2)^2 - 1 \]

PTS: 2 REF: 081619aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

20 ANS: 4
\[ (x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3 \]

PTS: 2 REF: 081620aii NAT: A.APR.C.4 TOP: Polynomial Identities

21 ANS: 3
Since \(x + 4\) is a factor of \(p(x)\), there is no remainder.

PTS: 2 REF: 081621aii NAT: A.APR.B.2 TOP: Remainder Theorem
22 ANS: 4  PTS: 2  REF: 081622aii  NAT: F.BF.A.1  TOP: Modeling Exponential Functions  KEY: AII

23 ANS: 2

Combining (1) and (3): $-6c = -18$

Combining (1) and (2): $5a + 3c = -1$

Using (3): $(-2) - 5b - 5(3) = 2$

$c = 3$

$5a + 3(3) = -1$

$2 - 5b - 15 = 2$

$5a = -10$

$b = -3$

$a = -2$

PTS: 2  REF: 081623aii  NAT: A.REI.C.6  TOP: Solving Linear Systems  KEY: three variables

24 ANS: 4  PTS: 2  REF: 081624aii  NAT: F.BF.A.2  TOP: Sequences

25 ANS:

Amplitude, because the height of the graph shows the volume of the air.

PTS: 2  REF: 081625aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions  KEY: mixed

26 ANS:

Applying the commutative property, $\left(\frac{1}{3}\right)^{\frac{1}{5}}$ can be rewritten as $\left(3^2\right)^{\frac{1}{5}}$ or $9^{\frac{1}{5}}$. A fractional exponent can be rewritten as a radical with the denominator as the index, or $9^{\frac{1}{5}} = \sqrt[5]{9}$.

PTS: 2  REF: 081626aii  NAT: N.RN.A.1  TOP: Radicals and Rational Exponents

27 ANS:

$x i(-6i)^2 = x i(36i^2) = 36xi^3 = -36xi$

PTS: 2  REF: 081627aii  NAT: N-CN.A.2  TOP: Operations with Complex Numbers

28 ANS:

$\sin^2 \theta + (-0.7)^2 = 1$

Since $\theta$ is in Quadrant II, $\sin \theta = \sqrt{.51}$ and $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{.51}}{-0.7} \approx -1.02$

$\sin^2 \theta = .51$

$\sin \theta = \pm \sqrt{.51}$

PTS: 2  REF: 081628aii  NAT: F.TF.C.8  TOP: Determining Trigonometric Functions

29 ANS:

Using a 95% level of confidence, $x \pm 2$ standard deviations sets the usual wait time as 150-302 seconds. 360 seconds is unusual.

PTS: 2  REF: 081629aii  NAT: S.IC.B.6  TOP: Analysis of Data
30 ANS: 
\[ 0 = \log_{10}(x - 4) \] The x-intercept of \( h \) is (2,0). \( f \) has the larger value.

\[ 10^0 = x - 4 \]
\[ 1 = x - 4 \]
\[ x = 5 \]

PTS: 2  REF: 081630aii  NAT: F.IF.C.9  TOP: Comparing Functions
KEY: AII

31 ANS:
\[ \frac{306.25 - 156.25}{70 - 50} = \frac{150}{20} = 7.5 \] Between 50-70 mph, each additional mph in speed requires 7.5 more feet to stop.

PTS: 2  REF: 081631aii  NAT: F.IF.B.6  TOP: Rate of Change
KEY: AII

32 ANS:
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ A \text{ and } B \text{ are independent since } P(A \cap B) = P(A) \cdot P(B) \]
\[ 0.8 = 0.6 + 0.5 - P(A \cap B) \]
\[ 0.3 = 0.6 \cdot 0.5 \]
\[ P(A \cap B) = 0.3 \]

PTS: 2  REF: 081632aii  NAT: S.CP.A.2  TOP: Probability of Compound Events
KEY: independence

33 ANS: 
\[ 0 = x^2(x + 1) - 4(x + 1) \]
\[ 0 = (x^2 - 4)(x + 1) \]
\[ 0 = (x + 2)(x - 2)(x + 1) \]
\[ x = -2, -1, 2 \]

PTS: 4  REF: 081633aii  NAT: F.IF.C.7  TOP: Graphing Polynomial Functions
34 ANS:
\[
7 = 20(0.5)^{\frac{t}{8.02}}
\]

\[
\log 0.35 = \log 0.5^{\frac{t}{8.02}}
\]

\[
\log 0.35 = \frac{t \log 0.5}{8.02}
\]

\[
\frac{8.02 \log 0.35}{\log 0.5} = t
\]

\[
t \approx 12
\]

PTS: 4 REF: 081634aii NAT: F.LE.A.4 TOP: Exponential Decay

35 ANS:
\[
\left( \sqrt{2x-7} \right)^2 = (5-x)^2 \quad \sqrt{2(4)-7} + 4 = 5 \quad \sqrt{2(8)-7} + 8 = 5
\]

\[
2x - 7 = 25 - 10x + x^2 \quad \sqrt{1} = 1 \quad \sqrt{9} \neq -3
\]

\[
2x - 7 = 25 - 10x + x^2
\]

\[
0 = x^2 - 12x + 32
\]

\[
0 = (x - 8)(x - 4)
\]

\[
x = 4, 8
\]

PTS: 4 REF: 081635aii NAT: A.REI.A.2 TOP: Solving Radicals
KEY: extraneous solutions

36 ANS:
Some of the students who did not drink energy drinks read faster than those who did drink energy drinks.

\[
17.7 - 19.1 = -1.4 \quad \text{Differences of -1.4 and less occur} \quad \frac{25}{232} \text{ or about 10\% of the time, so the difference is not unusual.}
\]

PTS: 4 REF: 081636aii NAT: S.IC.B.5 TOP: Analysis of Data

37 ANS:
\[
A = 5000(1.045)^n \quad 5000 \left( 1 + \frac{0.046}{4} \right)^{4(6)} - 5000(1.045)^6 \approx 6578.87 - 6511.30 \approx 67.57 \quad 10000 = 5000 \left( 1 + \frac{0.046}{4} \right)^{4n}
\]

\[
B = 5000 \left( 1 + \frac{0.046}{4} \right)^{4n}
\]

\[
2 = 1.0115^{4n} \quad \log 2 = 4n \cdot \log 1.0115
\]

\[
n = \frac{\log 2}{4 \log 1.0115} \approx 15.2
\]

PTS: 6 REF: 081637aii NAT: A.CED.A.1 TOP: Exponential Growth
0117AII Common Core State Standards
Answer Section

1 ANS: 2 PTS: 2 REF: 011701a
TOP: Graphing Trigonometric Functions
NAT: F.IF.B.4

2 ANS: 1

\[ P(28) = 5\left(\frac{98}{28}\right) \approx 56 \]

PTS: 2 REF: 011702a
NAT: F.LE.A.2
TOP: Modeling Exponential Functions
KEY: AII

3 ANS: 4

\[ m^5 + m^3 - 6m = m(m^4 + m^2 - 6) = m(m^2 + 3)(m^2 - 2) \]

PTS: 2 REF: 011703a
NAT: A.SSE.A.2
TOP: Factoring Polynomials
KEY: higher power AII

4 ANS: 1

\[ y = g(x) = (x - 2)^2 \quad (x - 2)^2 = 3x - 2 \quad y = 3(6) - 2 = 16 \]

\[ x^2 - 4x + 4 = 3x - 2 \quad y = 3(1) - 2 = 1 \]

\[ x^2 - 7x + 6 = 0 \]

\[ (x - 6)(x - 1) = 0 \]

\[ x = 6, 1 \]

PTS: 2 REF: 011705a
NAT: A.REI.C.7
TOP: Quadratic-Linear Systems
KEY: AII

5 ANS: 3

\[ \left(\frac{5}{3}\right)^{\frac{1}{2}} = m^{\frac{5}{6}} = \frac{1}{\sqrt[6]{m^5}} \]

PTS: 2 REF: 011707a
NAT: N.RN.A.2
TOP: Radicals and Rational Exponents
KEY: variables

6 ANS: 3

PTS: 2 REF: 011708a
NAT: F.BF.B.4
TOP: Inverse of Functions
KEY: other

7 ANS: 2


8 ANS: 3

PTS: 2 REF: 011709a
NAT: S.IC.B.5
TOP: Analysis of Data

9 ANS: 2

PTS: 2 REF: 011710a
NAT: F.BF.A.1
TOP: Operations with Functions

10 ANS: 3
11 ANS: 4

\[ x = \frac{8 \pm \sqrt{(-8)^2 - 4(6)(29)}}{2(6)} = \frac{8 \pm \sqrt{-632}}{12} = \frac{8 \pm i\sqrt{4 \times 158}}{12} = \frac{2}{3} \pm \frac{1}{6} i \sqrt{158} \]

PTS: 2  REF: 011711aii  NAT: A.REI.B.4  TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

12 ANS: 2

****************************************************************************

PTS: 2  REF: 011712aii  NAT: A.REI.D.11  TOP: Other Systems
KEY: AII

13 ANS: 3

The pattern suggests an exponential pattern, not linear or sinusoidal. A 4% growth rate is accurate, while a 43% growth rate is not.

