1. If \(a, b,\) and \(c\) are all positive real numbers, which graph could represent the sketch of the graph of \(p(x) = -a(x + b)(x^2 - 2cx + c^2)\)?

2. Which equation represents a parabola with a focus of \((0,4)\) and a directrix of \(y = 2\)?
   1) \(y = x^2 + 3\)
   2) \(y = -x^2 + 1\)
   3) \(y = \frac{x^2}{2} + 3\)
   4) \(y = \frac{x^2}{4} + 3\)

3. If the terminal side of angle \(\theta\), in standard position, passes through point \((-4,3)\), what is the numerical value of \(\sin \theta\)?
   1) \(\frac{3}{5}\)
   2) \(\frac{4}{5}\)
   3) \(\frac{-3}{5}\)
   4) \(\frac{-4}{5}\)

4. A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, \(B(t)\), can be represented by the function \(B(t) = 750(1.16)^t\), where the \(t\) represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function
   1) \(B(t) = 750(1.012)^t\)
   2) \(B(t) = 750(1.012)^{12t}\)
   3) \(B(t) = 750(1.16)^{12t}\)
   4) \(B(t) = 750(1.16)^{\frac{t}{12}}\)

5. Use the properties of rational exponents to determine the value of \(y\) for the equation:
   \[
   \sqrt[3]{x^8} = x^y, \quad x > 1
   \]
   \[
   \left(x^4\right)^{\frac{1}{3}}
   \]
6 Write \((5 + 2yi)(4 - 3i) - (5 - 2yi)(4 - 3i)\) in \(a + bi\) form, where \(y\) is a real number.

7 Use an appropriate procedure to show that \(x - 4\) is a factor of the function \(f(x) = 2x^3 - 5x^2 - 11x + 4\). Explain your answer.

8 Solve algebraically for all values of \(x\):
\[
\sqrt{x - 5} + x = 7
\]

9 Monthly mortgage payments can be found using the formula below:
\[
M = \frac{P \left(\frac{r}{12}\right) \left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}
\]
where:
- \(M\) = monthly payment
- \(P\) = amount borrowed
- \(r\) = annual interest rate
- \(n\) = number of monthly payments

The Banks family would like to borrow $120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the fewest number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than $720.

10 Solve the following system of equations algebraically for all values of \(x\), \(y\), and \(z\):
\[
\begin{align*}
&x + 3y + 5z = 45 \\
&6x - 3y + 2z = -10 \\
&-2x + 3y + 8z = 72
\end{align*}
\]

11 Write an explicit formula for \(a_n\), the \(n\)th term of the recursively defined sequence below.
\[
a_1 = x + 1 \\
a_n = x(a_{n-1})
\]
For what values of \(x\) would \(a_n = 0\) when \(n > 1\)?

12 Stephen’s Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products \(A\), \(B\), and the new product. Nine out of fifty participants preferred Stephen’s new cola to products \(A\) and \(B\). The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen’s new product, each of sample size 50, simulated 100 times.

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.
13 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on both issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

14 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form $f(t) = A \cos(Bt)$, where $A$ and $B$ are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in $t$ hours since 8:30 a.m. On the grid below, graph one cycle of this function.

People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.
15 What is the solution set of the equation \( \frac{3x + 25}{x + 7} - 5 = \frac{3}{x} \)?

1) \( \left\{ \frac{3}{2}, 7 \right\} \)
2) \( \left\{ \frac{7}{2}, -3 \right\} \)
3) \( \left\{ -\frac{3}{2}, 7 \right\} \)
4) \( \left\{ -\frac{7}{2}, -3 \right\} \)

16 Functions \( f, g, \) and \( h \) are given below.

\[
\begin{align*}
f(x) &= \sin(2x) \\
g(x) &= f(x) + 1
\end{align*}
\]

17 The expression \( \frac{6x^3 + 17x^2 + 10x + 2}{2x + 3} \) equals

1) \( 3x^2 + 4x - 1 + \frac{5}{2x + 3} \)
2) \( 6x^2 + 8x - 2 + \frac{5}{2x + 3} \)
3) \( 6x^2 - x + 13 - \frac{37}{2x + 3} \)
4) \( 3x^2 + 13x + \frac{49}{2} + \frac{151}{2x + 3} \)

18 The solutions to the equation \(-\frac{1}{2} x^2 = -6x + 20\) are

1) \(-6 \pm 2i\)
2) \(-6 \pm 2\sqrt{19}\)
3) \(6 \pm 2i\)
4) \(6 \pm 2\sqrt{19}\)

19 What is the completely factored form of \( k^4 - 4k^3 + 8k^2 - 32k + 12k^2 - 48 \)?

1) \( (k - 2)(k - 2)(k + 3)(k + 4) \)
2) \( (k - 2)(k - 2)(k + 6)(k + 2) \)
3) \( (k + 2)(k - 2)(k + 3)(k + 4) \)
4) \( (k + 2)(k - 2)(k + 6)(k + 2) \)

20 Which statement is incorrect for the graph of the function \( y = -3 \cos \left[ \frac{\pi}{3} (x - 4) \right] + 7 \)?

1) The period is 6.
2) The amplitude is 3.
3) The range is [4,10].
4) The midline is \( y = -4 \).

21 Algebraically determine the values of \( x \) that satisfy the system of equations below.

\[
\begin{align*}
y &= -2x + 1 \\
y &= -2x^2 + 3x + 1
\end{align*}
\]
22 The results of a poll of 200 students are shown in the table below:

<table>
<thead>
<tr>
<th></th>
<th>Preferred Music Style</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Techno</td>
</tr>
<tr>
<td>Female</td>
<td>54</td>
</tr>
<tr>
<td>Male</td>
<td>36</td>
</tr>
</tbody>
</table>

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

23 For the function \( f(x) = (x - 3)^3 + 1 \), find \( f^{-1}(x) \).

24 Given: \( h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2 \)

\( k(x) = -|0.7x| + 5 \)

State the solutions to the equation \( h(x) = k(x) \), rounded to the nearest hundredth.

25 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

26 Rewrite the expression

\[
\left(4x^2 + 5x\right)^2 - 5\left(4x^2 + 5x\right) - 6
\]

as a product of four linear factors.

27 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton’s Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

\[
T = T_a + \left(T_0 - T_a\right)e^{-kt}
\]

\( T_a = \) the temperature surrounding the object

\( T_0 = \) the initial temperature of the object

\( t = \) the time in hours

\( T = \) the temperature of the object after \( t \) hours

\( k = \) decay constant

The turkey reaches the temperature of approximately 100°F after 2 hours. Find the value of \( k \), to the nearest thousandth, and write an equation to determine the temperature of the turkey after \( t \) hours. Determine the Fahrenheit temperature of the turkey, to the nearest degree, at 3 p.m.
28 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups’ final grades is shown below:

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>80.16</td>
</tr>
<tr>
<td>Sx</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students’ final grades were rerandomized 500 times. The results are shown below.

Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

29 Given \( z(x) = 6x^3 + bx^2 - 52x + 15 \), \( z(2) = 35 \), and \( z(-5) = 0 \), algebraically determine all the zeros of \( z(x) \).

30 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the nearest ten thousandth, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

31 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the nearest tenth. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.
0616AII Common Core State Standards

1. When \(b > 0\) and \(d\) is a positive integer, the expression \((3b)^{2d}\) is equivalent to

1) \(\frac{1}{(\sqrt[4]{3b})^2}\)
2) \(\left(\sqrt[3]{3b}\right)^d\)
3) \(\frac{1}{\sqrt[3]{3b^d}}\)
4) \(\left(\sqrt[3]{3b}\right)^2\)

2. Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, \(T\), are left in the semester?

1) \(\frac{255 + 93T}{3T} = 90\)
2) \(\frac{255 + 90T}{3T} = 93\)
3) \(\frac{255 + 93T}{T + 3} = 90\)
4) \(\frac{255 + 90T}{T + 3} = 93\)

3. Given \(i\) is the imaginary unit, \((2 - yi)^2\) in simplest form is

1) \(y^2 - 4yi + 4\)
2) \(-y^2 - 4yi + 4\)
3) \(-y^2 + 4\)
4) \(y^2 + 4\)

4. Which graph has the following characteristics?

- three real zeros
- as \(x \to -\infty\), \(f(x) \to -\infty\)
- as \(x \to \infty\), \(f(x) \to \infty\)

1) 
2) 
3)
4)

5. The solution set for the equation \(\sqrt{56 - x} = x\) is

1) \{\(-8, 7\}\)
2) \{\(-7, 8\}\)
3) \{\(7\}\)
4) \{\}\
6 The zeros for \( f(x) = x^4 - 4x^3 - 9x^2 + 36x \) are
1) \{0, \pm 3, 4\}
2) \{0, 3, 4\}
3) \{0, \pm 3, -4\}
4) \{0, 3, -4\}

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Given the results of her coin flips and of her computer simulation, which statement is most accurate?
1) 73 of the computer's next 100 coin flips will be heads.
2) 50 of her next 100 coin flips will be heads.
3) Her coin is not fair.
4) Her coin is fair.

8 If \( g(c) = 1 - c^2 \) and \( m(c) = c + 1 \), then which statement is not true?
1) \( g(c) \cdot m(c) = 1 + c - c^2 - c^3 \)
2) \( g(c) + m(c) = 2 + c - c^2 \)
3) \( m(c) - g(c) = c + c^2 \)
4) \( \frac{m(c)}{g(c)} = \frac{-1}{1 - c} \)

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the nearest whole percent, is
1) 6
2) 48
3) 68
4) 95

10 The formula below can be used to model which scenario?
\[ a_1 = 3000 \]
\[ a_n = 0.80a_{n-1} \]
1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of $3000, and each year it grows by 80%.
4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are
1) independent
2) dependent
3) mutually exclusive
4) complements

12 A solution of the equation \( 2x^2 + 3x + 2 = 0 \) is
1) \( -\frac{3}{4} + \frac{1}{4}i\sqrt{7} \)
2) \( -\frac{3}{4} + \frac{1}{4}i \)
3) \( -\frac{3}{4} + \frac{1}{4}\sqrt{7} \)
4) \( \frac{1}{2} \)
13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, \( H \), in feet, above the ground of one of the six-person cars can be modeled by
\[
H(t) = 70 \sin \left( \frac{2\pi}{7} (t - 1.75) \right) + 80,
\]
where \( t \) is time, in minutes. Using \( H(t) \) for one full rotation, this car's minimum height, in feet, is
1) 150
2) 70
3) 10
4) 0

14 The expression \( \frac{4x^3 + 5x + 10}{2x + 3} \) is equivalent to
1) \( 2x^2 + 3x - 7 + \frac{31}{2x + 3} \)
2) \( 2x^2 - 3x + 7 - \frac{11}{2x + 3} \)
3) \( 2x^2 + 2.5x + 5 + \frac{15}{2x + 3} \)
4) \( 2x^2 - 2.5x - 5 - \frac{20}{2x + 3} \)

15 Which function represents exponential decay?
1) \( y = 2^{0.3t} \)
2) \( y = 1.2^t \)
3) \( y = \left( \frac{1}{2} \right)^t \)
4) \( y = 5^{-t} \)

16 Given \( f^{-1}(x) = -\frac{3}{4}x + 2 \), which equation represents \( f(x) \)?
1) \( f(x) = \frac{4}{3}x - \frac{8}{3} \)
2) \( f(x) = -\frac{4}{3}x + \frac{8}{3} \)
3) \( f(x) = \frac{3}{4}x - 2 \)
4) \( f(x) = -\frac{3}{4}x + 2 \)

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, \( \theta \), intercepts the circle in Quadrant II at point \( C \). The \( y \)-coordinate of point \( C \) is 8. What is the value of \( \cos \theta \)?
1) \( -\frac{3}{5} \)
2) \( -\frac{3}{4} \)
3) \( \frac{3}{5} \)
4) \( \frac{4}{5} \)

18 Which statement about the graph of \( c(x) = \log_6 x \) is false?
1) The asymptote has equation \( y = 0 \).
2) The graph has no \( y \)-intercept.
3) The domain is the set of positive reals.
4) The range is the set of all real numbers.

19 The equation \( 4x^2 - 24x + 4y^2 + 72y = 76 \) is equivalent to
1) \( 4(x - 3)^2 + 4(y + 9)^2 = 76 \)
2) \( 4(x - 3)^2 + 4(y + 9)^2 = 121 \)
3) \( 4(x - 3)^2 + 4(y + 9)^2 = 166 \)
4) \( 4(x - 3)^2 + 4(y + 9)^2 = 436 \)
20 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.

Which statement about this function is incorrect?
1) The degree of the polynomial is even.
2) There is a positive leading coefficient.
3) At two pH values, there is a relative maximum value.
4) There are two intervals where the function is decreasing.

22 Which value, to the nearest tenth, is not a solution of \( p(x) = q(x) \) if \( p(x) = x^3 + 3x^2 - 3x - 1 \) and \( q(x) = 3x + 8 \)?
1) \(-3.9\)
2) \(-1.1\)
3) \(2.1\)
4) \(4.7\)

23 The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>250,000</td>
</tr>
<tr>
<td>2011</td>
<td>250,937</td>
</tr>
<tr>
<td>2012</td>
<td>251,878</td>
</tr>
<tr>
<td>2013</td>
<td>252,822</td>
</tr>
</tbody>
</table>

How can this sequence be recursively modeled?
1) \( j_n = 250,000(1.00375)^{n-1} \)
2) \( j_n = 250,000 + 937^{(n-1)} \)
3) \( j_1 = 250,000 \)
4) \( j_1 = 250,000 \)
   \[ j_n = 1.00375j_{n-1} \]
   \[ j_n = j_{n-1} + 937 \]

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Which equation best represents the value of the voltage as it flows through the electric wires, where \( t \) is time in seconds?
1) \( V = 120 \sin(t) \)
2) \( V = 120 \sin(60t) \)
3) \( V = 120 \sin(60\pi t) \)
4) \( V = 120 \sin(120\pi t) \)

25 Solve for \( x \):
\[
\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}
\]

26 Describe how a controlled experiment can be created to examine the effect of ingredient \( X \) in a toothpaste.

27 Determine if \( x - 5 \) is a factor of \( 2x^3 - 4x^2 - 7x - 10 \). Explain your answer.
28 On the axes below, graph one cycle of a cosine function with amplitude 3, period \( \frac{\pi}{2} \), midline \( y = -1 \), and passing through the point (0,2).

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is \( \frac{974}{1376} \), what is the probability that a student participates in both sports and music?

30 The directrix of the parabola \( 12(y + 3) = (x - 4)^2 \) has the equation \( y = -6 \). Find the coordinates of the focus of the parabola.