PTS: 2  REF: 011713aii  NAT: S.ID.B.6  TOP: Regression
KEY: choose model

14 ANS: 1

\[ d = 18; \ r = \pm \frac{5}{4} \]

PTS: 2  REF: 011714aii  NAT: F.IF.A.3  TOP: Sequences
KEY: term

15 ANS: 3

\[ d = 10 \log \frac{6.3 \times 10^{-3}}{1.0 \times 10^{-12}} \approx 98 \]

PTS: 2  REF: 011715aii  NAT: F.IF.B.4  TOP: Evaluating Logarithmic Expressions

16 ANS: 2

****************************************************************************

PTS: 2  REF: 011716aii  NAT: A.REI.D.11  TOP: Other Systems
KEY: AII
17 ANS: 1

\[ \frac{2(x - 4)}{(x + 3)(x - 4)} + \frac{3(x + 3)}{(x - 4)(x + 3)} = \frac{2x - 2}{x^2 - x - 12} \]

\[ 2x - 8 + 3x + 9 = 2x - 2 \]

\[ 3x = -3 \]

\[ x = -1 \]


18 ANS: 4

\[ 496 \pm 2(115) \]

PTS: 2 REF: 01171aii NAT: S.ID.A.4 TOP: Normal Distributions KEY: interval

19 ANS: 2

\[ h(x) \text{ does not have a y-intercept.} \]

PTS: 2 REF: 01171aii NAT: F.IF.C.9 TOP: Comparing Functions

20 ANS: 2

PTS: 2 REF: 011720aii NAT: A.APR.B.2 TOP: Remainder Theorem

21 ANS: 4

(1) \[ \frac{B(60) - B(10)}{60 - 10} \approx 28\% \] (2) \[ \frac{B(69) - B(19)}{69 - 19} \approx 33\% \] (3) \[ \frac{B(72) - B(36)}{72 - 36} \approx 38\% \] (4) \[ \frac{B(73) - B(60)}{73 - 60} \approx 46\% \]

PTS: 2 REF: 011721aii NAT: F.IF.B.6 TOP: Rate of Change KEY: AII

22 ANS: 3

(3) repeats 3 times over 2\pi.

PTS: 2 REF: 011722aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions KEY: recognize | bimodal graph

23 ANS: 1

\[ \frac{A}{\rho} = e^{rt} \]

\[ 0.42 = e^{rt} \]

\[ \ln 0.42 = \ln e^{rt} \]

\[ -0.87 \approx rt \]

PTS: 2 REF: 011723aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions KEY: AII
ANS: 1

(1) \( \frac{9 - 0}{2 - 1} = 9 \)
(2) \( \frac{17 - 0}{3.5 - 1} = 6.8 \)
(3) \( \frac{0 - 0}{5 - 1} = 0 \)
(4) \( \frac{17 - 5}{3.5 - 1} = 6.3 \)

PTS: 2  REF: 011724aii  NAT: F.IF.B.6  TOP: Rate of Change
KEY: AII

25 ANS:

\( (1 - i)(1 - i)(1 - i) = (1 - 2i + i^2)(1 - i) = -2i(1 - i) = -2i + 2i^2 = -2 - 2i \)

PTS: 2  REF: 011725aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

26 ANS:

Sample: pails of oranges; population: truckload of oranges. It is likely that about 5% of all the oranges are unsatisfactory.

PTS: 2  REF: 011726aii  NAT: S.IC.A.2  TOP: Analysis of Data

27 ANS:

csc \( \theta = \frac{1}{\sin \theta} \), and \( \sin \theta \) on a unit circle represents the y value of a point on the unit circle. Since \( y = \sin \theta \),

csc \( \theta = \frac{1}{y} \).

PTS: 2  REF: 011727aii  NAT: F.TF.A.2  TOP: Reciprocal Trigonometric Relationships

28 ANS:

\( \left( \frac{\ln \left( \frac{1}{2} \right)}{1590} \right) \)

is negative, so \( M(t) \) represents decay.

PTS: 2  REF: 011728aii  NAT: F.IF.C.7  TOP: Graphing Exponential Functions

29 ANS:

PTS: 2  REF: 011729aii  NAT: F.IF.C.7  TOP: Graphing Polynomial Functions
30 ANS:
\[
\left( \frac{\frac{5}{3}}{x} \right)^{\frac{6}{5}} = \left( \frac{\frac{5}{y}}{x} \right)^{\frac{6}{5}}
\]

\[x^2 = y\]

PTS: 2  REF: 011730aii  NAT: N.RN.A.2  TOP: Radicals and Rational Exponents
KEY: variables

31 ANS:
No, because \(P(M/R) \neq P(M)\)

\[
\frac{70}{180} \neq \frac{230}{490}
\]

0.38 \(\neq\) 0.47

PTS: 2  REF: 011731aii  NAT: S.CP.A.4  TOP: Conditional Probability

32 ANS:
\[
\frac{3x + 13}{x - 2} \div \frac{3x^2 + 7x - 20}{3x + 13 + \frac{6}{x - 2}} = \frac{3x^2 - 6x}{13x - 20}
\]

\[13x - 26 \div 6\]

PTS: 2  REF: 011732aii  NAT: A.APR.D.6  TOP: Division of Polynomials

33 ANS:
\[2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k\]

\[h = -2\]

\[-2x^2 + 8x + 5 = hx^2 - 4hx + k\]

\[k = 5\]

PTS: 4  REF: 011733aii  NAT: A.APR.C.4  TOP: Polynomial Identities

34 ANS:
Jillian’s plan, because distance increases by one mile each week. \(a_1 = 10\)

\[a_n = n + 12\]

\[a_n = a_{n-1} + 1\]

PTS: 4  REF: 011734aii  NAT: F.LE.A.2  TOP: Sequences

35 ANS:
\[
P(P/K) = \frac{P(P^aK)}{P(K)} = \frac{1.9}{2.3} \approx 82.6\%\]

A key club member has an 82.6\% probability of being enrolled in AP Physics.

PTS: 4  REF: 011735aii  NAT: S.CP.B.6  TOP: Conditional Probability
36 ANS:

\[ 20000 = PMT \left( \frac{1 - (1 + .00625)^{-60}}{.00625} \right) \]

\[ 21000 - x = 300 \left( \frac{1 - (1 + .00625)^{-60}}{.00625} \right) \]

\[ PMT \approx 400.76 \]

\[ x \approx 6028 \]

PTS: 4 REF: 011736aii NAT: A.SSE.B.4 TOP: Series

37 ANS:

\[ 0 = \sqrt{t} - 2t + 6 \left( \frac{9}{4} \right) - 6 < 0 \]

\[ 2t - 6 = \sqrt{t} \]

\[ 4t^2 - 24t + 36 = t \]

\[ 4t^2 - 25t + 36 = 0 \]

\[ (4t - 9)(t - 4) = 0 \]

\[ t = \frac{9}{4}, 4 \]

\[ (\sqrt{1 - 2(1) + 6}) - (\sqrt{3 - 2(3) + 6}) = 5 - \sqrt{3} \approx 3.268 \ 327 \text{ mph} \]

PTS: 6 REF: 011737aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: context
0617a1i
Answer Section

1 ANS: 1 PTS: 2 REF: 061701a1i NAT: A.APR.B.3
TOP: Zeros of Polynomials KEY: AII
2 ANS: 1
\[ 8(2^{x^3}) = 48 \]
\[ 2^{x^3} = 6 \]
\[ (x + 3) \ln 2 = \ln 6 \]
\[ x + 3 = \frac{\ln 6}{\ln 2} \]
\[ x = \frac{\ln 6}{\ln 2} - 3 \]

PTS: 2 REF: 061702a1i NAT: F.LE.A.4 TOP: Exponential Equations
KEY: without common base
3 ANS: 3
Self selection causes bias.

PTS: 2 REF: 061703a1i NAT: S.IC.B.3 TOP: Analysis of Data
KEY: bias
4 ANS: 2
\[ 6xi(-4xi + 5) = -24x^2i^4 + 30x^3i^3 = -24x^2(1) + 30x(-1) = -24x^2 - 30xi \]

PTS: 2 REF: 061704a1i NAT: N.CN.A.2 TOP: Operations with Complex Numbers
5 ANS: 2

PTS: 2 REF: 061705a1i NAT: A.REI.D.11 TOP: Other Systems
KEY: AII
6 ANS: 4 PTS: 2 REF: 061706a1i NAT: F.IF.B.4 TOP: Graphing Trigonometric Functions
7 ANS: 4  
\[4x^2 = -98\]

\[x^2 = \frac{-98}{4}\]

\[x^2 = \frac{-49}{2}\]

\[x = \pm \sqrt{\frac{-49}{2}} = \pm \frac{7i\sqrt{2}}{\sqrt{2}} = \pm \frac{7i\sqrt{2}}{2}\]

PTS: 2  REF: 061707aii  NAT: A.REI.B.4  TOP: Solving Quadratics  
KEY: complex solutions | taking square roots

8 ANS: 1  PTS: 2  REF: 061708aii  NAT: F.TF.B.5  
TOP: Modeling Trigonometric Functions

9 ANS: 2  
\[x(30 - 0.01x) - (0.15x^3 + 0.01x^2 + 2x + 120) = 30x - 0.01x^2 - 0.15x^3 - 0.01x^2 + 2x - 120\]

\[= -0.15x^3 - 0.02x^2 + 28x - 120\]

PTS: 2  REF: 061709aii  NAT: F.BF.A.1  TOP: Operations with Functions

10 ANS: 3  PTS: 2  REF: 061710aii  NAT: S.IC.A.2  
TOP: Analysis of Data

11 ANS: 1  
\[
\begin{array}{cccc}
2 & 1 & 0 & -4 \\
& 2 & 4 & 0 \\
1 & 2 & 0 & -4 \\
& 1 & 2 & 0 & 0
\end{array}
\]

Since there is no remainder when the quartic is divided by \(x - 2\), this binomial is a factor.

PTS: 2  REF: 061711aii  NAT: A.APR.B.2  TOP: Remainder Theorem

12 ANS: 2  
\[\cos \theta = \pm \sqrt{1 - \left(-\frac{\sqrt{2}}{5}\right)^2} = \pm \sqrt{\frac{25 - 2}{25}} = \pm \frac{\sqrt{23}}{5}\]

PTS: 2  REF: 061712aii  NAT: F.TF.C.8  TOP: Determining Trigonometric Functions

13 ANS: 3  
\[
\frac{1}{0.75} \approx 0.9716
\]

PTS: 2  REF: 061713aii  NAT: A.SSE.B.3  TOP: Modeling Exponential Functions  
KEY: AII
The events are independent because 
\[ P(A \text{ and } B) = P(A) \cdot P(B). \]
\[ 0.125 = 0.5 \cdot 0.25 \]
If 
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.25 + 0.5 - 0.125 = 0.625, \]
then the events are not mutually exclusive because 
\[ P(A \text{ or } B) = P(A) + P(B) \]
\[ 0.625 \neq 0.5 + 0.25 \]

15 ANS: 4

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Bar Harbor</th>
<th>Phoenix</th>
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</thead>
<tbody>
<tr>
<td>31.386</td>
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<td></td>
</tr>
<tr>
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<td>Maximum</td>
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<td>Range</td>
<td>47.828</td>
<td>40.476</td>
</tr>
</tbody>
</table>

16 ANS: 4

The vertex is (1,0) and 
\[ p = 2. \]
\[ y = \frac{1}{4(2)}(x - 1)^2 + 0 \]

18 ANS: 2

The 2010 population is 110 million.

19 ANS: 1

\[ \frac{2x}{x - 2} \left( \frac{x}{x} - \frac{11}{x} \left( \frac{x - 2}{x - 2} \right) \right) = \frac{8}{x^2 - 2x} \]
\[ 2x^2 - 11x + 22 = 8 \]
\[ 2x^2 - 11x + 14 = 0 \]
\[ (2x - 7)(x - 2) = 0 \]
\[ x = \frac{7}{2}, 2 \]

20 ANS: 3

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21 ANS: 3

\[
\frac{f(7) - f(-7)}{7 - (-7)} = \frac{2^{-0.25(7)} \sin\left(\frac{\pi}{2} (7)\right) - 2^{-0.25(-7)} \sin\left(\frac{\pi}{2} (-7)\right)}{14} \approx -0.26
\]

PTS: 2 REF: 061721aii NAT: F.IF.B.6 TOP: Rate of Change

KEY: AII

22 ANS: 3 PTS: 2 REF: 061722aii NAT: A.CED.A.1

TOP: Modeling Rationals

23 ANS: 4

\[
\frac{-3x^2 - 5x + 2}{x^3 + 2x^2} = \frac{(-3x + 1)(x + 2)}{x^2(x + 2)} = \frac{-3x + 1}{x^2} + \frac{1}{x^2} = -3x^{-1} + x^{-2}
\]

PTS: 2 REF: 061723aii NAT: A.APR.D.6 TOP: Expressions with Negative Exponents

KEY: variables

24 ANS: 2 PTS: 2 REF: 061724aii NAT: A.SSE.B.4

TOP: Series

25 ANS:

\[r(2) = -6. \text{ Since there is a remainder when the cubic is divided by } x - 2, \text{ this binomial is not a factor.}\]

\[
\begin{array}{cccc}
2 & 1 & -4 & 4 & 6 \\
1 & 2 & -4 & 0 \\
1 & -2 & 0 & -6 \\
\end{array}
\]

PTS: 2 REF: 061725aii NAT: A.APR.B.2 TOP: Remainder Theorem

26 ANS:

\[
69
\]

PTS: 2 REF: 061726aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: percent

27 ANS:

\[x^2(4x - 1) + 4(4x - 1) = (x^2 + 4)(4x - 1)\]

PTS: 2 REF: 061727aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: factoring by grouping

28 ANS:

period is \(\frac{2}{3}\). The wheel rotates once every \(\frac{2}{3}\) second.

PTS: 2 REF: 061728aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions

KEY: period
29 ANS:

![Graph showing exponential function]

PTS: 2  REF: 061729a1i  NAT: F.IF.C.7  TOP: Graphing Exponential Functions
KEY: AII

30 ANS:

\[ \sqrt{x - 4} = -x + 6 \quad \sqrt{x - 4} = -8 + 6 = -2 \text{ is extraneous.} \]

\[ x - 4 = x^2 - 12x + 36 \]
\[ 0 = x^2 - 13x + 40 \]
\[ 0 = (x - 8)(x - 5) \]
\[ x = 5, 8 \]

PTS: 2  REF: 061730a1i  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: extraneous solutions

31 ANS:

\[ x \cdot \cdot x = x^{\frac{1}{3}} \cdot x^{\frac{1}{2}} = x^{\frac{1}{3}} \cdot x^{\frac{1}{2}} = x^{\frac{5}{6}} \]

PTS: 2  REF: 061731a1i  NAT: N.RN.A.2  TOP: Operations with Radicals
KEY: with variables, index > 2

32 ANS:

A student is more likely to jog if both siblings jog. 1 jogs: \( \frac{416}{2239} \approx 0.19 \). both jog: \( \frac{400}{1780} \approx 0.22 \)

PTS: 2  REF: 061732a1i  NAT: S.CP.A.4  TOP: Conditional Probability

33 ANS:

\[ x + y + z = 1 \]
\[ 2x + 2y + 2z = 2 \quad -2z - z = 3 \quad y - (-1) = 3 \quad x + 2 - 1 = 1 \]
\[ -x + 3y - 5z = 11 \]
\[ 2x + 4y + 6z = 2 \quad -3z = 3 \quad y = 2 \quad x = 0 \]
\[ 4y - 4z = 12 \]
\[ 2y + 4z = 0 \quad z = -1 \]
\[ y - z = 3 \]
\[ y + 2z = 0 \]
\[ y = -2z \]