31 Algebraically prove that \( \frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8} \), where \( x \neq -2 \).

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the nearest percent.

33 Solve the system of equations shown below algebraically.
\[
(x - 3)^2 + (y + 2)^2 = 16 \\
2x + 2y = 10
\]

34 Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, \( S_n \), for Alexa's total earnings over \( n \) years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the nearest cent.

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the nearest hundredth.

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% - 50% split. Explain what statistical evidence supports this concern.
36 Which function shown below has a greater average rate of change on the interval \([-2,4]\)? Justify your answer.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0.3125</td>
</tr>
<tr>
<td>-3</td>
<td>0.625</td>
</tr>
<tr>
<td>-2</td>
<td>1.25</td>
</tr>
<tr>
<td>-1</td>
<td>2.5</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>320</td>
</tr>
</tbody>
</table>

\[ g(x) = 4x^3 - 5x^2 + 3 \]

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function

\[ N(t) = N_0 (e^{-rt}) \]

where \( N(t) \) is the amount left in the body, \( N_0 \) is the initial dosage, \( r \) is the decay rate, and \( t \) is time in hours. Patient \( A \), \( A(t) \), is given 800 milligrams of a drug with a decay rate of 0.347. Patient \( B \), \( B(t) \), is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, \( A(t) \) and \( B(t) \), to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

To the nearest hour, \( t \), when does the amount of the given drug remaining in patient \( B \) begin to exceed the amount of the given drug remaining in patient \( A \)? The doctor will allow patient \( A \) to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the nearest tenth of an hour, how long patient \( A \) will have to wait to take another 800 milligram dose of the drug.
1 Which equation has $1 - i$ as a solution?
1) $x^2 + 2x - 2 = 0$
2) $x^2 + 2x + 2 = 0$
3) $x^2 - 2x - 2 = 0$
4) $x^2 - 2x + 2 = 0$

2 Which statement(s) about statistical studies is true?
I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don’t like math.
1) I, only
2) II, only
3) I and III
4) III and IV

3 To the nearest tenth, the value of $x$ that satisfies $2^x = -2x + 11$ is
1) 2.5
2) 2.6
3) 5.8
4) 5.9

4 The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?
1) 0.3803
2) 0.4612
3) 0.8415
4) 0.9612

5 Which factorization is incorrect?
1) $4k^2 - 49 = (2k + 7)(2k - 7)$
2) $a^3 - 8b^3 = (a - 2b)(a^2 + 2ab + 4b^2)$
3) $m^3 + 3m^2 - 4m + 12 = (m - 2)^2 (m + 3)$
4) $t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t + 1)(t + 2)(t + 3)$

6 Sally’s high school is planning their spring musical. The revenue, $R$, generated can be determined by the function $R(t) = -33t^2 + 360t$, where $t$ represents the price of a ticket. The production cost, $C$, of the musical is represented by the function $C(t) = 700 + 5t$. What is the highest ticket price, to the nearest dollar, they can charge in order to not lose money on the event?
1) $t = 3$
2) $t = 5$
3) $t = 8$
4) $t = 11$
7. The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Text Messages per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0–10</td>
</tr>
<tr>
<td>15–18</td>
<td>4</td>
</tr>
<tr>
<td>19–22</td>
<td>6</td>
</tr>
<tr>
<td>23–60</td>
<td>25</td>
</tr>
</tbody>
</table>

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

1) $\frac{157}{229}$
2) $\frac{157}{312}$
3) $\frac{157}{384}$
4) $\frac{157}{456}$

8. A recursive formula for the sequence 18, 9, 4.5, . . . is

1) $g_1 = 18$

$$g_n = \frac{1}{2} g_{n-1}$$

2) $g_1 = 18$

$$g_n = 18 \left( \frac{1}{2} \right)^{n-1}$$

3) $g_1 = 18$

$$g_n = 2g_{n-1}$$

4) $g_1 = 18(2)^{n-1}$

9. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1) $\sum_{n=1}^{6} 8(1.10)^{n-1}$
2) $\sum_{n=1}^{6} 8(1.10)^n$
3) $8 - 8(1.10)^6$
4) $\frac{8 - 8(0.10)^n}{1.10}$

10. A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave decreasing, only?

1) $(0, 200)$
2) $(100, 300)$
3) $(200, 400)$
4) $(300, 400)$

11. The expression $\frac{x^3 + 2x^2 + x + 6}{x + 2}$ is equivalent to

1) $x^2 + 3$
2) $x^2 + 1 + \frac{4}{x + 2}$
3) $2x^2 + x + 6$
4) $2x^2 + 1 + \frac{4}{x + 2}$
12 A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the diagram below.

![Graph showing frequency distribution of proportion votes.

Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to
1) 0.01
2) 0.03
3) 0.06
4) 0.12

13 An equation to represent the value of a car after \( t \) months of ownership is \( v = 32,000(0.81)^{\frac{t}{12}} \). Which statement is not correct?
1) The car lost approximately 19% of its value each month.
2) The car maintained approximately 98% of its value each month.
3) The value of the car when it was purchased was $32,000.
4) The value of the car 1 year after it was purchased was $25,920.

14 Which equation represents an odd function?
1) \( y = \sin x \)
2) \( y = \cos x \)
3) \( y = (x + 1)^3 \)
4) \( y = e^{5x} \)

15 The completely factored form of 
\( 2d^4 + 6d^3 - 18d^2 - 54d \) is
1) \( 2d(d^2 - 9)(d + 3) \)
2) \( 2d(d^2 + 9)(d + 3) \)
3) \( 2d(d + 3)^2(d - 3) \)
4) \( 2d(d - 3)^2(d + 3) \)
16. Which diagram shows an angle rotation of 1 radian on the unit circle?

17. The focal length, \( F \), of a camera’s lens is related to the distance of the object from the lens, \( J \), and the distance to the image area in the camera, \( W \), by the formula below.

\[
\frac{1}{J} + \frac{1}{W} = \frac{1}{F}
\]

When this equation is solved for \( J \) in terms of \( F \) and \( W \), \( J \) equals

1) \( F - W \)
2) \( \frac{FW}{F - W} \)
3) \( \frac{FW}{W - F} \)
4) \( \frac{1}{F} - \frac{1}{W} \)

18. The sequence \( a_1 = 6, a_n = 3a_{n-1} \) can also be written as

1) \( a_n = 6 \cdot 3^n \)
2) \( a_n = 6 \cdot 3^{n+1} \)
3) \( a_n = 2 \cdot 3^n \)
4) \( a_n = 2 \cdot 3^{n+1} \)
19 Which equation represents the set of points equidistant from line \( \ell \) and point \( R \) shown on the graph below?

1) \( y = -\frac{1}{8} (x + 2)^2 + 1 \)

2) \( y = -\frac{1}{8} (x + 2)^2 - 1 \)

3) \( y = -\frac{1}{8} (x - 2)^2 + 1 \)

4) \( y = -\frac{1}{8} (x - 2)^2 - 1 \)

20 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I \( (m + p)^2 = m^2 + 2mp + p^2 \)

II \( (x + y)^3 = x^3 + 3xy + y^3 \)

III \( (a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2 \)

1) I, only

2) I and II

3) II and III

4) I and III

21 The graph of \( p(x) \) is shown below.

What is the remainder when \( p(x) \) is divided by \( x + 4 \)?

1) \( x - 4 \)

2) \( -4 \)

3) 0

4) 4

22 A payday loan company makes loans between \$100 and \$1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a \$300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

1) \( 300(.30)^{\frac{14}{365}} \)

2) \( 300(1.30)^{\frac{14}{365}} \)

3) \( 300(.30)^{\frac{365}{14}} \)

4) \( 300(1.30)^{\frac{365}{14}} \)
23 Which value is not contained in the solution of the system shown below?
\[ a + 5b - c = -20 \]
\[ 4a - 5b + 4c = 19 \]
\[ -a - 5b - 5c = 2 \]
1) \(-2\)
2) 2
3) 3
4) \(-3\)

24 In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State \(t\) years after 2010?
1) \(P_t = 19,378,000(1.5)^t\)
2) \(P_0 = 19,378,000\)
   \[ P_t = 19,378,000 + 1.015P_{t-1} \]
3) \(P_t = 19,378,000(1.015)^{t-1}\)
4) \(P_0 = 19,378,000\)
   \[ P_t = 1.015P_{t-1} \]

25 The volume of air in a person’s lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

26 Explain how \(\left(\frac{1}{3}\right)^2\) can be written as the equivalent radical expression \(\sqrt[2]{9}\).

27 Simplify \(xi(i - 7i)^2\), where \(i\) is the imaginary unit.

28 Using the identity \(\sin^2 \theta + \cos^2 \theta = 1\), find the value of \(\tan \theta\), to the nearest hundredth, if \(\cos \theta = -0.7\) and \(\theta\) is in Quadrant II.

29 Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth’s wait time unusual? Justify your answer.

30 The \(x\)-value of which function’s \(x\)-intercept is larger, \(f\) or \(h\)? Justify your answer.

\[ f(x) = \log(x - 4) \]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(h(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
</tr>
</tbody>
</table>
31 The distance needed to stop a car after applying the brakes varies directly with the square of the car’s speed. The table below shows stopping distances for various speeds.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>6.25</td>
<td>25</td>
<td>56.25</td>
<td>100</td>
<td>156.25</td>
<td>225</td>
<td>306.25</td>
</tr>
</tbody>
</table>

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

32 Given events $A$ and $B$, such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether $A$ and $B$ are independent or dependent.

33 Find algebraically the zeros for $p(x) = x^3 + x^2 - 4x - 4$. On the set of axes below, graph $y = p(x)$.

34 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the nearest day, the amount of time needed before the amount of I–131 in the patient’s body is approximately 7 milligrams.

35 Solve the equation $\sqrt{2x - 7} + x = 5$ algebraically, and justify the solution set.
Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

<table>
<thead>
<tr>
<th>Group 1 (seconds)</th>
<th>Group 2 (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.4</td>
<td>23.3</td>
</tr>
<tr>
<td>18.1</td>
<td>18.8</td>
</tr>
<tr>
<td>18.2</td>
<td>22.1</td>
</tr>
<tr>
<td>19.6</td>
<td>12.7</td>
</tr>
<tr>
<td>18.6</td>
<td>16.9</td>
</tr>
<tr>
<td>16.2</td>
<td>24.4</td>
</tr>
<tr>
<td>16.1</td>
<td>21.2</td>
</tr>
<tr>
<td>15.3</td>
<td>21.2</td>
</tr>
<tr>
<td>17.8</td>
<td>16.3</td>
</tr>
<tr>
<td>19.7</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Mean = 17.7
Mean = 19.1

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva’s hypothesis may be incorrect. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.

Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.
Seth’s parents gave him $5000 to invest for his 16th birthday. He is considering two investment options. Option A will pay him 4.5% interest compounded annually. Option B will pay him 4.6% compounded quarterly. Write a function of option A and option B that calculates the value of each account after \( n \) years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option B will earn than option A to the nearest cent. Algebraically determine, to the nearest tenth of a year, how long it would take for option B to double Seth’s initial investment.
1. Relative to the graph of $y = 3\sin x$, what is the shift of the graph of $y = 3\sin \left( x + \frac{\pi}{3} \right)$?

1) $\frac{\pi}{3}$ right
2) $\frac{\pi}{3}$ left
3) $\frac{\pi}{3}$ up
4) $\frac{\pi}{3}$ down

2. A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If $t$ represents the time, in weeks, and $P(t)$ is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?

1) 56
2) 152
3) 3688
4) 81,920

3. Factored completely, $m^5 + m^3 - 6m$ is equivalent to

1) $(m + 3)(m - 2)$
2) $(m^2 + 3m)(m^2 - 2)$
3) $m(m^4 + m^2 - 6)$
4) $m(m^2 + 3)(m^2 - 2)$

4. If $\sin^2 (32^\circ) + \cos^2 (M) = 1$, then $M$ equals

1) $32^\circ$
2) $58^\circ$
3) $68^\circ$
4) $72^\circ$

5. What is the solution to the system of equations $y = 3x - 2$ and $y = g(x)$ where $g(x)$ is defined by the function below?

![Graph of $y = g(x)$]

1) $\{(0, -2)\}$
2) $\{(0, -2), (1, 6)\}$
3) $\{(1, 6)\}$
4) $\{(1, 1), (6, 16)\}$

6. Which statement about statistical analysis is false?

1) Experiments can suggest patterns and relationships in data.
2) Experiments can determine cause and effect relationships.
3) Observational studies can determine cause and effect relationships.
4) Observational studies can suggest patterns and relationships in data.
7 The expression $\left( \frac{m^2}{\sqrt[3]{m}} \right)^{\frac{1}{2}}$ is equivalent to

1) $-\sqrt[m^5]{m^5}$
2) $\frac{1}{\sqrt[6]{m^5}}$
3) $-m\sqrt{m}$
4) $\frac{1}{m^5\sqrt{m}}$

8 What is the inverse of the function $y = \log_3 x$?

1) $y = x^3$
2) $y = \log_3 3$
3) $y = 3^x$
4) $x = 3^y$

9 Gabriel performed an experiment to see if planting 13 tomato plants in black plastic mulch leads to larger tomatoes than if 13 plants are planted without mulch. He observed that the average weight of the tomatoes from tomato plants grown in black plastic mulch was 5 ounces greater than those from the plants planted without mulch. To determine if the observed difference is statistically significant, he rerandomized the tomato groups 100 times to study these random differences in the mean weights. The output of his simulation is summarized in the dotplot below.

Given these results, what is an appropriate inference that can be drawn?

1) There was no effect observed between the two groups.
2) There was an effect observed that could be due to the random assignment of plants to the groups.
3) There is strong evidence to support the hypothesis that tomatoes from plants planted in black plastic mulch are larger than those planted without mulch.
4) There is strong evidence to support the hypothesis that tomatoes from plants planted without mulch are larger than those planted in black plastic mulch.