PTS: 4  REF: 061733a1i  NAT: A.REI.C.6  TOP: Solving Linear Systems
KEY: three variables
34 ANS:

\[ M = 172600 \cdot \frac{0.00305(1 + 0.00305)^{12 \cdot 15}}{(1 + 0.00305)^{12 \cdot 15} - 1} \approx 1247 \]

\[ 1100 = (172600 - x) \cdot \frac{0.00305(1 + 0.00305)^{12 \cdot 15}}{(1 + 0.00305)^{12 \cdot 15} - 1} \]

\[ 1100 \approx (172600 - x) \cdot (0.007228) \]

\[ 152193 \approx 172600 - x \]

\[ 20407 \approx x \]

PTS: 4 REF: 061734aii NAT: A.SSE.B.4 TOP: Series

35 ANS:

\[ \text{As } x \to -3, y \to -\infty. \text{ As } x \to \infty, y \to \infty. \]

PTS: 4 REF: 061735aii NAT: F.IF.C.7 TOP: Graphing Logarithmic Functions

36 ANS:

\[ 0.506 \pm 2 \cdot 0.078 = 0.35 - 0.66. \text{ The 32.5\% value falls below the 95\% confidence level.} \]

PTS: 4 REF: 061736aii NAT: S.IC.B.5 TOP: Analysis of Data

37 ANS:

\[ 100 = 140 \left( \frac{1}{2} \right)^{\frac{5}{h}} \]

\[ \log \frac{100}{140} = \log \left( \frac{1}{2} \right)^{\frac{5}{h}} \]

\[ \log 5 \log 2 = 5 \log 1 \]

\[ \log 2 = \log \left( \frac{1}{2} \right)^{\frac{5}{h}} \]

\[ h = \frac{5 \log 1}{\log 5} \approx 10.3002 \]

\[ \log 2 = \frac{t \log \left( \frac{1}{2} \right)}{10.3002} \]

\[ t = \frac{10.3002 \log 2}{\log \left( \frac{1}{2} \right)} \approx 18.6 \]

PTS: 6 REF: 061737aii NAT: F.LE.A.4 TOP: Exponential Decay
0817AII Common Core State Standards
Answer Section

1  ANS: 1
   \( x^2 + 2x - 8 = 0 \)
   \((x + 4)(x - 2) = 0\)
   \(x = -4, 2\)

   PTS: 2       REF: 081701aii   NAT: A.APR.D.6   TOP: Undefined Rationals

2  ANS: 3
   \((3k - 2i)^2 = 9k^2 - 12ki + 4i^2 = 9k^2 - 12ki - 4\)

   PTS: 2       REF: 081702aii   NAT: N.CN.A.2   TOP: Operations with Complex Numbers

3  ANS: 3
   \(x^2 + 2x + 1 = -5 + 1\)
   \((x + 1)^2 = -4\)
   \(x + 1 = \pm 2i\)
   \(x = -1 \pm 2i\)

   PTS: 2       REF: 081703aii   NAT: A.REI.B.4   TOP: Solving Quadratics
   KEY: complex solutions | completing the square

4  ANS: 2
   \(\sqrt{x + 14} = \sqrt{2x + 5} + 1\)
   \(\sqrt{22 + 14} - \sqrt{2(22) + 5} = 1\)
   \(x + 14 = 2x + 5 + 2\sqrt{2x + 5} + 1\)
   \(6 - 7 \neq 1\)
   \(-x + 8 = 2\sqrt{2x + 5}\)

   \(x^2 - 16x + 64 = 8x + 20\)
   \(x^2 - 24x + 44 = 0\)
   \((x - 22)(x - 2) = 0\)
   \(x = 2, 22\)

   PTS: 2       REF: 081704aii   NAT: A.REI.A.2   TOP: Solving Radicals
   KEY: advanced

5  ANS: 3
   PTS: 2
   REF: 081705aii   NAT: F.IF.B.4
   TOP: Graphing Trigonometric Functions
   KEY: increasing/decreasing
6  ANS: 2 
The vertex of the parabola is (0,0). The distance, \( p \), between the vertex and the focus or the vertex and the directrix is 1. 
\[ y = \frac{-1}{4p} (x - h)^2 + k \]
\[ y = \frac{-1}{4(1)} (x - 0)^2 + 0 \]
\[ y = \frac{-1}{4} x^2 \]

PTS: 2  
REF: 081706aii  
NAT: G.GPE.A.2  
TOP: Graphing Quadratic Functions

7  ANS: 4  
PTS: 2  
REF: 081707aii  
NAT: F.TF.A.2  
TOP: Reference Angles

8  ANS: 4  
PTS: 2  
REF: 081708aii  
NAT: A.APR.B.3  
TOP: Zeros of Polynomials

9  ANS: 3  
\[ \log_{0.8} \left( \frac{V}{17000} \right) = t \]
\[ \frac{17,000(0.8)^3 - 17,000(0.8)^1}{3 - 1} \approx -2450 \]
\[ 0.8^t = \frac{V}{17000} \]
\[ V = 17000(0.8)^t \]

PTS: 2  
REF: 081709aii  
NAT: F.IF.B.6  
TOP: Rate of Change

KEY: AII

10  ANS: 3  
\[ \left( \frac{1}{2} \right)^{13.83} \approx 0.990656 \]

PTS: 2  
REF: 081710aii  
NAT: A.SSE.B.3  
TOP: Modeling Exponential Functions

KEY: AII

11  ANS: 1  

PTS: 2  
REF: 081711aii  
NAT: S.ID.A.4  
TOP: Normal Distributions

KEY: percent
12 ANS: 4
The maximum volume of \( p(x) = -(x + 2)(x - 10)(x - 14) \) is about 56, at \( x = 12.1 \)

PTS: 2 REF: 081712aii NAT: F.IF.B.4 TOP: Graphing Polynomial Functions

13 ANS: 1

\[
\begin{align*}
\frac{2x^2 + x + 5}{2x - 1} &= \frac{4x^3 + 0x^2 + 9x - 5}{2x - 1} \\
4x^3 - 2x^2 &\\
2x^2 + 9x &\\
2x^2 - x &\\
10x - 5 &\\
10x - 5 &
\end{align*}
\]

PTS: 2 REF: 081713aii NAT: A.APR.D.6 TOP: Rational Expressions

14 ANS: 2

\[
x = \frac{y + 1}{y - 2}
\]

\[
xy - 2x = y + 1
\]

\[
xy - y = 2x + 1
\]

\[
y(x - 1) = 2x + 1
\]

\[
y = \frac{2x + 1}{x - 1}
\]

PTS: 2 REF: 081714aii NAT: F.BF.B.4 TOP: Inverse of Functions

KEY: equations

15 ANS: 1

1) let \( y = x + 2 \), then \( y^2 + 2y - 8 \)
\[
(y + 4)(y - 2)
\]

\[
(x + 2 + 4)(x + 2 - 2)
\]

\[
(x + 6)x
\]

PTS: 2 REF: 081715aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: multivariable

16 ANS: 2

\[
ME = \left[ z \sqrt{\frac{p(1-p)}{n}} \right] = \left[ 1.96 \sqrt{\frac{(0.16)(0.84)}{1334}} \right] \approx 0.02
\]

PTS: 2 REF: 081716aii NAT: S.IC.B.4 TOP: Analysis of Data
17 ANS: 2 PTS: 2 REF: 081717aii NAT: S.IC.B.3
TOP: Analysis of Data KEY: type
18 ANS: 4 PTS: 2 REF: 081718aii NAT: F.IF.C.7
TOP: Graphing Trigonometric Functions KEY: amplitude
19 ANS: 1
\[(x+3)^2 + (2x-4)^2 = 8 \quad b^2 - 4ac\]
\[x^2 + 6x + 9 + 4x^2 - 16x + 16 = 8 \quad 100 - 4(5)(17) < 0\]
\[5x^2 - 10x + 17 = 0\]

20 ANS: 2
\[
\begin{array}{ccc}
-4 & 1 & -11 \\
1 & 60 & -304 \\
1 & -15 & 76 \\
\end{array}
\]
Since there is a remainder when the cubic is divided by \(x + 4\), this binomial is not a factor.