10 If $p(x) = ab^x$ and $r(x) = cd^x$, then $p(x) \cdot r(x)$ equals

1) $ac(b + d)^x$
2) $ac(b + d)^{2x}$
3) $ac(bd)^x$
4) $ac(bd)^{x^2}$
11 The solution to the equation $18x^2 - 24x + 87 = 0$ is
1) $\frac{2}{3} \pm 6i\sqrt{158}$
2) $\frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$
3) $\frac{2}{3} \pm 6i\sqrt{158}$
4) $\frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$

12 When $g(x) = \frac{2}{x+2}$ and $h(x) = \log(x+1) + 3$ are graphed on the same set of axes, which coordinates best approximate their point of intersection?
1) $(-0.9, 1.8)$
2) $(-0.9, 1.9)$
3) $(1.4, 3.3)$
4) $(1.4, 3.4)$

13 The price of a postage stamp in the years since the end of World War I is shown in the scatterplot below.

The equation that best models the price, in cents, of a postage stamp based on these data is
1) $y = 0.59x - 14.82$
2) $y = 1.04(1.43)^x$
3) $y = 1.43(1.04)^x$
4) $y = 24\sin(14x) + 25$

14 The eighth and tenth terms of a sequence are 64 and 100. If the sequence is either arithmetic or geometric, the ninth term can not be
1) -82
2) -80
3) 80
4) 82
15 The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity $I_0$ to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, $I$, and the decibel rating, $d$, of this sound is found using $d = 10 \log \frac{I}{I_0}$. The threshold sound audible to the average person is $1.0 \times 10^{-12}$ W/m$^2$ (watts per square meter). Consider the following sound level classifications:

<table>
<thead>
<tr>
<th>Classification</th>
<th>Decibel Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderate</td>
<td>45-69 dB</td>
</tr>
<tr>
<td>Loud</td>
<td>70-89 dB</td>
</tr>
<tr>
<td>Very loud</td>
<td>90-109 dB</td>
</tr>
<tr>
<td>Deafening</td>
<td>&gt;110 dB</td>
</tr>
</tbody>
</table>

How would a sound with intensity $6.3 \times 10^{-3}$ W/m$^2$ be classified?
1) moderate 3) very loud
2) loud 4) deafening

16 Pedro and Bobby each own an ant farm. Pedro starts with 100 ants and says his farm is growing exponentially at a rate of 15% per month. Bobby starts with 350 ants and says his farm is steadily decreasing by 5 ants per month. Assuming both boys are accurate in describing the population of their ant farms, after how many months will they both have approximately the same number of ants?
1) 7
2) 8
3) 13
4) 36

17 What is the solution, if any, of the equation
$$\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}?$$
1) -1
2) -5
3) all real numbers
4) no real solution

18 In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects 95% of the Critical Reading scores?
1) $430 \pm 115$
2) $430 \pm 230$
3) $496 \pm 115$
4) $496 \pm 230$

19 Which statement regarding the graphs of the functions below is untrue?

\[ f(x) = 3 \sin 2x, \text{ from } -\pi < x < \pi \]
\[ g(x) = (x - 0.5)(x + 4)(x - 2) \]
\[ h(x) = \log_2 x \]
\[ j(x) = -4x - 2|+3 \]
1) $f(x)$ and $j(x)$ have a maximum $y$-value of 3.
2) $f(x)$, $h(x)$, and $j(x)$ have one $y$-intercept.
3) $g(x)$ and $j(x)$ have the same end behavior as $x \to -\infty$.
4) $g(x)$, $h(x)$, and $j(x)$ have rational zeros.

20 When $g(x)$ is divided by $x + 4$, the remainder is 0. Given $g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8$, which conclusion about $g(x)$ is true?
1) $g(4) = 0$
2) $g(-4) = 0$
3) $x - 4$ is a factor of $g(x)$.
4) No conclusion can be made regarding $g(x)$. 
21 Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of $B$ dollars after $m$ months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after $m$ months.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.00</td>
</tr>
<tr>
<td>10</td>
<td>1172.00</td>
</tr>
<tr>
<td>19</td>
<td>1352.00</td>
</tr>
<tr>
<td>36</td>
<td>1770.80</td>
</tr>
<tr>
<td>60</td>
<td>2591.90</td>
</tr>
<tr>
<td>69</td>
<td>2990.00</td>
</tr>
<tr>
<td>72</td>
<td>3135.80</td>
</tr>
<tr>
<td>73</td>
<td>3186.00</td>
</tr>
</tbody>
</table>

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?
1) month 10 to month 60  
2) month 19 to month 69  
3) month 36 to month 72  
4) month 60 to month 73

22 Which graph represents a cosine function with no horizontal shift, an amplitude of 2, and a period of $\frac{2\pi}{3}$?

23 According to a pricing website, Indroid phones lose 58% of their cash value over 1.5 years. Which expression can be used to estimate the value of a $300 Indroid phone in 1.5 years?
1) $300e^{-0.87}$  
2) $300e^{-0.63}$  
3) $300e^{-0.58}$  
4) $300e^{-0.42}$

24 A cardboard box manufacturing company is building boxes with length represented by $x + 1$, width by $5 - x$, and height by $x - 1$. The volume of the box is modeled by the function below.

Over which interval is the volume of the box changing at the fastest average rate?
1) $[1,2]$  
2) $[1,3.5]$  
3) $[1,5]$  
4) $[0,3.5]$  

25 Express $(1 - i)^3$ in $a + bi$ form.
26 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario. State one conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

27 Using the unit circle below, explain why \( \csc \theta = \frac{1}{y} \).

![Unit Circle Diagram](image)

28 The function \( M(t) \) represents the mass of radium over time, \( t \), in years.

\[
M(t) = 100e^{\frac{\ln 2}{1590} t}
\]

Determine if the function \( M(t) \) represents growth or decay. Explain your reasoning.

29 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

![Graph Paper Grid](image)

30 Given the equal terms \( \sqrt[3]{x^5} \) and \( \frac{5}{y} \), determine and state \( y \), in terms of \( x \).

31 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

<table>
<thead>
<tr>
<th></th>
<th>Comedy Series</th>
<th>Drama Series</th>
<th>Reality Series</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>95</td>
<td>65</td>
<td>70</td>
<td>230</td>
</tr>
<tr>
<td>Females</td>
<td>80</td>
<td>70</td>
<td>110</td>
<td>260</td>
</tr>
<tr>
<td>Total</td>
<td>175</td>
<td>135</td>
<td>180</td>
<td>490</td>
</tr>
</tbody>
</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.
32 Given \( f(x) = 3x^2 + 7x - 20 \) and \( g(x) = x - 2 \), state the quotient and remainder of \( \frac{f(x)}{g(x)} \), in the form \( q(x) + \frac{r(x)}{g(x)} \).

33 Algebraically determine the values of \( h \) and \( k \) to correctly complete the identity stated below.

\[
2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k
\]

34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer. Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose. Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

35 The guidance department has reported that of the senior class, 2.3% are members of key club, \( K \), 8.6% are enrolled in AP Physics, \( P \), and 1.9% are in both. Determine the probability of \( P \) given \( K \), to the nearest tenth of a percent. The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.
36 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

- \( P_n \) = present amount borrowed
- \( n \) = number of monthly pay periods
- \( PMT \) = monthly payment
- \( i \) = interest rate per month

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

37 The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation

\[ s = \sqrt{t - 2t + 6} \]

where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \). How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.
1 The graph of the function $p(x)$ is sketched below.

Which equation could represent $p(x)$?

1) $p(x) = (x^2 - 9)(x - 2)$
2) $p(x) = x^3 - 2x^2 + 9x + 18$
3) $p(x) = (x^2 + 9)(x - 2)$
4) $p(x) = x^3 + 2x^2 - 9x - 18$

2 What is the solution to $8(2^{x^3}) = 48$?

1) $x = \frac{\ln 6}{\ln 2} - 3$
2) $x = 0$
3) $x = \frac{\ln 48}{\ln 16} - 3$
4) $x = \ln 4 - 3$

3 Cheap and Fast gas station is conducting a consumer satisfaction survey. Which method of collecting data would most likely lead to a biased sample?

1) interviewing every 5th customer to come into the station
2) interviewing customers chosen at random by a computer at the checkout
3) interviewing customers who call an 800 number posted on the customers' receipts
4) interviewing every customer who comes into the station on a day of the week chosen at random out of a hat

4 The expression $6x^3(-4xi + 5)$ is equivalent to

1) $2x - 5i$
2) $-24x^2 - 30xi$
3) $-24x^2 + 30x - i$
4) $26x - 24x^2 i - 5i$

5 If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is

1) 1.96
2) 11.29
3) (-0.99, 1.96)
4) (11.29, 32.87)

6 Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if $a$ and $b$ are positive constants?

1) right $a$ units, up $b$ units
2) right $a$ units, down $b$ units
3) left $a$ units, up $b$ units
4) left $a$ units, down $b$ units

7 The solution to the equation $4x^2 + 98 = 0$ is

1) $\pm 7$
2) $\pm 7i$
3) $\pm \frac{7\sqrt{2}}{2}$
4) $\pm \frac{7i\sqrt{2}}{2}$
8  Which equation is represented by the graph shown below?

1) \( y = \frac{1}{2} \cos 2x \)
2) \( y = \cos x \)
3) \( y = \frac{1}{2} \cos x \)
4) \( y = 2 \cos \frac{1}{2}x \)

9  A manufacturing company has developed a cost model, \( C(x) = 0.15x^3 + 0.01x^2 + 2x + 120 \), where \( x \) is the number of items sold, in thousands. The sales price can be modeled by \( S(x) = 30 - 0.01x \). Therefore, revenue is modeled by \( R(x) = x \cdot S(x) \). The company's profit, \( P(x) = R(x) - C(x) \), could be modeled by

1) \( 0.15x^3 + 0.02x^2 - 28x + 120 \)
2) \( -0.15x^3 - 0.02x^2 + 28x - 120 \)
3) \( -0.15x^3 + 0.01x^2 - 2.01x - 120 \)
4) \( -0.15x^3 + 32x + 120 \)

10  A game spinner is divided into 6 equally sized regions, as shown in the diagram below.

For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times, assuming the probability of winning each spin is \( \frac{1}{6} \). The output of the simulation is shown in the diagram below.

Which explanation is appropriate for Miles and his dad to make?

1) The spinner was likely unfair, since the number 6 failed to occur in about 20% of the simulations.
2) The spinner was likely unfair, since the spinner should have landed on the number 6 by the sixth spin.
3) The spinner was likely not unfair, since the number 6 failed to occur in about 20% of the simulations.
4) The spinner was likely not unfair, since in the output the player wins once or twice in the majority of the simulations.

11  Which binomial is a factor of \( x^4 - 4x^2 - 4x + 8 \)?

1) \( x - 2 \)
2) \( x + 2 \)
3) \( x - 4 \)
4) \( x + 4 \)
12 Given that $\sin^2 \theta + \cos^2 \theta = 1$ and $\sin \theta = -\frac{2}{5}$, what is a possible value of $\cos \theta$?

1) $\frac{5 + \sqrt{2}}{5}$
2) $\frac{\sqrt{23}}{5}$
3) $\frac{3\sqrt{3}}{5}$
4) $\frac{\sqrt{35}}{5}$

13 A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P = 714(0.75)^d$, where $P$ is the population, in thousands, $d$ decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after $y$ years. Suzanne's model is best represented by

1) $P = 714(0.6500)^y$
2) $P = 714(0.8500)^y$
3) $P = 714(0.9716)^y$
4) $P = 714(0.9750)^y$

14 The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

I: independent
II: dependent
III: mutually exclusive

1) I, only
2) II, only
3) I and III
4) II and III

15 Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914 \sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238 \sin(0.525x - 2.148) + 86.729$. Which statement can not be concluded based on the average monthly temperature models $x$ months after starting data collection?

1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.
2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.
3) The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.
4) The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree.

16 For $x \neq 0$, which expressions are equivalent to one divided by the sixth root of $x$?

I. $\frac{\sqrt[6]{x}}{\sqrt[3]{x}}$
II. $\frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}$
III. $x^{-\frac{1}{6}}$

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

17 A parabola has its focus at $(1,2)$ and its directrix is $y = -2$. The equation of this parabola could be

1) $y = 8(x + 1)^2$
2) $y = \frac{1}{8} (x + 1)^2$
3) $y = 8(x - 1)^2$
4) $y = \frac{1}{8} (x - 1)^2$
18 The function \( p(t) = 110e^{0.0392t} \) models the population of a city, in millions, \( t \) years after 2010. As of today, consider the following two statements:
I. The current population is 110 million.
II. The population increases continuously by approximately 3.9% per year.
This model supports
1) I, only
2) II, only
3) both I and II
4) neither I nor II

19 To solve \( \frac{2x}{x-2} - \frac{11}{x} = \frac{8}{x^2 - 2x} \), Ren multiplied both sides by the least common denominator. Which statement is true?
1) 2 is an extraneous solution.
2) \( \frac{7}{2} \) is an extraneous solution.
3) 0 and 2 are extraneous solutions.
4) This equation does not contain any extraneous solutions.

20 Given \( f(9) = -2 \), which function can be used to generate the sequence \(-8, -7.25, -6.5, -5.75, \ldots\)?
1) \( f(n) = -8 + 0.75n \)
2) \( f(n) = -8 - 0.75(n - 1) \)
3) \( f(n) = -8.75 + 0.75n \)
4) \( f(n) = -0.75 + 8(n - 1) \)

21 The function \( f(x) = 2^{-0.25x} \cdot \sin \left( \frac{\pi}{2} x \right) \) represents a damped sound wave function. What is the average rate of change for this function on the interval \([-7, 7]\), to the nearest hundredth?
1) -3.66
2) -0.30
3) -0.26
4) 3.36

22 Mallory wants to buy a new window air conditioning unit. The cost for the unit is $329.99. If she plans to run the unit three months out of the year for an annual operating cost of $108.78, which function models the cost per year over the lifetime of the unit, \( C(n) \), in terms of the number of years, \( n \), that she owns the air conditioner?
1) \( C(n) = 329.99 + 108.78n \)
2) \( C(n) = 329.99 + 326.34n \)
3) \( C(n) = \frac{329.99 + 108.78n}{n} \)
4) \( C(n) = \frac{329.99 + 326.34n}{n} \)

23 The expression \( \frac{-3x^2 - 5x + 2}{x^3 + 2x^2} \) can be rewritten as
1) \( -3x - 3 \)
2) \( \frac{-3x - 1}{x^2} \)
3) \( -3x^{-1} + 1 \)
4) \( -3x^{-1} + x^{-2} \)

24 Jasmine decides to put $100 in a savings account each month. The account pays 3% annual interest, compounded monthly. How much money, \( S \), will Jasmine have after one year?
1) \( S = 100(1.03)^{12} \)
2) \( S = \frac{100 - 100(1.0025)^{12}}{1 - 1.0025} \)
3) \( S = 100(1.0025)^{12} \)
4) \( S = \frac{100 - 100(1.03)^{12}}{1 - 1.03} \)

25 Given \( r(x) = x^3 - 4x^2 + 4x - 6 \), find the value of \( r(2) \). What does your answer tell you about \( x - 2 \) as a factor of \( r(x) \)? Explain.
26 The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the nearest integer, weighed less than 8.25 pounds.

27 Over the set of integers, factor the expression \( 4x^3 - x^2 + 16x - 4 \) completely.

28 The graph below represents the height above the ground, \( h \), in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, \( t \), in seconds.