PTS: 2 REF: 081720aii NAT: A.APR.B.2 TOP: Remainder Theorem
21 ANS: 4
\[d = 32(.8)^{b-1} \quad S_n = \frac{32 - 32(.8)^{12}}{1 - .8} \approx 149\]

PTS: 2 REF: 081721aii NAT: A.SSE.B.4 TOP: Series
TOP: Analysis of Data
22 ANS: 1 PTS: 2 REF: 081722aii NAT: S.IC.B.6
TOP: Analysis of Data
23 ANS: 4
\[
\left(\frac{-54x^9}{y^4}\right)^\frac{2}{3} = (2 \cdot -27)^{\frac{2}{3}} \cdot x^{\frac{18}{3}} = 2^{\frac{2}{3}} \cdot 9x^6 = \frac{9x^6\sqrt[3]{4}}{y^2} \cdot \frac{2}{3}\sqrt[3]{y^2}
\]

PTS: 2 REF: 081723aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents KEY: variables
24 ANS: 3 PTS: 2 REF: 081724aii NAT: F.BF.A.2
TOP: Sequences
25 ANS:
Rewrite \(\frac{4}{3}\) as \(\frac{1}{3} \cdot 4\), using the power of a power rule.

PTS: 2 REF: 081725aii NAT: N.RN.A.1 TOP: Radicals and Rational Exponents
26 ANS:
\[P(W/D) = \frac{P(W \cap D)}{P(D)} = \frac{.4}{.5} \approx .8\]

PTS: 2 REF: 081726aii NAT: S.CP.B.6 TOP: Conditional Probability
27 ANS:
\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\]
\[x^4 + 2x^2y^2 + y^4 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2\]
\[x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4\]

PTS: 2 REF: 081727aii NAT: A.APR.C.4 TOP: Polynomial Identities

28 ANS:
Since there are six flavors, each flavor can be assigned a number, 1-6. Use the simulation to see the number of times the same number is rolled 4 times in a row.

PTS: 2 REF: 081728aii NAT: S.IC.A.2 TOP: Analysis of Data

29 ANS:
\[a_1 = 4 \quad a_8 = 639\]
\[a_n = 2a_{n-1} + 1\]

PTS: 2 REF: 081729aii NAT: F.LE.A.2 TOP: Sequences

30 ANS:
\[8.75 = 1.25x^{49} \quad 4\]
\[7 = x^{49}\]
\[x = \sqrt[49]{7} \approx 1.04\]

PTS: 2 REF: 081730aii NAT: F.LE.A.4 TOP: Exponential Growth

31 ANS:
\[j(-x) = (-x)^4 - 3(-x)^2 - 4 = x^2 - 3x^2 - 4\] Since \(j(x) = j(-x)\), the function is even.

PTS: 2 REF: 081731aii NAT: F.BF.B.3 TOP: Even and Odd Functions

32 ANS:

PTS: 2 REF: 081732aii NAT: F.IF.C.7 TOP: Graphing Polynomial Functions

KEY: AII
33 ANS:
\[
\frac{3p}{p - 5} = \frac{p + 2}{p + 3}
\]
\[3p^2 + 9p = p^2 - 3p - 10\]
\[2p^2 + 12p + 10 = 0\]
\[p^2 + 6p + 5 = 0\]
\[(p + 5)(p + 1) = 0\]
\[p = -5, -1\]


34 ANS:
\[
\frac{6.25 - 2.25}{21 - 5} = \frac{4}{16} = \$0.25\text{ fine per day. } 2.25 - 5(0.25) = \$1\text{ replacement fee. } a_n = 1.25 + (n - 1)(0.25). \ a_{60} = \$16
\]

PTS: 4  REF: 081734aii  NAT: F.LE.A.2  TOP: Sequences

35 ANS:

Part a sketch is shifted \(\frac{\pi}{3}\) units right.

PTS: 4  REF: 081735aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions  KEY: graph
36 ANS:
\[ y = 4.168(3.981)^x. \]
\[ 100 = 4.168(3.981)^x \]
\[ \frac{\log 100}{4.168} = \log(3.981)^x \]
\[ \frac{\log 100}{4.168} = x \log(3.981) \]
\[ \frac{\log 100}{\log(3.981)} = x \]
\[ x \approx 2.25 \]

PTS: 4  REF: 081736aii  NAT: S.ID.B.6  TOP: Regression
KEY: exponential AII

37 ANS:
At 1.95 years, the value of the car equals the loan balance. Zach can cancel the policy after 6 years.

PTS: 4  REF: 081737aii  NAT: A.REI.D.11  TOP: Other Systems
KEY: AII
### 0118AII Common Core State Standards

**Answer Section**

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</table>
9 ANS: 4

\[
\begin{align*}
\frac{5x^2 + x - 3}{2x - 1} & = \frac{10x^3 - 3x^2 - 7x + 3}{10x^3 - 5x^2} \\
& = \frac{2x^2 - 7x}{2x^2 - x} \\
& = \frac{-6x + 3}{-6x + 3}
\end{align*}
\]

PTS: 2 REF: 011809aii NAT: A.APR.D.6 TOP: Rational Expressions

10 ANS: 1

\[
\frac{9110}{5000} = 5000e^{30r}
\]

\[
\ln \frac{911}{500} = \ln e^{30r}
\]

\[
\frac{\ln 911}{\ln 500} = r
\]

\[
r \approx 0.02
\]

PTS: 2 REF: 011810aii NAT: F.LE.A.4 TOP: Exponential Growth

11 ANS: 4

\[
\frac{n}{m} = \frac{\sqrt[5]{a^5}}{a^2} = \frac{a^{\frac{5}{2}}}{a^2} = a^{\frac{3}{2}} = \sqrt[3]{a^3}
\]

PTS: 2 REF: 011811aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: variables

12 ANS: 1

\[
x - \frac{4}{x - 1} = 2
\]

\[
x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)} = \frac{3 \pm \sqrt{17}}{2}
\]

\[
x(x - 1) - 4 = 2(x - 1)
\]

\[
x^2 - x - 4 = 2x - 2
\]

\[
x^2 - 3x - 2 = 0
\]

PTS: 2 REF: 011812aii NAT: A.REI.A.2 TOP: Solving Rationals

KEY: rational solutions
13 ANS: 3
\[ e^{bt} = \frac{c}{a} \]
\[ \ln e^{bt} = \ln \frac{c}{a} \]
\[ bt \ln e = \ln \frac{c}{a} \]
\[ t = \frac{\ln \frac{c}{a}}{b} \]

PTS: 2  REF: 011813aii  NAT: F.LE.A.4  TOP: Exponential Growth

14 ANS: 1  PTS: 2  REF: 011814aii  NAT: A.REI.D.11
TOP: Other Systems  KEY: AII

15 ANS: 1  PTS: 2  REF: 011815aii  NAT: F.TF.A.2
TOP: Unit Circle

16 ANS: 1
In vertex form, the parabola is \[ y = -\frac{1}{4(2)} (x+4)^2 + 3 \]. The vertex is \((-4,3)\) and \( p = 2 \). \(3 + 2 = 5\)

PTS: 2  REF: 011816aii  NAT: G.GPE.A.2  TOP: Graphing Quadratic Functions

17 ANS: 3

PTS: 2  REF: 011817aii  NAT: F.IF.B.4  TOP: Graphing Polynomial Functions

18 ANS: 3
\[ \frac{c^2 - d^2}{d^2 + cd - 2c^2} = \frac{(c+d)(c-d)}{(d+2c)(d-c)} = \frac{-c+d}{d+2c} = \frac{-c-d}{d+2c} \]

PTS: 2  KEY: a > 0  REF: 011818aii  NAT: A.APR.D.6  TOP: Rational Expressions

19 ANS: 4
\[ p(5) = 2(5)^3 - 3(5) + 5 = 240 \]

PTS: 2  REF: 011819aii  NAT: A.APR.B.2  TOP: Remainder Theorem

20 ANS: 2  PTS: 2  REF: 011820aii  NAT: S.IC.A.2  TOP: Analysis of Data
21 ANS: 2
\[ x = -6(y - 2) \]
\[ \frac{x}{6} = y - 2 \]
\[ \frac{x}{6} + 2 = y \]

PTS: 2  REF: 011821aii  NAT: F.BF.B.4  TOP: Inverse of Functions
KEY: equations

22 ANS: 2
\[ S_{20} = \frac{0.01 - 0.01(3)^{20}}{1 - 3} = 17,433,922 \]

PTS: 2  REF: 011822aii  NAT: A.SSE.B.4  TOP: Series

23 ANS: 4
There is no \( x \)-intercept.

PTS: 2  REF: 011823aii  NAT: F.IF.C.7  TOP: Graphing Exponential Functions
KEY: AII

24 ANS: 3  PTS: 2  REF: 011824aii  NAT: F.BF.A.2  TOP: Sequences

25 ANS:
\[ i^2 = -1, \text{ and not } 1; \ 10 + 10i \]

PTS: 2  REF: 011825aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

26 ANS:
\[ D = 1.223(2.652)^4 \]

PTS: 2  REF: 011826aii  NAT: S.ID.B.6  TOP: Regression
KEY: exponential AII

27 ANS:
\[ \frac{1}{8} + \frac{1}{6} = \frac{1}{t_b}, \quad \frac{24t_b}{8} + \frac{24t_b}{6} = \frac{24t_b}{t_b} \]
\[ 3t_b + 4t_b = 24 \]
\[ t_b = \frac{24}{7} \approx 3.4 \]

PTS: 2  REF: 011827aii  NAT: A.CED.A.1  TOP: Modeling Rationals

28 ANS:
\[ 3x^3 + x^2 + 3xy + y = x^2(3x + 1) + y(3x + 1) = \left(x^2 + y\right)(3x + 1) \]

PTS: 2  REF: 011828aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials
KEY: factoring by grouping
29 ANS:
\[ 20e^{0.05t} = 30e^{0.03t} \]
\[ \frac{2}{3}e^{0.05t} = \frac{e^{0.03t}}{e^{0.05t}} \]
\[ \ln \frac{2}{3} = \ln e^{-0.02t} \]
\[ \frac{2}{3} = -0.02t \ln e \]
\[ \frac{2}{3} = -0.02t \]
\[ t = -0.02 \ln 2 \]
\[ 20.3 \approx t \]

PTS: 2 REF: 011829aP NAT: A.REI.D.11 TOP: Other Systems
KEY: AII

30 ANS:
q has the smaller minimum value for the domain [-2, 2]. h’s minimum is \(-1 \left(2(-1) + 1\right)\) and q’s minimum is \(-8\).

PTS: 2 REF: 011830aP NAT: F.IF.C.9 TOP: Comparing Functions
KEY: AII

31 ANS:

\[ \sqrt[4]{81} = 3 \]

PTS: 2 REF: 011831aP NAT: F.IF.C.7 TOP: Graphing Polynomial Functions
KEY: AII

32 ANS:
The denominator of the rational exponent represents the index of a root, and the 4th root of 81 is 3 and \(3^3\) is 27.

PTS: 2 REF: 011832aP NAT: N.RN.A.1 TOP: Radicals and Rational Exponents
33 ANS: 
\[
\left(2x^2 + x - 3\right) \cdot (x - 1) - \left[\left(2x^2 + x - 3\right) + (x - 1)\right]
\]
\[
\left(2x^3 - 2x^2 + x^2 - x - 3x + 3\right) - \left(2x^2 + 2x - 4\right)
\]
\[
2x^3 - 3x^2 - 6x + 7
\]

PTS: 4 REF: 011833aii NAT: F.BF.A.1 TOP: Operations with Functions

34 ANS: 
\[
\frac{47}{108} = \frac{1}{4} + \frac{116}{459} = P(M \text{ and } J); \text{ No, because } \frac{31}{459} \neq \frac{1}{4} \cdot \frac{116}{459}
\]
\[
P(M \text{ and } J) = \frac{31}{459}
\]

PTS: 4 REF: 011834aii NAT: S.CP.A.3 TOP: Conditional Probability

35 ANS: 
138.905 ± 7.95 = 123 – 155. No, since 125 (50% of 250) falls within the 95% interval.

PTS: 4 REF: 011835aii NAT: S.IC.A.2 TOP: Analysis of Data

36 ANS: 
\[
f(x) = x^2(x + 4)(x - 3); \ g(x) = (x + 2)^2(x + 6)(x - 1)
\]

PTS: 4 REF: 011836aii NAT: A.APR.B.3 TOP: Zeros of Polynomials

37 ANS: 

The period of \(P\) is \(\frac{2}{3}\), which means the patient’s blood pressure reaches a high every \(\frac{2}{3}\) second and a low every \(\frac{2}{3}\) second. The patient’s blood pressure is high because 144 over 96 is greater than 120 over 80.

PTS: 6 REF: 011837aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions

KEY: graph
0618aii

Answer Section

1 ANS: 2
\( x^2 + 4x - 1 = x - 3 \quad y + 3 = -1 \)
\( x^2 + 3x + 2 = 0 \quad y = -4 \)
\( (x + 2)(x + 1) = 0 \)
\( x = -2, -1 \)

PTS: 2  REF: 061801aii  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems
KEY: All

2 ANS: 2
PTS: 2  REF: 061802aii  NAT: F.IF.C.7  TOP: Graphing Exponential Functions
KEY: All

3 ANS: 3
\( \frac{x^2(x + 2) - 9(x + 2)}{x(x^2 - x - 6)} = \frac{(x^2 - 9)(x + 2)}{x(x - 3)(x + 2)} = \frac{(x + 3)(x - 3)}{x(x - 3)} = \frac{x + 3}{x} \)

PTS: 2  REF: 061803aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: factoring

4 ANS: 2
PTS: 2  REF: 061804aii  NAT: S.ID.B.6  TOP: Regression
KEY: choose model

5 ANS: 3
\( (x + 3i)^2 - (2x - 3i)^2 = x^2 + 6xi + 9i^2 - \left( 4x^2 - 12xi + 9i^2 \right) = -3x^2 - 18xi \)

PTS: 2  REF: 061805aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

6 ANS: 2
\( f(x) = f(-x) \)
\( x^2 - 4 = (-x)^2 - 4 \)
\( x^2 - 4 = x^2 - 4 \)

PTS: 2  REF: 061806aii  NAT: F.BF.B.3  TOP: Even and Odd Functions

7 ANS: 1
\( \frac{N(10) - N(1)}{10 - 1} \approx -2.03, \quad \frac{N(20) - N(10)}{20 - 10} \approx -1.63, \quad \frac{N(25) - N(15)}{25 - 15} \approx -1.46, \quad \frac{N(30) - N(1)}{30 - 1} \approx -1.64 \)

PTS: 2  REF: 061807aii  NAT: F.IF.B.6  TOP: Rate of Change
KEY: All

8 ANS: 1
\( (x + 7)(x - 1) = x^2 + 6x - 7 = x^2 + 6x + 9 - 7 - 9 = (x + 3)^2 - 16 \)

PTS: 2  REF: 061808aii  NAT: A.APR.C.4  TOP: Polynomial Identities
9 ANS: 4
\[
\frac{2}{x} = \frac{4x}{x + 3}
\]
\[
2x + 6 = 4x^2
\]
\[
4x^2 - 2x - 6 = 0
\]
\[
2(2x^2 - x - 3) = 0
\]
\[
(2x - 3)(x + 1) = 0
\]
\[
x = \frac{3}{2}, -1
\]
PTS: 2 REF: 061809aii NAT: A.REI.A.2 TOP: Solving Rationals

10 ANS: 4
\[
a = \frac{14 - 4}{2} = 5, \quad d = \frac{14 + 4}{2} = 9
\]
PTS: 2 REF: 061810aii NAT: F.TF.B.5 TOP: Modeling Trigonometric Functions

11 ANS: 4
\[
0.48 \cdot 0.25 = 0.12
\]
PTS: 1 REF: 061811aii NAT: S.CP.A.2 TOP: Probability of Compound Events
KEY: probability

12 ANS: 3
\[
1^3 - k(1)^2 + 2(1) = 0
\]
\[
k = 3
\]
PTS: 2 REF: 061812aii NAT: A.APR.B.3 TOP: Zeros of Polynomials
KEY: AII