![Graph of a triathlete's bike wheel height over time](image)

Identify the period of the graph and describe what the period represents in this context.

29 Graph \( y = 400(0.85)^x - 6 \) on the set of axes below.

![Graph of the function \( y = 400(0.85)^x - 6 \)](image)

30 Solve algebraically for all values of \( x \):

\[ \sqrt{x} - 4 + x = 6 \]

31 Write \( \sqrt[3]{x} \cdot \sqrt{x} \) as a single term with a rational exponent.
32. Data collected about jogging from students with two older siblings are shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Neither Sibling Jogs</th>
<th>One Sibling Jogs</th>
<th>Both Siblings Jogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Does Not Jog</td>
<td>1168</td>
<td>1823</td>
<td>1380</td>
</tr>
<tr>
<td>Student Jogs</td>
<td>188</td>
<td>416</td>
<td>400</td>
</tr>
</tbody>
</table>

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.

33. Solve the following system of equations algebraically for all values of \(x\), \(y\), and \(z\):

\[
\begin{align*}
  x + y + z &= 1 \\
  2x + 4y + 6z &= 2 \\
  -x + 3y - 5z &= 11
\end{align*}
\]

34. Jim is looking to buy a vacation home for $172,600 near his favorite southern beach. The formula to compute a mortgage payment, \(M\), is

\[
M = P \cdot \frac{r(1+r)^N}{(1+r)^N - 1}
\]

where \(P\) is the principal amount of the loan, \(r\) is the monthly interest rate, and \(N\) is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With no down payment, determine Jim's mortgage payment, rounded to the nearest dollar. Algebraically determine and state the down payment, rounded to the nearest dollar, that Jim needs to make in order for his mortgage payment to be $1100.

35. Graph \(y = \log_2(x + 3) - 5\) on the set of axes below. Use an appropriate scale to include both intercepts.

Describe the behavior of the given function as \(x\) approaches -3 and as \(x\) approaches positive infinity.
Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if 50% or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that 50% of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40, simulated 100 times.

Assume the set of data is approximately normal and the dealership wants to be 95% confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the nearest hundredth. Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is 32.5%. The dealership decides not to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.

A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form

\[ A = A_0 \left( \frac{1}{2} \right)^{\frac{t}{h}} \]

that models this situation, where \( h \) is the constant representing the number of hours in the half-life, \( A_0 \) is the initial mass, and \( A \) is the mass \( t \) hours after 3 p.m. Using this equation, solve for \( h \), to the nearest ten thousandth. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the nearest tenth of an hour.
1 The function \( f(x) = \frac{x - 3}{x^2 + 2x - 8} \) is undefined when
\( x \) equals
1) 2 or –4
2) 4 or –2
3) 3, only
4) 2, only

2 Which expression is equivalent to \((3k - 2i)^2\), where \( i \) is the imaginary unit?
1) \( 9k^2 - 4 \)
2) \( 9k^2 + 4 \)
3) \( 9k^2 - 12ki - 4 \)
4) \( 9k^2 - 12ki + 4 \)

3 The roots of the equation \( x^2 + 2x + 5 = 0 \) are
1) –3 and 1
2) –1, only
3) \(-1 + 2i \) and \(-1 - 2i \)
4) \(-1 + 4i \) and \(-1 - 4i \)

4 The solution set for the equation \( \sqrt{x + 14} - \sqrt{2x + 5} = 1 \) is
1) \{–6\}
2) \{2\}
3) \{18\}
4) \{2, 22\}

5 As \( x \) increases from 0 to \( \frac{\pi}{2} \), the graph of the equation \( y = 2 \tan x \) will
1) increase from 0 to 2
2) decrease from 0 to –2
3) increase without limit
4) decrease without limit

6 Which equation represents a parabola with the focus at \((0, -1)\) and the directrix of \( y = 1 \)?
1) \( x^2 = -8y \)
2) \( x^2 = -4y \)
3) \( x^2 = 8y \)
4) \( x^2 = 4y \)

7 Which diagram represents an angle, \( \alpha \), measuring \( \frac{13\pi}{20} \) radians drawn in standard position, and its reference angle, \( \theta \)?

8 What are the zeros of \( P(m) = (m^2 - 4)(m^2 + 1) \)?
1) 2 and –2, only
2) 2, –2, and –4
3) –4, \( i \), and \(-i \)
4) 2, –2, \( i \), and \(-i \)
9 The value of a new car depreciates over time. Greg purchased a new car in June 2011. The value, \( V \), of his car after \( t \) years can be modeled by the equation
\[
\log_{0.8} \left( \frac{V}{1700} \right) = t.
\]
What is the average decreasing rate of change per year of the value of the car from June 2012 to June 2014, to the nearest ten dollars per year?
1) 1960
2) 2180
3) 2450
4) 2770

10 Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, \( A \), of Iridium-192 present after \( t \) days would be \( A = 100 \left( \frac{1}{2} \right)^{\frac{t}{73.83}} \). Which equation approximates the amount of Iridium-192 present after \( t \) days?
1) \( A = 100 \left( \frac{73.83}{2} \right)^t \)
2) \( A = 100 \left( \frac{1}{147.66} \right)^t \)
3) \( A = 100(0.990656)^t \)
4) \( A = 100(0.116381)^t \)

11 The distribution of the diameters of ball bearings made under a given manufacturing process is normally distributed with a mean of 4 cm and a standard deviation of 0.2 cm. What proportion of the ball bearings will have a diameter less than 3.7 cm?
1) 0.0668
2) 0.4332
3) 0.8664
4) 0.9500

12 A polynomial equation of degree three, \( p(x) \), is used to model the volume of a rectangular box. The graph of \( p(x) \) has \( x \) intercepts at \(-2\), 10, and 14. Which statements regarding \( p(x) \) could be true?
A. The equation of \( p(x) = (x - 2)(x + 10)(x + 14) \).
B. The equation of \( p(x) = -(x + 2)(x - 10)(x - 14) \).
C. The maximum volume occurs when \( x = 10 \).
D. The maximum volume of the box is approximately 56.
1) \( A \) and \( C \)
2) \( A \) and \( D \)
3) \( B \) and \( C \)
4) \( B \) and \( D \)

13 Which expression is equivalent to \( \frac{4x^3 + 9x - 5}{2x - 1} \), where \( x \neq \frac{1}{2} \)?
1) \( 2x^2 + x + 5 \)
2) \( 2x^2 + \frac{11}{2} + \frac{1}{2(2x - 1)} \)
3) \( 2x^2 - x + 5 \)
4) \( 2x^2 - x + 4 + \frac{1}{2x - 1} \)

14 The inverse of the function \( f(x) = \frac{x + 1}{x - 2} \) is
1) \( f^{-1}(x) = \frac{x + 1}{x + 2} \)
2) \( f^{-1}(x) = \frac{2x + 1}{x - 1} \)
3) \( f^{-1}(x) = \frac{x + 1}{x - 2} \)
4) \( f^{-1}(x) = \frac{x - 1}{x + 1} \)

15 Which expression has been rewritten correctly to form a true statement?
1) \( (x + 2)^2 + 2(x + 2) - 8 = (x + 6)x \)
2) \( x^4 + 4x^2 + 9x^2y^2 - 36y^2 = (x + 3y)^2(x - 2)^2 \)
3) \( x^3 + 3x^2 - 4xy^2 - 12y^2 = (x - 2y)(x + 3)^2 \)
4) \( (x^2 - 4)^2 - 5(x^2 - 4) - 6 = (x^2 - 7)(x^2 - 6) \)
A study conducted in 2004 in New York City found that 212 out of 1334 participants had hypertension. Kim ran a simulation of 100 studies based on these data. The output of the simulation is shown in the diagram below.

At a 95% confidence level, the proportion of New York City residents with hypertension and the margin of error are closest to
1) proportion ≈ .16; margin of error ≈ .01
2) proportion ≈ .16; margin of error ≈ .02
3) proportion ≈ .01; margin of error ≈ .16
4) proportion ≈ .02; margin of error ≈ .16

Which scenario is best described as an observational study?
1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.
3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.

Which sinusoid has the greatest amplitude?
1) $y = 3 \sin(\theta - 3) + 5$
2) $y = -5 \sin(\theta - 1) - 3$

Consider the system shown below.

\[ 2x - y = 4 \]
\[ (x + 3)^2 + y^2 = 8 \]

The two solutions of the system can be described as
1) both imaginary
2) both irrational
3) both rational
4) one rational and one irrational

Which binomial is not a factor of the expression
\[ x^3 - 11x^2 + 16x + 84? \]
1) $x + 2$
2) $x + 4$
3) $x - 6$
4) $x - 7$

A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?
1) 29
2) 58
3) 120
4) 149
22 A public opinion poll was conducted on behalf of Mayor Ortega's reelection campaign shortly before the election. 264 out of 550 likely voters said they would vote for Mayor Ortega; the rest said they would vote for his opponent. Which statement is least appropriate to make, according to the results of the poll?

1) There is a 48% chance that Mayor Ortega will win the election.
2) The point estimate (\(\hat{p}\)) of voters who will vote for Mayor Ortega is 48%.
3) It is most likely that between 44% and 52% of voters will vote for Mayor Ortega.
4) Due to the margin of error, an inference cannot be made regarding whether Mayor Ortega or his opponent is most likely to win the election.

23 What does \(\left(\frac{-54x^9}{y^4}\right)^\frac{2}{3}\) equal?

1) \(\frac{9x^6\sqrt[3]{4}}{y^{\frac{2}{3}}\sqrt[3]{y^2}}\)
2) \(\frac{9x^6\sqrt[3]{4}}{y^{\frac{2}{3}}\sqrt[3]{y^2}}\)
3) \(\frac{9x^6\sqrt[3]{4}}{y^{\frac{2}{3}}\sqrt[3]{y^2}}\)
4) \(\frac{9x^6\sqrt[3]{4}}{y^{\frac{2}{3}}\sqrt[3]{y^2}}\)

24 The Rickerts decided to set up an account for their daughter to pay for her college education. The day their daughter was born, they deposited $1000 in an account that pays 1.8% compounded annually. Beginning with her first birthday, they deposit an additional $750 into the account on each of her birthdays. Which expression correctly represents the amount of money in the account \(n\) years after their daughter was born?

1) \(a_n = 1000(1.018)^n + 750\)
2) \(a_n = 1000(1.018)^n + 750n\)
3) \(a_0 = 1000\)
   \(a_n = a_{n-1}(1.018) + 750\)
4) \(a_0 = 1000\)
   \(a_n = a_{n-1}(1.018) + 750n\)

25 Explain how \((-8)^\frac{4}{3}\) can be evaluated using properties of rational exponents to result in an integer answer.

26 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

27 Verify the following Pythagorean identity for all values of \(x\) and \(y\):

\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\]

28 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said "What are the odds I got all of that kind?" Mrs. Jones replied, "simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual." Explain how this simulation could be used to solve the problem.
29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79, .... Write a recursive formula for Candy's sequence. Determine the eighth term in Candy's sequence.

30 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was $1.25 an hour and in 2015, it was $8.75. Algebraically determine the rate of growth to the nearest percent.

31 Algebraically determine whether the function \( j(x) = x^4 - 3x^2 - 4 \) is odd, even, or neither.

32 On the axes below, sketch a possible function \( p(x) = (x - a)(x - b)(x + c) \), where \( a, b, \) and \( c \) are positive, \( a > b \), and \( p(x) \) has a positive \( y \)-intercept of \( d \). Label all intercepts.

33 Solve for all values of \( p \):
\[
\frac{3p}{p - 5} - \frac{2}{p + 3} = \frac{p}{p + 3}
\]

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed $2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed $6.25 to replace his library card and pay the fine for the overdue book. Suppose the total amount Simon owes when the book is \( n \) days late can be determined by an arithmetic sequence. Determine a formula for \( a_n \), the \( n \)th term of this sequence. Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

35 a) On the axes below, sketch at least one cycle of a sine curve with an amplitude of 2, a midline at \( y = -\frac{3}{2} \), and a period of \( 2\pi \).

b) Explain any differences between a sketch of \( y = 2 \sin \left(x - \frac{\pi}{3}\right) - \frac{3}{2} \) and the sketch from part a.
36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

<table>
<thead>
<tr>
<th>Hours (x)</th>
<th>Average Number of Spores (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>260</td>
</tr>
<tr>
<td>4</td>
<td>1130</td>
</tr>
<tr>
<td>6</td>
<td>16,380</td>
</tr>
</tbody>
</table>

Using these data, write an exponential regression equation, rounding all values to the nearest thousandth. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the nearest quarter hour, that the meat can be kept at room temperature safely.

37 The value of a certain small passenger car based on its use in years is modeled by \( V(t) = 28482.698(0.684)^t \), where \( V(t) \) is the value in dollars and \( t \) is the time in years. Zach had to take out a loan to purchase the small passenger car. The function \( Z(t) = 22151.327(0.778)^t \), where \( Z(t) \) is measured in dollars, and \( t \) is the time in years, models the unpaid amount of Zach's loan over time. Graph \( V(t) \) and \( Z(t) \) over the interval \( 0 \leq t \leq 5 \), on the set of axes below.

State when \( V(t) = Z(t) \), to the nearest hundredth, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a $3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the nearest year, how long will it take Zach to cancel this policy? Justify your answer.
2015 Algebra II Common Core State Standards Sample Items
Answer Section

1 ANS: 1
The zeros of the polynomial are at \(-b\), and \(c\). The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as \(x\) goes to negative infinity, \(f(x)\) goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

PTS: 2 REF: spr1501aii NAT: A.APR.B.3 TOP: Zeros of Polynomials
KEY: AII

2 ANS: 4

A parabola with a focus of \((0,4)\) and a directrix of \(y=2\) is sketched as follows:
By inspection, it is determined that the vertex of the parabola is \((0,3)\). It is also evident that the distance, \(p\), between the vertex and the focus is 1. It is possible to use the formula \((x-h)^2 = 4p(y-k)\) to derive the equation of the parabola as follows: 
\[(x-0)^2 = 4(1)(y-3)\]
\[x^2 = 4y - 12\]
\[x^2 + 12 = 4y\]
\[\frac{x^2}{4} + 3 = y\]

or A point \((x,y)\) on the parabola must be the same distance from the focus as it is from the directrix. For any such point \((x,y)\), the distance to the focus is \(\sqrt{(x-0)^2 + (y-4)^2}\) and the distance to the directrix is \(y-2\). Setting this equal leads to: 
\[x^2 + y^2 - 8y + 16 = y^2 - 4y + 4\]
\[x^2 + 16 = 4y + 4\]
\[\frac{x^2}{4} + 3 = y\]

PTS: 2 REF: spr1502aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

3 ANS: 1
A reference triangle can be sketched using the coordinates \((-4,3)\) in the second quadrant to find the value of \(\sin \theta\).

PTS: 2 REF: spr1503aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions
KEY: extension to reals
4 ANS: 2

\[ B(t) = 750 \left( 1.16 \right)^{\frac{t}{12}} \approx 750(1.012)^{12t} \]

\[ B(t) = 750 \left( 1 + \frac{0.16}{12} \right)^{12t} \]

is wrong, because the growth is an annual rate that is not compounded monthly.

PTS: 2
KEY: AII
REF: spr1504aii
NAT: A.SSE.B.3
TOP: Modeling Exponential Functions

5 ANS:

\[ \frac{8}{3} = x^y \]
\[ \frac{4}{3} = x^y \]
\[ \frac{4}{3} = y \]

PTS: 2
KEY: numbers
REF: spr1505aii
NAT: N.RN.A.2
TOP: Radicals and Rational Exponents

6 ANS:

\[ (4 - 3i)(5 + 2yi - 5 + 2yi) \]
\[ (4 - 3i)(4yi) \]
\[ 16yi - 12yi^2 \]
\[ 12y + 16yi \]

PTS: 2
KEY: numbers
REF: spr1506aii
NAT: N.CN.A.2
TOP: Operations with Complex Numbers

7 ANS:

\[ f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 128 - 80 - 44 - 4 = 0 \]

Any method that demonstrates 4 is a zero of \( f(x) \) confirms that \( x - 4 \) is a factor, as suggested by the Remainder Theorem.

PTS: 2
KEY: numbers
REF: spr1507aii
NAT: A.APR.B.2
TOP: Remainder Theorem
8 ANS:

\[ \sqrt{x - 5} = -x + 7 \quad \sqrt{x - 5} = -9 + 7 = -2 \text{ is extraneous.} \]

\[ x - 5 = x^2 - 14x + 49 \]
\[ 0 = x^2 - 15x + 54 \]
\[ 0 = (x - 6)(x - 9) \]
\[ x = 6, 9 \]

PTS: 2  REF: spr1508aii  NAT: A.REI.A.2  TOP: Solving Radicals

KEY: extraneous solutions

9 ANS:

\[ 720 = \frac{120000 \left( \frac{-0.048}{12} \right) \left( 1 + \frac{0.048}{12} \right)^n}{1 + \frac{0.048}{12}} - 1 \]
\[ \frac{275.2}{12} \approx 23 \text{ years} \]

\[ 720(1.004)^n - 720 = 480(1.004)^n \]
\[ 240(1.004)^n = 720 \]
\[ 1.004^n = 3 \]
\[ n \log 1.004 = \log 3 \]
\[ n \approx 275.2 \text{ months} \]

PTS: 4  REF: spr1509aii  NAT: A.CED.A.1  TOP: Exponential Growth
10 ANS:

\[ 6x - 3y + 2z = -10 \quad 6x + 3y + 5z = 45 \quad 4x + 10z = 62 \quad 4x + 4(7) = 20 \]

\[ -2x + 3y + 8z = 72 \quad 6x - 3y + 2z = -10 \quad 4x + 4z = 20 \quad 4x = -8 \]

\[ 4x + 10z = 62 \quad 7x + 7z = 35 \quad 6z = 42 \quad x = -2 \]

\[ 4x + 4z = 20 \quad z = 7 \]

\[ 6(-2) - 3y + 2(7) = -10 \]

\[ -3y = -12 \]

\[ y = 4 \]

PTS: 4  REF: spr1510aii  NAT: A.REI.C.6  TOP: Solving Linear Systems

KEY: three variables

11 ANS:

\[ a_n = x^{n-1} (x + 1) \quad x^{n-1} = 0 \quad x + 1 = 0 \]

\[ x = 0 \quad x = -1 \]

PTS: 4  REF: spr1511aii  NAT: F.BF.A.2  TOP: Sequences

12 ANS:

Yes. The margin of error from this simulation indicates that 95% of the observations fall within ±0.12 of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to be 0.06 in the dotplot, by 2, or applying the estimated standard error formula,

\[ \sqrt{\frac{p(1-p)}{n}} \quad \text{or} \quad \sqrt{\frac{(0.25)(0.75)}{50}} \]

and multiplying by 2. The interval 0.25 ± 0.12 includes plausible values for the true proportion of people who prefer Stephen’s new product. The company has evidence that the population proportion could be at least 25%. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 (9 out of 50) or less several times, even when the population proportion is 0.25, due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

PTS: 4  REF: spr1512aii  NAT: S.IC.B.4  TOP: Analysis of Data
This scenario can be modeled with a Venn Diagram: \[ P(S \cup I) = 0.2, \quad P(S \cup I) = 0.8. \] Then, \[ P(S \cap I) = P(S) + P(I) - P(S \cup I) \] If \( S \) and \( I \) are independent, then the \[ = 0.5 + 0.7 - 0.8 \] \[ = 0.4 \] Product Rule must be satisfied. However, \((0.5)(0.7) \neq 0.4\). Therefore, salary and insurance have not been treated independently.

The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of \(-12\) and a maximum of 12. The value of \( A \) is \(-12\) since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter \( B \). By experimentation with technology or using the relation \( P = \frac{2\pi}{B} \) (where \( P \) is the period), it is determined that \( B = \frac{2\pi}{13} \).

In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from \( t = 13 \) to \( t = 19.5 \) (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.
\[ x(x + 7) \left[ \frac{3x + 25}{x + 7} - 5 \right] = \frac{3}{x} \]

\[ x(3x + 25) - 5x(x + 7) = 3(x + 7) \]
\[ 3x^2 + 25x - 5x^2 - 35x = 3x + 21 \]
\[ 2x^2 + 13x + 21 = 0 \]
\[ (2x + 7)(x + 3) = 0 \]
\[ x = \frac{-7}{2}, -3 \]

**PTS:** 2  **REF:** fall1501aii  **NAT:** A.REI.A.2  **TOP:** Solving Rationals
**KEY:** rational solutions

16  **ANS:** 3
\[ f(x) = -f(x), \text{ so } f(x) \text{ is odd. } g(-x) \neq g(x), \text{ so } g(x) \text{ is not even. } g(-x) \neq -g(x), \text{ so } g(x) \text{ is not odd. } h(-x) = h(x), \text{ so } h(x) \text{ is even.} \]

**PTS:** 2  **REF:** fall1502aii  **NAT:** F.BF.B.3  **TOP:** Even and Odd Functions

17  **ANS:** 1
\[ \frac{3x^2 + 4x - 1}{2x + 3} \]
\[ \frac{6x^3 + 9x^2}{6x^3 + 9x^2} \]
\[ \frac{8x^2 + 16x}{8x^2 + 16x} \]
\[ \frac{-2x - 2}{-2x - 3} \]
\[ \frac{5}{5} \]

**PTS:** 2  **REF:** fall1503aii  **NAT:** A.APR.D.6  **TOP:** Rational Expressions
18 ANS: 3

\[-2 \left( \frac{1}{2} x^2 = -6x + 20 \right) \]

\[x^2 - 12x = -40\]

\[x^2 - 12x + 36 = -40 + 36\]

\[(x - 6)^2 = -4\]

\[x - 6 = \pm 2i\]

\[x = 6 \pm 2i\]

PTS: 2  REF: fall1504aii  NAT: A.REI.B.4  TOP: Solving Quadratics
KEY: complex solutions | completing the square

19 ANS: 4

\[k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48\]

\[k^2(k^2 - 4) + 8k(k^2 - 4) + 12(k^2 - 4)\]

\[(k^2 - 4)(k^2 + 8k + 12)\]

\[(k + 2)(k - 2)(k + 6)(k + 2)\]

PTS: 2  REF: fall1505aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials
KEY: factoring by grouping

20 ANS: 4

As the range is \([4,10]\), the midline is \(y = \frac{4 + 10}{2} = 7\).

PTS: 2  REF: fall1506aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions
KEY: mixed
21 ANS:

\[-2x + 1 = -2x^2 + 3x + 1\]

\[2x^2 - 5x = 0\]

\[x(2x - 5) = 0\]

\[x = 0, \frac{5}{2}\]

PTS: 2  
REF: fall1507a  
NAT: A.REI.C.7  
TOP: Quadratic-Linear Systems  
KEY: AII

22 ANS:

Based on these data, the two events do not appear to be independent. \(P(F) = \frac{106}{200} = 0.53\), while \(P(F|T) = \frac{54}{90} = 0.6\), \(P(F|R) = \frac{25}{65} = 0.39\), and \(P(F|C) = \frac{27}{45} = 0.6\). The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.

PTS: 2  
REF: fall1508a  
NAT: S.CP.A.4  
TOP: Conditional Probability

23 ANS:

\[x = (y - 3)^3 + 1\]

\[x - 1 = (y - 3)^3\]

\[\sqrt[3]{x - 1} = y - 3\]

\[\sqrt[3]{x - 1} + 3 = y\]

\[f^{-1}(x) = \sqrt[3]{x - 1} + 3\]

PTS: 2  
REF: fall1509a  
NAT: F.BF.B.4  
TOP: Inverse of Functions  
KEY: equations
24 ANS:

Let \( x \) equal the first integer and \( x + 1 \) equal the next. \((x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1\). \(2x + 1\) is an odd integer.

25 ANS:

The expression is of the form \( y^2 - 5y - 6 \) or \((y - 6)(y + 1)\). Let \( y = 4x^2 + 5x \):

\[
\left( 4x^2 + 5x - 6 \right) \left( 4x^2 + 5x + 1 \right)
\]

\((4x - 3)(x + 2)(4x + 1)(x + 1)\)

26 ANS:

\[
100 = 325 + (68 - 325)e^{-2k} \quad T = 325 - 257e^{-0.066t}
\]

\[
-225 = -257e^{-2k} \quad T = 325 - 257e^{-0.066(7)} \approx 163
\]

\[
k = \frac{\ln\left(\frac{-225}{-257}\right)}{-2}
\]

\[
k \approx 0.066
\]
The mean difference between the students’ final grades in group 1 and group 2 is –3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription. One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students’ final grades many (500) times. If the observed difference –3.64 is the result of the assignment of students to groups alone, then a difference of –3.64 or less should be observed fairly regularly in the simulation output. However, a difference of –3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups’ mean final grades.

\[ 0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 \quad z(x) = 6x^3 + 19x^2 - 52x + 15 \]

\[ 0 = -750 + 25b + 260 + 15 \]

\[ 475 = 25b \]

\[ b = \frac{19}{5} \]

\[ x = \frac{3}{2}, 1, -3, -5 \]

The actual mass is \( A(40) = 100(0.5)^{40/63} \approx 64.3976 \). The estimated mass is less than the actual mass.
0616AII Common Core State Standards  
Answer Section

1. ANS: 4  PTS: 2  REF: 061601aii  NAT: N.RN.A.2 
   TOP: Radicals and Rational Exponents  KEY: variables

2. ANS: 3  PTS: 2  REF: 061602aii  NAT: A.CED.A.1 
   TOP: Modeling Rationals

3. ANS: 2 
   
   \[(2 - yi)(2 - yi) = 4 - 4yi + y^2i^2 = -y^2 - 4yi + 4\]  
   PTS: 2  REF: 061603aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

4. ANS: 3
   The graph shows three real zeros, and has end behavior matching the given end behavior.  
   PTS: 2  REF: 061604aii  NAT: A.APR.B.3  TOP: Zeros of Polynomials  
   KEY: AII

5. ANS: 3
   \[\sqrt{56 - x} = x\]  
   \[-8 \text{ is extraneous.}\]
   
   \[56 - x = x^2\]
   \[0 = x^2 + x - 56\]
   \[0 = (x + 8)(x - 7)\]
   \[x = 7\]  
   PTS: 2  REF: 061605aii  NAT: A.REI.A.2  TOP: Solving Radicals  
   KEY: extraneous solutions
6 ANS: 1

\[ x^4 - 4x^3 - 9x^2 + 36x = 0 \]
\[ x^3(x - 4) - 9x(x - 4) = 0 \]
\[ (x^3 - 9x)(x - 4) = 0 \]
\[ x(x^2 - 9)(x - 4) = 0 \]
\[ x(x + 3)(x - 3)(x - 4) = 0 \]
\[ x = 0, \pm 3, 4 \]

PTS: 2
REF: 061606a
NAT: A.APR.B.3
TOP: Zeros of Polynomials

7 ANS: 3
TOP: Analysis of Data

8 ANS: 4

\[ \frac{m(c)}{g(c)} = \frac{c + 1}{1 - c^2} = \frac{c + 1}{(1 + c)(1 - c)} = \frac{1}{1 - c} \]

PTS: 2
REF: 061608a
NAT: F.BF.A.1
TOP: Operations with Functions

9 ANS: 2

\[ x + 2\sigma \text{ represents approximately } 48\% \text{ of the data.} \]

PTS: 2
REF: 061609a
NAT: S.ID.A.4
TOP: Normal Distributions

10 ANS: 4

The scenario represents a decreasing geometric sequence with a common ratio of 0.80.

PTS: 2
REF: 061610a
NAT: F.BF.A.2
TOP: Sequences

11 ANS: 1

The probability of rain equals the probability of rain, given that Sean pitches.