13 ANS: 1
\[
p(x) = r(x) - c(x)
\]
\[
-0.5x^2 + 250x - 300 = -0.3x^2 + 150x - c(x)
\]
\[
c(x) = 0.2x^2 - 100x + 300
\]
PTS: 2 REF: 061813aii NAT: F.BF.A.1 TOP: Operations with Functions

14 ANS: 1
\[
1240(1.06)^x = 890(1.11)^x
\]
\[
x \approx 7
\]
PTS: 2 REF: 061814aii NAT: A.REI.D.11 TOP: Other Systems
KEY: AII
15 ANS: 3
\[ y = x^3 - 2 \]
\[ x = y^3 - 2 \]
\[ x + 2 = y^3 \]
\[ \sqrt[3]{x + 2} = y \]

PTS: 2 REF: 061815aii NAT: F.BF.B.4 TOP: Inverse of Functions
KEY: other

16 ANS: 2 PTS: 2 REF: 061816aii NAT: F.IF.C.7
TOP: Graphing Polynomial Functions KEY: bimodalgraph

17 ANS: 2

PTS: 2 REF: 061817aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: probability

18 ANS: 1
\[ 100 \left( \frac{1}{2} \right)^{\frac{k}{s}} = 100e^{kd} \]
\[ \left( \frac{1}{2} \right)^{\frac{1}{s}} = e^k \]
\[ k \approx -0.087 \]

PTS: 2 REF: 061818aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions
KEY: AII

19 ANS: 4
\[ \log_2(x - 1) - 1 = 0 \]
\[ \log_2(x - 1) = 1 \]
\[ x - 1 = 2^1 \]
\[ x = 3 \]

PTS: 2 REF: 061819aii NAT: F.IF.C.7 TOP: Graphing Logarithmic Functions
20 ANS: 2
\[ 4x \cdot x^\frac{2}{3} + 2x^\frac{5}{3} = 6x^\frac{5}{3} = 6\sqrt[3]{x^5} \]

KEY: with variables, index > 2

21 ANS: 4
\[ \frac{5 + 9}{2} = 7, \text{ vertex: } (-2,7); \ p = 7 - 9 = -2, \ y = \frac{1}{4(-2)} (x + 2)^2 + 7 \]
\[ y - 7 = \frac{1}{-8} (x + 2)^2 \]
\[ -8(y - 7) = (x + 2)^2 \]

PTS: 2 RE: 061821aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

22 ANS: 4
\[ (a + b + c)^2 = a^2 + ab + ac + ab + b^2 + bc + ac + ab + c^2 \]
\[ x = a^2 + b^2 + c^2 + 2(ab + bc + ac) \]
\[ x = y + 2z \]

PTS: 2 RE: 061822aii NAT: A.APR.C.4 TOP: Polynomial Identities

23 ANS: 4
1 year = 365 days

PTS: 2 RE: 061823aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions
KEY: AII

24 ANS: 3 PTS: 2 RE: 061824aii NAT: A.CED.A.1 TOP: Modeling Rationals

25 ANS:
\[ \frac{103}{110 + 103} = \frac{103}{213} \]

PTS: 2 RE: 061825aii NAT: S.CP.A.4 TOP: Conditional Probability

26 ANS:

PTS: 2 RE: 061826aii NAT: F.IF.C.7 TOP: Graphing Polynomial Functions
27 ANS:

\[ x = \frac{-5 \pm \sqrt{5^2 + 4(2)(8)}}{2(2)} = \frac{-5 \pm \frac{i\sqrt{39}}{4}}{4} \]

PTS: 2 REF: 061827aii NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

28 ANS:

Self selection is a cause of bias because people with more free time are more likely to respond.

PTS: 2 REF: 061828aii NAT: S.IC.B.3 TOP: Analysis of Data
KEY: bias

29 ANS:

\[ \frac{2a^2 + 5a + 2}{3a - 2} = \frac{6a^3 + 11a^2 - 4a - 9}{6a^3 - 4a^2} \]

\[ = \frac{2a^2 + 5a + 2 - \frac{5}{3a - 2}}{6a^3 - 4a^2} \]

\[ = \frac{15a^2 - 4a}{6a^3 - 4a^2} \]

\[ = \frac{15a^2 - 10a}{6a - 9} \]

\[ = \frac{6a - 4}{6a - 9} \]

\[ = -\frac{5}{3} \]

PTS: 2 REF: 061829aii NAT: A.APR.D.6 TOP: Division of Polynomials

30 ANS:

\[ a_1 = 3 \quad a_2 = 7 \quad a_3 = 15 \quad a_4 = 31; \] No, because there is no common ratio: \( \frac{7}{3} \neq \frac{15}{7} \)

PTS: 2 REF: 061830aii NAT: F.IF.A.3 TOP: Sequences
KEY: term

31 ANS:

\[ M = \frac{(152500 - 152500)(\frac{0.036}{12})(1 + \frac{0.036}{12})^{360}}{\left(1 + \frac{0.036}{12}\right)^{360} - 1} \approx 624 \]

PTS: 2 REF: 061831aii NAT: A.SSE.B.4 TOP: Series
32 ANS: \[
\frac{-1}{\sqrt{2^2 + (-1)^2}} = \frac{-1}{\sqrt{5}}
\]

PTS: 2 REF: 061832aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions
KEY: extension to reals

33 ANS: \[
\sqrt{6 - 2x + x} = 2x + 30 - 9 \quad \sqrt{6 - 2(-29)} \neq -29 + 21, \text{ so } -29 \text{ is extraneous.}
\]
\[
\sqrt{6 - 2x} = x + 21 \quad \sqrt{64} = -8
\]
\[
6 - 2x = x^2 + 42x + 441
\]
\[
x^2 + 44x + 435 = 0
\]
\[
(x + 29)(x + 15) = 0
\]
\[
x = -29, -15
\]

PTS: 4 REF: 061833aii NAT: A.REI.A.2 TOP: Solving Radicals
KEY: extraneous solutions

34 ANS: \[
23 - 18 = 5, \quad x \pm 2\sigma = -3.07 - 3.13, \text{ Yes, a difference of 5 or more occurred three times out of a thousand, which is statistically significant.}
\]

PTS: 4 REF: 061834aii NAT: S.IC.B.5 TOP: Analysis of Data

35 ANS: \[
C(t) = 63000 \left(1 + \frac{0.0255}{12}\right)^{12t} \quad \frac{63000 \left(1 + \frac{0.0255}{12}\right)^{12t}}{12t} = \frac{100}{\log(1.002125)}
\]
\[
t \approx 18.14
\]

PTS: 4 REF: 061835aii NAT: A.CED.A.1 TOP: Exponential Growth

36 ANS: \[
\frac{h(2) - h(1)}{2 - 1} = -12, \quad h(t) = 0 \text{ at } t \approx 2.2, 3.8, \text{ using a graphing calculator to find where } h(t) = 0.
\]

PTS: 4 REF: 061836aii NAT: F.IF.B.4 TOP: Graphing Trigonometric Functions
$P(16) = \log(16 - 4) \approx 1.1, \quad 14000$
1. \( \ln e^{0.3x} = \ln \frac{5918}{87} \)

\[
x = \frac{\ln \frac{5918}{87}}{0.3}
\]

PTS: 2  REF: 081801aii  NAT: F.LE.A.4  TOP: Exponential Equations
KEY: without common base

2. \( x = 2 \)

PTS: 2  REF: 081802aii  NAT: S.IC.B.3  TOP: Analysis of Data
KEY: type

3. \( x = 4 \)

PTS: 2  REF: 081803aii  NAT: F.BF.A.1  TOP: Operations with Functions

4. \( x = 1 \)

PTS: 2  REF: 081804aii  NAT: F.IF.C.9  TOP: Comparing Functions

5. \[
2x^3 - 4x^2 - x + \frac{14}{x + 6}
\]

\[
\begin{array}{l}
2x^3 + 12x^2 \\
- 4x^3 - 25x^2 \\
- 4x^3 - 24x^2 \\
-x^2 - 6x \\
-x^2 - 6x \\
\end{array}
\]

PTS: 2  REF: 081805aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: division

6. \( y = \frac{1}{2}x + 8 \quad x = \frac{1}{2}y + 8 \)

\[
2x = y + 16
\]

\[
y = 2x - 16
\]