PTS: 2
REF: 061611a
NAT: S.CP.A.3
TOP: Conditional Probability
12 ANS: 1

\[ x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)} = \frac{-3 \pm \sqrt{-7}}{4} = \frac{-3}{4} \pm \frac{i\sqrt{7}}{4} \]

PTS: 2   REF: 061612aii   NAT: A.REI.B.4   TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

13 ANS: 3

\[ H(t) \text{ is at a minimum at } 70(-1) + 80 = 10 \]

PTS: 2   REF: 061613aii   NAT: F.IF.B.4   TOP: Graphing Trigonometric Functions
KEY: maximum/minimum

14 ANS: 2

\[
\begin{align*}
2x^2 - 3x + 7 \\
2x + 3 & \bigg[ 4x^3 + 0x^2 + 5x + 10 \\
4x^3 + 6x^2 & \\
-6x^2 + 5x & \\
-6x^2 - 9x & \\
14x + 10 & \\
14x + 21 & \\
-11 & 
\end{align*}
\]

PTS: 2   REF: 061614aii   NAT: A.APR.D.6   TOP: Rational Expressions

15 ANS: 4

\[ y = 5^{-t} = \left( \frac{1}{5} \right)^t \]

PTS: 2   REF: 061615aii   NAT: F.IF.C.8   TOP: Modeling Exponential Functions
16 ANS: 2
\[
x = \frac{-3}{4}y + 2
\]
\[-4x = 3y - 8
\]
\[-4x + 8 = 3y
\]
\[\frac{4}{3}x + \frac{8}{3} = y\]

PTS: 2 REF: 061616aii NAT: F.BF.B.4 TOP: Inverse of Functions
KEY: equations

17 ANS: 1

PTS: 2 REF: 061617aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions
KEY: extension to reals

18 ANS: 1

PTS: 2 REF: 061618aii NAT: F.IF.C.7 TOP: Graphing Logarithmic Functions

19 ANS: 4
\[
4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324
\]
\[4(x - 3)^2 + 4(y + 9)^2 = 436\]

PTS: 2 REF: 061619aii NAT: G.GPE.A.1 TOP: Equations of Circles
KEY: completing the square

20 ANS: 2 PTS: 2 REF: 061620aii NAT: F.IF.B.4 TOP: Graphing Polynomial Functions
21 ANS: 3
\[ \frac{1}{12} \approx 1.00427 \]

PTS: 2 REF: 061621aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions
KEY: AII

22 ANS: 4

PTS: 2 REF: 061622aii NAT: A.REI.D.11 TOP: Other Systems
KEY: AII

23 ANS: 3
PTS: 2 REF: 061623aii NAT: F.BF.A.2 TOP: Sequences

24 ANS: 4
\[ \text{period} = \frac{2\pi}{B} \]
\[ \frac{1}{60} = \frac{2\pi}{B} \]
\[ B = 120\pi \]

PTS: 2 REF: 061624aii NAT: F.TF.B.5 TOP: Modeling Trigonometric Functions

25 ANS:
\[ \frac{1}{x} - \frac{1}{3} = -\frac{1}{3x} \]
\[ \frac{3-x}{3x} = -\frac{1}{3x} \]
\[ 3-x = -1 \]
\[ x = 4 \]

PTS: 2 REF: 061625aii NAT: A.REI.A.2 TOP: Solving Rationals
KEY: rational solutions
26 ANS:
Randomly assign participants to two groups. One group uses the toothpaste with ingredient X and the other group uses the toothpaste without ingredient X.

PTS: 2  REF: 061626aii  NAT: S.IC.B.3  TOP: Analysis of Data

27 ANS:
\[
x - 5 \left( \frac{2x^2 + 6x + 23}{x - 5} \right) \frac{2x^3 - 4x^2 - 7x - 10}{2x^3 - 10x^2} \frac{6x^2 - 7x}{6x^2 - 30x} \frac{23x - 10}{23x - 115} = 105
\]

Since there is a remainder, \( x - 5 \) is not a factor.

PTS: 2  REF: 061627aii  NAT: A.APR.B.2  TOP: Remainder Theorem

28 ANS:

PTS: 2  REF: 061628aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions

29 ANS:
\[
P(S \cap M) = P(S) + P(M) - P(S \cup M) = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376}
\]

PTS: 2  REF: 061629aii  NAT: S.CP.B.7  TOP: Theoretical Probability
30 ANS:

The vertex of the parabola is (4, -3). The x-coordinate of the focus and the vertex is the same. Since the distance from the vertex to the directrix is 3, the distance from the vertex to the focus is 3, so the y-coordinate of the focus is 0. The coordinates of the focus are (4, 0).

31 ANS:

\[
\frac{x^3 + 9}{x^3 + 8} = \frac{x^3 + 8 + 1}{x^3 + 8} = \frac{x^3 + 9}{x^3 + 8}
\]

32 ANS:

\[
A = Pe^{rt}
\]

\[
135000 = 100000e^{5r}
\]

\[
1.35 = e^{5r}
\]

\[
ln1.35 = ln e^{5r}
\]

\[
ln1.35 = 5r
\]

\[
.06 \approx r \text{ or } 6\%
\]
33 ANS:

\[ y = -x + 5 \quad y = -7 + 5 = -2 \]

\[ (x - 3)^2 + (-x + 5 + 2)^2 = 16 \quad y = -3 + 5 = 2 \]

\[ x^2 - 6x + 9 + x^2 - 14x + 49 = 16 \]

\[ 2x^2 - 20x + 42 = 0 \]

\[ x^2 - 10x + 21 = 0 \]

\[ (x - 7)(x - 3) = 0 \]

\[ x = 7, 3 \]

PTS: 4  REF: 061633aii  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems

KEY: AII

34 ANS:

\[ S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04} \]

\[ S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04} \approx 660778.39 \]

PTS: 4  REF: 061634aii  NAT: A.SSE.B.4  TOP: Series

35 ANS:

\[ 0.602 \pm 2 \cdot 0.066 = 0.47 - 0.73. \] Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

PTS: 4  REF: 061635aii  NAT: S.IC.B.5  TOP: Analysis of Data

36 ANS:

\[ \frac{f(4) - f(-2)}{4 - (-2)} = \frac{80 - 1.25}{6} = 13.125 \] \( g(x) \) has a greater rate of change

\[ \frac{g(4) - g(-2)}{4 - (-2)} = \frac{179 - 49}{6} = 38 \]

PTS: 4  REF: 061636aii  NAT: F.IF.B.6  TOP: Rate of Change

KEY: AII
\[ A(t) = 800e^{-0.347t} \]
\[ B(t) = 400e^{-0.231t} \]

\[ 800e^{-0.347t} = 400e^{-0.231t} \]
\[ 0.15 = e^{-0.347t} \]
\[ \ln 2 = \ln e^{-0.347t} = \ln e^{-0.231t} \]
\[ \ln 0.15 = \ln e^{-0.347t} \]
\[ \ln 2 + \ln e^{-0.347t} = \ln e^{-0.231t} \]
\[ \ln 0.15 = -0.347t \cdot \ln e \]
\[ \ln 2 - 0.347t = -0.231t \]
\[ 5.5 \approx t \]
\[ \ln 2 = 0.116t \]
\[ 6 \approx t \]
1 ANS: 4
If $1 - i$ is one solution, the other is $1 + i$.

$$(x - (1 - i))(x - (1 + i)) = 0$$

$$x^2 - x - ix - x + ix + (1 - i^2) = 0$$

$$x^2 - 2x + 2 = 0$$

PTS: 2    REF: 081601a    NAT: A.REI.B.4    TOP: Complex Conjugate Root Theorem

2 ANS: 1
II. Ninth graders drive to school less often; III. Students know little about adults; IV. Calculus students love math!

PTS: 2    REF: 081602a    NAT: S.IC.B.3    TOP: Analysis of Data

3 ANS: 2

PTS: 2    REF: 081603a    NAT: A.REI.D.11    TOP: Other Systems

4 ANS: 3

PTS: 2    REF: 081604a    NAT: S.ID.A.4    TOP: Normal Distributions

5 ANS: 3
$$(m - 2)^2(m + 3) = (m^2 - 4m + 4)(m + 3) = m^3 + 3m^2 - 4m^2 - 12m + 4m + 12 = m^3 - m^2 - 8m + 12$$

PTS: 2    REF: 081605a    NAT: A.SSE.A.2    TOP: Factoring Polynomials

KEY: factoring by grouping
6 ANS: 3

\[-33t^2 + 360t = 700 + 5t\]
\[-33t^2 + 355t - 700 = 0\]

\[t = \frac{-355 \pm \sqrt{355^2 - 4(-33)(-700)}}{2(-33)} \approx 3, 8\]

KEY: AII

7 ANS: 1

\[\frac{157}{25 + 47 + 157}\]

PTS: 2 REF: 081607aii NAT: S.CP.A.4 TOP: Conditional Probability

8 ANS: 1

(2) is not recursive

PTS: 2 REF: 081608aii NAT: F.BF.A.2 TOP: Sequences

9 ANS: 1 PTS: 2 REF: 081609aii NAT: F.BF.B.6

TOP: Sigma Notation KEY: represent

10 ANS: 2 PTS: 2 REF: 081610aii NAT: F.IF.B.4

TOP: Graphing Trigonometric Functions KEY: increasing/decreasing

11 ANS: 2

\[x^2 + 0x + 1\]
\[x + 2\]
\[x^3 + 2x^2 + x + 6\]

\[x^3 + 2x^2\]
\[0x^2 + x\]
\[0x^2 + 0x\]
\[x + 6\]
\[x + 2\]
\[4\]

PTS: 2 REF: 081611aii NAT: A.APR.D.6 TOP: Rational Expressions

12 ANS: 2

\[ME = \left( z \sqrt{\frac{p(1-p)}{n}} \right) = \left( 1.96 \sqrt{\frac{(0.55)(0.45)}{900}} \right) \approx 0.03\]

PTS: 2 REF: 081612aii NAT: S.IC.B.4 TOP: Analysis of Data
13 ANS: 1
The car lost approximately 19% of its value each year.

PTS: 2 REF: 081613aii NAT: F.LE.B.5 TOP: Modeling Exponential Functions

14 ANS: 1
The graph of \( y = \sin x \) is unchanged when rotated 180º about the origin.

PTS: 2 REF: 081614aii NAT: F.BF.B.3 TOP: Even and Odd Functions

15 ANS: 3
\[ 2d(d^3 + 3d^2 - 9d - 27) \]
\[ 2d(d^2(d + 3) - 9(d + 3)) \]
\[ 2d(d^2 - 9)(d + 3) \]
\[ 2d(d + 3)(d - 3)(d + 3) \]
\[ 2d(d + 3)^2(d - 3) \]

PTS: 2 REF: 081615aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: factoring by grouping

16 ANS: 1
PTS: 2 REF: 081616aii NAT: F.TF.A.1
TOP: Unit Circle

17 ANS: 3
\[ \frac{1}{J} = \frac{1}{F} - \frac{1}{W} \]
\[ \frac{1}{J} = \frac{W - F}{FW} \]
\[ J = \frac{FW}{W - F} \]

PTS: 2 REF: 081617aii NAT: A.REI.A.2 TOP: Solving Rationals
KEY: rational solutions

18 ANS: 3
PTS: 2 REF: 081618aii NAT: F.LE.A.2
TOP: Sequences

19 ANS: 4
The vertex is \((2, -1)\) and \(p = 2\). 
\[ y = \frac{-1}{4(2)}(x - 2)^2 - 1 \]

PTS: 2 REF: 081619aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

20 ANS: 4
\[ (x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3 \]

PTS: 2 REF: 081620aii NAT: A.APR.C.4 TOP: Polynomial Identities

21 ANS: 3
Since \(x + 4\) is a factor of \(p(x)\), there is no remainder.

PTS: 2 REF: 081621aii NAT: A.APR.B.2 TOP: Remainder Theorem
Combining (1) and (3): 
\[-6c = -18\] Combining (1) and (2): 
\[5a + 3c = -1\] Using (3): 
\[-(-2) - 5b - 5(3) = 2\] 
\[c = 3\] \[5a + 3(3) = -1\] \[2 - 5b - 15 = 2\] \[5a = -10\] \[b = -3\] \[a = -2\]

Amplitude, because the height of the graph shows the volume of the air.

Applying the commutative property, 
\[\left(\frac{1}{3^5}\right)^2\] can be rewritten as \(\left(3^2\right)^{\frac{1}{5}}\) or \(9^{\frac{1}{5}}\). A fractional exponent can be
rewritten as a radical with the denominator as the index, or \(9^{\frac{1}{5}} = \sqrt[5]{9}\).

\[x(-6i)^2 = x(36i^2) = 36xi^3 = -36xi\]

\[\sin^2 \theta + (-0.7)^2 = 1\] Since \(\theta\) is in Quadrant II, \(\sin \theta = \sqrt{0.51}\) and \(\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{0.51}}{-0.7} \approx -1.02\)
\[\sin^2 \theta = 0.51\]
\[\sin \theta = \pm \sqrt{0.51}\]

Using a 95% level of confidence, \(x \pm 2\) standard deviations sets the usual wait time as 150-302 seconds. 360 seconds is unusual.
30. ANS: 
\[ 0 = \log_{10}(x - 4) \] The x-intercept of \( h \) is \((2, 0)\). \( f \) has the larger value.

\[
\begin{align*}
10^0 &= x - 4 \\
1 &= x - 4 \\
x &= 5
\end{align*}
\]

PTS: 2  
REF: 081630aii  
NAT: F.IF.C.9  
TOP: Comparing Functions  
KEY: AII

31. ANS: 
\[
\frac{156.25 - 56.25}{70 - 50} = \frac{150}{20} = 7.5 \] Between 50-70 mph, each additional mph in speed requires 7.5 more feet to stop.

PTS: 2  
REF: 081631aii  
NAT: F.IF.B.6  
TOP: Rate of Change  
KEY: AII

32. ANS: 
\[
P(A \cup B) = P(A) + P(B) - P(A \cap B) \]  
\( A \) and \( B \) are independent since \( P(A \cap B) = P(A) \cdot P(B) \)

\[
\begin{align*}
0.8 &= 0.6 + 0.5 - P(A \cap B) \\
0.3 &= 0.6 \cdot 0.5 \\
P(A \cap B) &= 0.3
\end{align*}
\]

PTS: 2  
REF: 081632aii  
NAT: S.CP.A.2  
TOP: Theoretical Probability

33. ANS: 
\[
0 = x^2(x + 1) - 4(x + 1)
\]

\[
0 = (x^2 - 4)(x + 1)
\]

\[
0 = (x + 2)(x - 2)(x + 1)
\]

\[x = -2, -1, 2\]

PTS: 4  
REF: 081633aii  
NAT: F.IF.C.7  
TOP: Graphing Polynomial Functions
34 ANS: 

\[
7 = 20(0.5)^{\frac{t}{8.02}}, \\
\log 0.35 = \log 0.5^{\frac{t}{8.02}}, \\
\log 0.35 = \frac{t \log 0.5}{8.02}, \\
8.02 \log 0.35 = t, \\
t \approx 12
\]

PTS: 4 REF: 081634aii NAT: F.LE.A.4 TOP: Exponential Decay

35 ANS: 

\[
\left(\sqrt{2x - 7}\right)^2 = (5 - x)^2, \quad \sqrt{2(4) - 7 + 4} = 5, \quad \sqrt{2(8) - 7 + 8} = 5 \\
2x - 7 = 25 - 10x + x^2, \quad \sqrt{1} = 1, \quad \sqrt{9} \neq -3 \\
0 = x^2 - 12x + 32 \\
0 = (x - 8)(x - 4) \\
x = 4, 8
\]


36 ANS: 

Some of the students who did not drink energy drinks read faster than those who did drink energy drinks. 

17.7 – 19.1 = −1.4 Differences of -1.4 and less occur \(\frac{25}{232}\) or about 10% of the time, so the difference is not unusual.