PTS: 2  REF: 081806aii  NAT: F.BF.B.4  TOP: Inverse of Functions
KEY: linear
7 ANS: 3
\[ x^2 - 4x - 5 = 4x^2 - 40x + 100 \]
\[ 3x^2 - 36x + 105 = 0 \]
\[ x^2 - 12x + 35 = 0 \]
\[ (x - 7)(x - 5) = 0 \]
\[ x = 5, 7 \]

PTS: 2 REF: 081807aii NAT: A.REI.A.2 TOP: Solving Radicals
KEY: extraneous solutions

8 ANS: 2
\[ 1.0064312 \approx 1.08 \]

PTS: 2 REF: 081808aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions

9 ANS: 3
\[ x = \frac{-2 \pm \sqrt{2^2 - 4(3)(7)}}{2(3)} = \frac{-2 \pm \sqrt{-80}}{6} = \frac{-2 \pm i\sqrt{16 \cdot 5}}{6} = \frac{1}{3} \pm \frac{2i\sqrt{5}}{3} \]

PTS: 2 REF: 081809aii NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

10 ANS: 4 PTS: 2 REF: 081810aii NAT: F.LE.A.2 TOP: Sequences

11 ANS: 2
If \( \cos \theta = \frac{7}{25} \), \( \sin \theta = \pm \frac{24}{25} \), and \( \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-24}{7} \)

PTS: 2 REF: 081811aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions

12 ANS: 3
\[ \frac{\frac{2}{3} \cdot x^{\frac{5}{2}}}{\frac{1}{x^{\frac{1}{6}}}} = \frac{x^\frac{1}{6}}{\frac{1}{x^{\frac{1}{6}}}} \cdot x^{\frac{1}{6}} = x^3 \]

KEY: with variables, index > 2

13 ANS: 1 PTS: 2 REF: 081813aii NAT: A.SSE.B.4 TOP: Series
14 ANS: 4

\((x^6y^4 - 9)(x^4 - 16)\)

\((x^3y^2 + 3)(x^3y^2 - 3)(x^2 + 4)(x^2 - 4)\)


15 ANS: 3

\(-3 + 5i - \left(4 + 24i - 2i - 12i^2\right) = -3 + 5i - (16 + 22i) = -19 - 17i\)

PTS: 2  REF: 081815aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

16 ANS: 2  PTS: 2  REF: 081816aii  NAT: F.IF.C.7  TOP: Graphing Logarithmic Functions  KEY: bimodalgraph

17 ANS: 4  PTS: 2  REF: 081817aii  NAT: F.BF.B.3  TOP: Transformations with Functions

18 ANS: 2

\[
\frac{85}{210 + 85}
\]

PTS: 2  REF: 081818aii  NAT: S.CP.A.1  TOP: Venn Diagrams

19 ANS: 3  PTS: 2  REF: 081819aii  NAT: A.REI.D.11  TOP: Other Systems

20 ANS: 1

\[-4(-1) - 3 = 1  \quad 8 = \frac{2\pi}{b}\]

\[b = \frac{\pi}{4}\]

PTS: 2  REF: 081820aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions  KEY: maximum/minimum

21 ANS: 4

\[m^3 - 2m^2 + 4m - 8 = 0\]

\[m^2(m - 2) + 4(m - 2) = 0\]

\[\left(m^2 + 4\right)(m - 2) = 0\]

PTS: 2  REF: 081821aii  NAT: A.APR.D.6  TOP: Solving Polynomial Equations

22 ANS: 2

\[
P = \frac{\frac{2\pi}{\pi}}{45} = 90
\]

PTS: 2  REF: 081822aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions  KEY: period
23. ANS: 4  
   The vertex is (2,2) and \( p = 3 \).  \( 3 + 2 = 5 \)  
   PTS: 2  
   REF: 081823aii  
   NAT: G.GPE.A.2  
   TOP: Graphing Quadratic Functions

24. ANS: 4  
   PTS: 2  
   REF: 081824aii  
   NAT: S.CP.A.3  
   TOP: Conditional Probability

25. ANS:  
   \( (x^2 - 6)(x^2 + 2) \)  
   PTS: 2  
   REF: 081825aii  
   NAT: A.SSE.A.2  
   TOP: Factoring Polynomials  
   KEY: higher power

26. ANS:  
   \( \frac{\frac{3}{2}x^{\frac{3}{2}}}{\frac{3}{2}} = x^\frac{1}{2} = \sqrt{x} \)  
   \( 2x^2 \)  
   PTS: 2  
   REF: 081826aii  
   NAT: N.RN.A.2  
   TOP: Radicals and Rational Exponents  
   KEY: variables

27. ANS:  
   \( \frac{p(8) - p(4)}{8 - 4} \approx 48.78 \)  
   PTS: 2  
   REF: 081827aii  
   NAT: F.IF.B.6  
   TOP: Rate of Change

28. ANS:  
   \( 1200 \cdot 0.784 \approx 941 \)  
   PTS: 2  
   REF: 081828aii  
   NAT: S.ID.A.4  
   TOP: Normal Distributions  
   KEY: predict

29. ANS:  
   \(-6(x + 3) \left( \frac{-3}{x + 3} - \frac{x}{6} + 1 = 0 \right) \)  
   \( 18 + x(x + 3) - 6(x + 3) = 0 \)  
   \( 18 + x^2 + 3x - 6x - 18 = 0 \)  
   \( x^2 - 3x = 0 \)  
   \( x(x - 3) = 0 \)  
   \( x = 0, 3 \)  
   PTS: 2  
   REF: 081829aii  
   NAT: A.REI.A.2  
   TOP: Solving Rationals  
   KEY: rational solutions
30 ANS: 

\[ x^2 + (x - 28)^2 = 400 \]
\[ y = 12 - 28 = -16 \quad y = 16 - 28 = -12 \]
\[ x^2 + x^2 - 56x + 784 = 400 \]
\[ 2x^2 - 56x + 384 = 0 \]
\[ x^2 - 28x + 192 = 0 \]
\[ (x - 16)(x - 12) = 0 \]
\[ x = 12, 16 \]

31 ANS: 

\[ 2(0.042) = 0.084 \approx 0.08 \] The percent of users making in-app purchases will be within 4% of 35%.

32 ANS: 

\[ 4x + 6y - 8z = -2 \quad 4x - 8y + 20z = 12 \quad z + 2 = 3z - 4 \quad y = 3 + 2 = 5 \quad -4x + 5 + 3 = 16 \]
\[ -4x + y + z = 16 \quad -4x + y + z = 16 \quad 6 = 2z \quad -4x = 8 \]
\[ 7y - 7z = 14 \quad -7y + 21z = 28 \quad z = 3 \quad x = -2 \]
\[ y - z = 2 \quad y - 3z = -4 \]
\[ y = z + 2 \quad y = 3z - 4 \]
ANS:
\[ j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48 = 2 + 1 - 35 - 16 + 48 = 0; \ x + 1 \text{ is a factor of } j(x); \]
\[ 2x^3 - 3x^2 - 32x + 48 = 0 \]
\[ x^2(2x - 3) - 16(2x - 3) = 0 \]
\[ \left(x^2 - 16\right)(2x - 3) = 0 \]
\[ x = \pm 4, \frac{3}{2} \]

PTS: 4  REF: 081834aii  NAT: A.APR.B.2  TOP: Remainder Theorem

ANS:
\[ 2 = e^{0.0375t} \]
\[ t \approx 18.5 \]

PTS: 4  REF: 081835aii  NAT: F.LE.A.4  TOP: Exponential Growth

ANS:
John found the means of the scores of the two rooms and subtracted the means. The mean score for the classical room was 7 higher than the rap room (82-75). Yes, there is less than a 5% chance this difference occurring due to random chance. It is likely the difference was due to the music.

PTS: 4  REF: 081836aii  NAT: S.IC.B.5  TOP: Analysis of Data

ANS:
\[ P(x) = R(x) - C(x) = -330x^3 + 9000x^2 - 67000x + 167000 \]
Least profitable at year 5 because there is a minimum in \( P(x) \). Most profitable at year 13 because there is a maximum in \( P(x) \).

PTS: 6  REF: 081837aii  NAT: F.IF.C.7  TOP: Graphing Polynomial Functions