PTS: 4 REF: 081636aii NAT: S.IC.B.5 TOP: Analysis of Data

37 ANS: 

\[
A = 5000(1.045)^n, \quad 5000 \left(1 + \frac{0.046}{4}\right)^{4n} - 5000(1.045)^6 \approx 6578.87 - 6511.30 \approx 67.57, \quad 10000 = 5000 \left(1 + \frac{0.046}{4}\right)^{4n} \\
B = 5000 \left(1 + \frac{0.046}{4}\right)^{4n} \\
2 = 1.0115^{4n} \\
\log 2 = 4n \log 1.0115 \\
\frac{\log 2}{4 \log 1.0115} = n \\
n \approx 15.2
\]

PTS: 6 REF: 081637aii NAT: A.CED.A.1 TOP: Exponential Growth
**0117AII Common Core State Standards**  
**Answer Section**

1. **ANS:** 2  
   **PTS:** 2  
   **REF:** 011701aii  
   **NAT:** F.IF.B.4  
   **TOP:** Graphing Trigonometric Functions

2. **ANS:** 1  
   
   \[ P(28) = 5(2)^{98} / 28 \approx 56 \]

   **PTS:** 2  
   **REF:** 011702aii  
   **NAT:** F.LE.A.2  
   **TOP:** Modeling Exponential Functions

3. **ANS:** 4  
   
   \[ m^5 + m^3 - 6m = m(m^4 + m^2 - 6) = m(m^2 + 3)(m^2 - 2) \]

   **PTS:** 2  
   **REF:** 011703aii  
   **NAT:** A.SSE.A.2  
   **TOP:** Factoring Polynomials

4. **ANS:** 1  
   **PTS:** 2  
   **REF:** 011704aii  
   **NAT:** F.TF.C.8  
   **TOP:** Simplifying Trigonometric Expressions

5. **ANS:** 4  
   
   \[ y = g(x) = (x - 2)^2 \]

   \[ (x - 2)^2 = 3x - 2 \]

   \[ y = 3(6) - 2 = 16 \]

   \[ x^2 - 4x + 4 = 3x - 2 \]

   \[ y = 3(1) - 2 = 1 \]

   \[ x^2 - 7x + 6 = 0 \]

   \[ (x - 6)(x - 1) = 0 \]

   \[ x = 6, 1 \]

   **PTS:** 2  
   **REF:** 011705aii  
   **NAT:** A.REI.C.7  
   **TOP:** Quadratic-Linear Systems

6. **ANS:** 3  
   **PTS:** 2  
   **REF:** 011706aii  
   **NAT:** S.IC.B.3  
   **TOP:** Analysis of Data

7. **ANS:** 2  
   
   \[ \left( \frac{5}{3} \right)^{\frac{1}{2}} = \frac{\sqrt{5}}{\sqrt{3}} = \frac{1}{\sqrt{m^6}} \]

   **PTS:** 2  
   **REF:** 011707aii  
   **NAT:** N.RN.A.2  
   **TOP:** Radicals and Rational Exponents

8. **ANS:** 3  
   **PTS:** 2  
   **REF:** 011708aii  
   **NAT:** F.BF.B.4  
   **TOP:** Inverse of Functions

9. **ANS:** 2  
   **PTS:** 2  
   **REF:** 011709aii  
   **NAT:** S.IC.B.5  
   **TOP:** Analysis of Data

10. **ANS:** 3  
    **PTS:** 2  
    **REF:** 011710aii  
    **NAT:** F.BF.A.1  
    **TOP:** Operations with Functions
11 ANS: 4

\[ x = \frac{8 \pm \sqrt{(-8)^2 - 4(6)(29)}}{2(6)} = \frac{8 \pm \sqrt{-632}}{12} = \frac{8 \pm i\sqrt{4 \cdot 158}}{12} = \frac{2}{3} \pm \frac{1}{6} i\sqrt{158} \]

PTS: 2  REF: 011711aii  NAT: A.REI.B.4  TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

12 ANS: 2

PTS: 2  REF: 011712aii  NAT: A.REI.D.11  TOP: Other Systems
KEY: AII

13 ANS: 3
The pattern suggests an exponential pattern, not linear or sinusoidal. A 4% growth rate is accurate, while a 43% growth rate is not.

PTS: 2  REF: 011713aii  NAT: S.ID.B.6  TOP: Regression
KEY: choose model

14 ANS: 1
\[ d = 18; \quad r = \pm \frac{5}{4} \]

PTS: 2  REF: 011714aii  NAT: F.IF.A.3  TOP: Sequences
KEY: term

15 ANS: 3
\[ d = 10\log\left(\frac{6.3 \times 10^{-3}}{1.0 \times 10^{-12}}\right) \approx 98 \]

PTS: 2  REF: 011715aii  NAT: F.IF.B.4  TOP: Evaluating Logarithmic Expressions

16 ANS: 2

PTS: 2  REF: 011716aii  NAT: A.REI.D.11  TOP: Other Systems
KEY: AII
17 ANS: 1
\[
\frac{2(x - 4)}{(x + 3)(x - 4)} + \frac{3(x + 3)}{(x - 4)(x + 3)} = \frac{2x - 2}{x^2 - x - 12}
\]
\[
2x - 8 + 3x + 9 = 2x - 2
\]
\[
3x = -3
\]
\[
x = -1
\]

PTS: 2 REF: 011717aii NAT: A.REI.A.2 TOP: Solving Rationals
KEY: rational solutions

18 ANS: 4
496 ± 2(115)

PTS: 2 REF: 011718aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: interval

19 ANS: 2
\( h(x) \) does not have a \( y \)-intercept.

PTS: 2 REF: 011719aii NAT: F.IF.C.9 TOP: Comparing Functions

20 ANS: 2 PTS: 2 REF: 011720aii NAT: A.APR.B.2 TOP: Remainder Theorem

21 ANS: 4
(1) \( \frac{B(60) - B(10)}{60 - 10} \approx 28\% \) (2) \( \frac{B(69) - B(19)}{69 - 19} \approx 33\% \) (3) \( \frac{B(72) - B(36)}{72 - 36} \approx 38\% \) (4) \( \frac{B(73) - B(60)}{73 - 60} \approx 46\% \)

PTS: 2 REF: 011721aii NAT: F.IF.B.6 TOP: Rate of Change
KEY: AII

22 ANS: 3
(3) repeats 3 times over \( 2\pi \).

PTS: 2 REF: 011722aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions
KEY: recognize

23 ANS: 1
\[
\frac{A}{p} = e^r
\]
\[
0.42 = e^r
\]
\[
\ln 0.42 = \ln e^r
\]
\[
-0.87 \approx rt
\]

PTS: 2 REF: 011723aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions
KEY: AII
24 ANS: 1

\[
\begin{align*}
(1) \quad \frac{9 - 0}{2 - 1} &= 9 \\
(2) \quad \frac{17 - 0}{3.5 - 1} &= 6.8 \\
(3) \quad \frac{0 - 0}{5 - 1} &= 0 \\
(4) \quad \frac{17 - 5}{3.5 - 1} &= 6.3
\end{align*}
\]

PTS: 2 
REF: 011724a
NAT: F.IF.B.6 
TOP: Rate of Change
KEY: AII

25 ANS:

\[
(1 - i)(1 - i)(1 - i) = (1 - 2i + i^2)(1 - i) = -2i(1 - i) = -2i + 2i^2 = -2 - 2i
\]

PTS: 2 
REF: 011725a
NAT: N.CN.A.2 
TOP: Operations with Complex Numbers

26 ANS:

time: pails of oranges; population: truckload of oranges. It is likely that about 5% of all the oranges are unsatisfactory.

PTS: 2 
REF: 011726a
NAT: S.IC.A.2 
TOP: Analysis of Data

27 ANS:

csc \theta = \frac{1}{\sin \theta}, \quad \text{and } \sin \theta \text{ on a unit circle represents the } y \text{ value of a point on the unit circle. Since } y = \sin \theta,

\[
csc \theta = \frac{1}{y}
\]

PTS: 2 
REF: 011727a
NAT: F.TF.A.2 
TOP: Reciprocal Trigonometric Relationships

28 ANS:

\[
\left\{ \ln \frac{1}{\sqrt{2}} \right\} \quad \text{is negative, so } M(t) \text{ represents decay.}
\]

PTS: 2 
REF: 011728a
NAT: F.IF.C.8 
TOP: Modeling Exponential Functions

29 ANS:

\[
\text{Graph}
\]

PTS: 2 
REF: 011729a
NAT: A.APR.B.3 
TOP: Zeros of Polynomials
30 ANS:
\[
\begin{pmatrix}
\frac{5}{3} \\
\frac{6}{5}
\end{pmatrix}
= \begin{pmatrix}
\frac{5}{3} \\
\frac{6}{5}
\end{pmatrix}
\]
\[x^2 = y\]

PTS: 2 REF: 011730aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents KEY: variables

31 ANS:
No, because \( P(M/R) \neq P(M) \)
\[
\frac{70}{\frac{180}{490}} \neq \frac{230}{490}
\]
\[0.38 \neq 0.47\]

PTS: 2 REF: 011731aii NAT: S.CP.A.4 TOP: Conditional Probability

32 ANS:
\[
\begin{array}{c}
\frac{3x + 13}{x - 2} \\
\frac{3x^2 + 7x - 20}{3x + 13} \\
\frac{6}{x - 2}
\end{array}
\]
\[
\begin{array}{c}
3x^2 - 6x \\
13x - 20 \\
13x - 26 \\
6
\end{array}
\]

PTS: 2 REF: 011732aii NAT: A.APR.D.6 TOP: Rational Expressions

33 ANS:
\[2x^3 - 10x^3 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k \quad h = -2
\]
\[-2x^3 + 8x + 5 = hx^2 - 4hx + k \quad k = 5
\]

PTS: 4 REF: 011733aii NAT: A.APR.C.4 TOP: Polynomial Identities

34 ANS:
Jillian’s plan, because distance increases by one mile each week. \( a_1 = 10 \quad a_n = n + 12 \)
\[a_n = a_{n-1} + 1
\]

PTS: 4 REF: 011734aii NAT: F.LE.A.2 TOP: Sequences

35 ANS:
\[
P(P/K) = \frac{P(P \land K)}{P(K)} = \frac{1.9}{\frac{2.3}{2}} \approx 82.6\%\quad \text{A key club member has an 82.6\% probability of being enrolled in AP Physics.}
\]

PTS: 4 REF: 011735aii NAT: S.CP.B.6 TOP: Conditional Probability
36 ANS:

\[
20000 = PMT \left( 1 - (1 + 0.00625)^{-60} \right) / 0.00625 = 21000 - x = 300 \left( 1 - (1 + 0.00625)^{-60} \right) / 0.00625
\]

\[
PMT \approx 400.76 \quad x \approx 6028
\]

PTS: 4 REF: 011736aii NAT: A.SSE.B.4 TOP: Series

37 ANS:

\[
0 = \sqrt{t} - 2t + 6 \quad 2 \left( \frac{9}{4} \right) - 6 < 0, \text{ so } \frac{9}{4} \text{ is extraneous.}
\]

\[
2t - 6 = \sqrt{t}
\]

\[
4t^2 - 24t + 36 = t
\]

\[
4t^2 - 25t + 36 = 0
\]

\[
(4t - 9)(t - 4) = 0
\]

\[
t = \frac{9}{4}, 4
\]

\[
(\sqrt{t} - 2(1) + 6) - (\sqrt{3} - 2(3) + 6) = 5 - \sqrt{3} \approx 3.268 \quad 327 \text{ mph}
\]

PTS: 6 REF: 011737aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: context
0617a11i

Answer Section

1. ANS: 1  PTS: 2  REF: 061701a1ii  NAT: A.APR.B.3
   TOP: Zeros of Polynomials  KEY: AII
   \[8(2^{x+3}) = 48\]
   \[2^{x+3} = 6\]
   \[(x + 3) \ln 2 = \ln 6\]
   \[x + 3 = \frac{\ln 6}{\ln 2}\]
   \[x = \frac{\ln 6}{\ln 2} - 3\]

2. ANS: 1  PTS: 2  REF: 061702a1ii  NAT: F.LE.A.4
   TOP: Exponential Equations  KEY: without common base
   Self selection causes bias.

3. ANS: 3  PTS: 2  REF: 061703a1ii  NAT: S.IC.B.3
   TOP: Analysis of Data  KEY: bias

4. ANS: 2  PTS: 2  REF: 061704a1ii  NAT: N.CN.A.2
   TOP: Operations with Complex Numbers
   \[6xi(-4xi + 5) = -24x^2i^3 + 30x^3 = -24x^2(1) + 30x(-1) = -24x^2 - 30xi\]

5. ANS: 2  PTS: 2  REF: 061705a1ii  NAT: A.REI.D.11
   TOP: Other Systems  KEY: AII

6. ANS: 4  PTS: 2  REF: 061706a1ii  NAT: F.IF.B.4
   TOP: Graphing Trigonometric Functions
7 ANS: 4
\[ 4x^2 = -98 \]
\[ x^2 = \frac{-98}{4} \]
\[ x^2 = \frac{-49}{2} \]
\[ x = \pm \sqrt{\frac{-49}{2}} = \pm \frac{7i}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm \frac{7i}{2} \]

PTS: 2 REF: 061707aii NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: complex solutions | taking square roots
8 ANS: 1 PTS: 2 REF: 061708aii NAT: F.IF.C.7
TOP: Graphing Trigonometric Functions KEY: identify
9 ANS: 2
\[ (30 - 0.01x) - (0.15x^3 + 0.01x^2 + 2x + 120) = 30x - 0.01x^2 - 0.15x^3 - 0.01x^2 - 2x - 120 \]
\[ = -0.15x^3 - 0.02x^2 + 28x - 120 \]

PTS: 2 REF: 061709aii NAT: F.BF.A.1 TOP: Operations with Functions
10 ANS: 3 PTS: 2 REF: 061710aii NAT: S.IC.A.2
TOP: Analysis of Data
11 ANS: 1
\[
\begin{array}{cccc}
2 & 1 & 0 & -4 & -4 & 8 \\
 & 2 & 4 & 0 & -8 \\
1 & 2 & 0 & -4 & 0 \\
\end{array}
\]
Since there is no remainder when the quartic is divided by \( x - 2 \), this binomial is a factor.

PTS: 2 REF: 061711aii NAT: A.APR.B.2 TOP: Remainder Theorem
12 ANS: 2
\[ \cos \theta = \pm \sqrt{1 - \left( \frac{-\sqrt{2}}{2} \right)^2} = \pm \sqrt{\frac{25}{25} - \frac{25}{25}} = \pm \frac{\sqrt{23}}{5} \]

PTS: 2 REF: 061712aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions
13 ANS: 3
\[ \frac{1}{0.75^{10}} \approx 9.761 \]

PTS: 2 REF: 061713aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions
KEY: AII
14  ANS: 2
The events are independent because $P(A \text{ and } B) = P(A) \cdot P(B)$.

$$0.125 = 0.5 \cdot 0.25$$

If $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.25 + 0.5 - 0.125 = 0.625$, then the events are not mutually exclusive because $P(A \text{ or } B) = P(A) + P(B)$

$$0.625 \neq 0.5 + 0.25$$

PTS: 2  REF: 061714aii  NAT: S.CP.B.7  TOP: Theoretical Probability

15  ANS: 4

| Minimum | 31.386 | 66.491 |
| Midline | 55.3   | 86.729 |
| Maximum | 79.214 | 106.967|
| Range   | 47.828 | 40.476 |

PTS: 2  REF: 061715aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions
KEY: maximum/minimum

16  ANS: 4  PTS: 2  REF: 061716aii  NAT: N.RN.A.2  TOP: Radicals and Rational Exponents
KEY: variables

17  ANS: 4
The vertex is $(1,0)$ and $p = 2$. $y = \frac{1}{4(2)}(x - 1)^2 + 0$

PTS: 2  REF: 061717aii  NAT: G.GPE.A.2  TOP: Graphing Quadratic Functions

18  ANS: 2
The 2010 population is 110 million.

PTS: 2  REF: 061718aii  NAT: F.LE.B.5  TOP: Modeling Exponential Functions

19  ANS: 1
$$\frac{2x}{x-2} \left( \frac{x}{x} \right) - \frac{11}{x} \left( \frac{x-2}{x} \right) = \frac{8}{x^2 - 2x}$$

$2x^2 - 11x + 22 = 8$

$2x^2 - 11x + 14 = 0$

$(2x - 7)(x - 2) = 0$

$x = \frac{7}{2} , 2$

PTS: 2  REF: 061719aii  NAT: A.REI.A.2  TOP: Solving Rationals

20  ANS: 3  PTS: 2  REF: 061720aii  NAT: F.LE.A.2  TOP: Sequences
KEY: AII
21 ANS: 3
\[
\frac{f(7) - f(-7)}{7 - (-7)} = \frac{2^{-0.25(7)} \cdot \sin\left(\frac{\pi}{2} (7)\right) - 2^{-0.25(-7)} \cdot \sin\left(\frac{\pi}{2} (-7)\right)}{14} \approx -0.26
\]

PTS: 2  REF: 061721aii  NAT: F.IF.B.6  TOP: Rate of Change
KEY: AII

22 ANS: 3

PTS: 2  REF: 061722aii  NAT: A.CED.A.1  TOP: Modeling Rationals

23 ANS: 4
\[
\frac{-3x^2 - 5x + 2}{x^3 + 2x^2} = \frac{(-3x + 1)(x + 2)}{x^2(x + 2)} = \frac{-3x}{x^2} + \frac{1}{x^2} = -3x^{-1} + x^{-2}
\]

PTS: 2  REF: 061723aii  NAT: A.APR.D.6  TOP: Expressions with Negative Exponents
KEY: variables

24 ANS: 2

TOP: Series

25 ANS:
r(2) = -6. Since there is a remainder when the cubic is divided by \(x - 2\), this binomial is not a factor.

\[
\begin{array}{c|cccc}
\# & 1 & -4 & 4 & 6 \\
\hline
& 2 & -4 & 0 & \hline
1 & -2 & 0 & -6 & \hline
\end{array}
\]

PTS: 2  REF: 061725aii  NAT: A.APR.B.2  TOP: Remainder Theorem

26 ANS:

PTS: 2  REF: 061726aii  NAT: S.ID.A.4  TOP: Normal Distributions
KEY: percent

27 ANS:
\[
x^2(4x - 1) + 4(4x - 1) = (x^2 + 4)(4x - 1)
\]

PTS: 2  REF: 061727aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials
KEY: factoring by grouping

28 ANS:
period is \(\frac{2}{3}\). The wheel rotates once every \(\frac{2}{3}\) second.

PTS: 2  REF: 061728aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions
KEY: period
29 ANS:

![Graph of exponential function](image)

PTS: 2  REF: 061729aii  NAT: F.IF.C.7  TOP: Graphing Exponential Functions
KEY: AII

30 ANS:
\[
\sqrt{x - 4} = -x + 6 \quad \text{is extraneous.}
\]
\[
x - 4 = x^2 - 12x + 36
\]
\[
0 = x^2 - 13x + 40
\]
\[
0 = (x - 8)(x - 5)
\]
\[
x = 5, 8
\]

PTS: 2  REF: 061730aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: extraneous solutions

31 ANS:
\[
\sqrt[3]{x} \cdot \sqrt{x} = x^{1/3} \cdot x^{1/2} = x^{5/6}
\]

KEY: with variables, index > 2

32 ANS:
A student is more likely to jog if both siblings jog.  1 jogs: \( \frac{416}{2239} \approx 0.19 \).  both jog: \( \frac{400}{1780} \approx 0.22 \)

PTS: 2  REF: 061732aii  NAT: S.CP.A.4  TOP: Conditional Probability

33 ANS:
\[
x + y + z = 1 \quad 2x + 2y + 2z = 2 \quad -2z - z = 3 \quad y - (-1) = 3 \quad x + 2 - 1 = 1
\]
\[
-x + 3y - 5z = 11 \quad 2x + 4y + 6z = 2 \quad -3z = 3 \quad y = 2 \quad x = 0
\]
\[
4y - 4z = 12 \quad 2y + 4z = 0 \quad z = -1
\]
\[
y - z = 3 \quad y + 2z = 0
\]
\[
y = -2z
\]

PTS: 4  REF: 061733aii  NAT: A.REL.C.6  TOP: Solving Linear Systems
KEY: three variables
34 ANS:
\[ M = 172600 \cdot \frac{0.00305(1 + 0.00305)^{12 \cdot 15}}{(1 + 0.00305)^{12 \cdot 15} - 1} \approx 1247 \]
\[ 1100 = (172600 - x) \cdot \frac{0.00305(1 + 0.00305)^{12 \cdot 15}}{(1 + 0.00305)^{12 \cdot 15} - 1} \]
\[ 1100 \approx (172600 - x) \cdot (0.007228) \]
\[ 152193 \approx 172600 - x \]
\[ 20407 \approx x \]

PTS: 4 REF: 061734aii NAT: A.SSE.B.4 TOP: Series

35 ANS:
\[
\text{As } x \to -3, y \to -\infty. \quad \text{As } x \to \infty, y \to \infty.
\]

PTS: 4 REF: 061735aii NAT: F.IF.C.7 TOP: Graphing Logarithmic Functions

36 ANS:
\[ 0.506 \pm 2 \cdot 0.078 = 0.35 - 0.66. \] The 32.5% value falls below the 95% confidence level.

PTS: 4 REF: 061736aii NAT: S.IC.B.5 TOP: Analysis of Data

37 ANS:
\[ 100 = 140 \left( \frac{1}{2} \right)^{\frac{5}{h}} \rightarrow \log \frac{100}{140} = \log \left( \frac{1}{2} \right)^{\frac{5}{h}} \]
\[ 40 = 140 \left( \frac{1}{2} \right)^{\frac{t}{10.3002}} \rightarrow \log 2 = \log \left( \frac{1}{2} \right)^{\frac{t}{10.3002}} \]
\[ h = \frac{5 \log \frac{1}{2}}{\log \frac{5}{7}} \approx 10.3002 \]
\[ t = \frac{10.3002 \log \frac{2}{7}}{\log \frac{1}{2}} \approx 18.6 \]

PTS: 6 REF: 061737aii NAT: F.LE.A.4 TOP: Exponential Decay
0817AII Common Core State Standards

Answer Section

1. ANS: 1
   \[ x^2 + 2x - 8 = 0 \]
   \[(x + 4)(x - 2) = 0 \]
   \[ x = -4, 2 \]

   PTS: 2   REF: 081701aii   NAT: A.APR.D.6   TOP: Undefined Rationals

2. ANS: 3
   \[(3k - 2i)^2 = 9k^2 - 12ki - 4 \]

   PTS: 2   REF: 081702aii   NAT: N.CN.A.2   TOP: Operations with Complex Numbers

3. ANS: 3
   \[ x^2 + 2x + 1 = -5 + 1 \]
   \[(x + 1)^2 = -4 \]
   \[ x = -1 \pm 2i \]

   PTS: 2   REF: 081703aii   NAT: A.REI.B.4   TOP: Solving Quadratics
   KEY: complex solutions | completing the square

4. ANS: 2
   \[ \sqrt{x + 14} = \sqrt{2x + 5} + 1 \]
   \[ \sqrt{22 + 14} - \sqrt{2(22) + 5} = 1 \]
   \[ x + 14 = 2x + 5 + 2\sqrt{2x + 5} + 1 \]
   \[ -x + 8 = 2\sqrt{2x + 5} \]
   \[ x^2 - 16x + 64 = 8x + 20 \]
   \[ x^2 - 24x + 44 = 0 \]
   \[(x - 22)(x - 2) = 0 \]
   \[ x = 2, 22 \]

   PTS: 2   REF: 081704aii   NAT: A.REI.A.2   TOP: Solving Radicals
   KEY: advanced

5. ANS: 3
   PTS: 2

   REF: 081705aii   NAT: F.IF.B.4   TOP: Graphing Trigonometric Functions
   KEY: increasing/decreasing
6 ANS: 2
The vertex of the parabola is (0,0). The distance, \( p \), between the vertex and the focus or the vertex and the
directrix is 1. \( y = \frac{-1}{4p} (x - h)^2 + k \)
\[ y = \frac{-1}{4(1)} (x - 0)^2 + 0 \]
\[ y = -\frac{1}{4} x^2 \]

PTS: 2 REF: 081706aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

7 ANS: 4 PTS: 2 REF: 081707aii NAT: F.TF.A.2
TOP: Reference Angles

8 ANS: 4 PTS: 2 REF: 081708aii NAT: A.APR.B.3
TOP: Zeros of Polynomials KEY: AII

9 ANS: 3
\[ \log_{0.8}\left(\frac{V}{1700}\right) = t \]
\[ \frac{17,000(0.8)^3 - 17,000(0.8)^1}{3 - 1} \approx -2450 \]
\[ 0.8^t = \frac{V}{17000} \]
\[ V = 17000(0.8)^t \]

PTS: 2 REF: 081709aii NAT: F.IF.B.6 TOP: Rate of Change KEY: AII

10 ANS: 3
\[ \left(\frac{1}{2}\right)^{\frac{1}{73.83}} \approx 0.990656 \]

PTS: 2 REF: 081710aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions KEY: AII

11 ANS: 1

PTS: 2 REF: 081711aii NAT: S.ID.A.4 TOP: Normal Distributions KEY: percent
The maximum volume of \( p(x) = -(x + 2)(x - 10)(x - 14) \) is about 56, at \( x = 12.1 \)

\[
\frac{2x^2 + x + 5}{2x - 1} = \frac{4x^3 + 0x^2 + 9x - 5}{4x^3 - 2x^2} - \frac{2x^2 + 9x}{2x^2 - x} - \frac{10x - 5}{10x - 5}
\]

\[
x = \frac{y + 1}{y - 2}
\]
\[
xy - 2x = y + 1
\]
\[
xy - y = 2x + 1
\]
\[
y(x - 1) = 2x + 1
\]
\[
y = \frac{2x + 1}{x - 1}
\]

\( ME = \left( z \sqrt{\frac{p(1-p)}{n}} \right) = \left( 1.96 \sqrt{\frac{(0.16)(0.84)}{1334}} \right) \approx 0.02 \)
17 ANS: 2   PTS: 2   REF: 081717aii   NAT: S.IC.B.3
   TOP: Analysis of Data   KEY: type
18 ANS: 4   PTS: 2   REF: 081718aii   NAT: F.IF.C.7
   TOP: Graphing Trigonometric Functions   KEY: amplitude
19 ANS: 1
   \[(x + 3)^2 + (2x - 4)^2 = 8 \quad b^2 - 4ac \]
   \[x^2 + 6x + 9 + 4x^2 - 16x + 16 = 8 \quad 100 - 4(5)(17) < 0 \]
   \[5x^2 - 10x + 17 = 0 \]
   PTS: 2   REF: 081719aii   NAT: A.REI.C.7   TOP: Quadratic-Linear Systems
   KEY: AII
20 ANS: 2
   \[
   \begin{array}{ccc}
   -4 & 1 & -11 \\
   -60 & 16 & 84 \\
   1 & -15 & 76
   \end{array}
   \]
   Since there is a remainder when the cubic is divided by \(x + 4\), this binomial is not a factor.
   PTS: 2   REF: 081720aii   NAT: A.APR.B.2   TOP: Remainder Theorem
21 ANS: 4
   \[d = 32(0.8)^{b-1} \quad S_n = \frac{32 - 32(0.8)^{12}}{1 - 0.8} \approx 149\]
   PTS: 2   REF: 081721aii   NAT: A.SSE.B.4   TOP: Series
22 ANS: 1   PTS: 2   REF: 081722aii   NAT: S.IC.B.6
   TOP: Analysis of Data
23 ANS: 4
   \[
   \left(\frac{-54x^9}{y^4}\right)^\frac{2}{3} = \left(\frac{2 \cdot -27}{x^3}\right)^{\frac{2}{3}} = \frac{2^\frac{2}{3} \cdot 9x^6}{y^2 \cdot \sqrt[3]{y^2}} = \frac{9x^6 \sqrt[3]{4}}{y^2 \sqrt[3]{y^2}}
   \]
   PTS: 2   REF: 081723aii   NAT: N.RN.A.2   TOP: Radicals and Rational Exponents
   KEY: variables
24 ANS: 3   PTS: 2   REF: 081724aii   NAT: F.BF.A.2
   TOP: Sequences
25 ANS:
   Rewrite \(\frac{4}{3}\) as \(\frac{1}{3} \cdot \frac{4}{1}\), using the power of a power rule.
   PTS: 2   REF: 081725aii   NAT: N.RN.A.1   TOP: Radicals and Rational Exponents
26 ANS:
   \[P(W/D) = \frac{P(W \cap D)}{P(D)} = \frac{0.4}{0.5} \approx 0.8\]
   PTS: 2   REF: 081726aii   NAT: S.CP.B.6   TOP: Conditional Probability
27 ANS:
\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\]
\[x^4 + 2x^2y^2 + y^4 = x^4 - 2x^2y^2 + y^4 + 4x^2y^4\]
\[x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4\]

PTS: 2 REF: 081727aii NAT: A.APR.C.4 TOP: Polynomial Identities

28 ANS:
Since there are six flavors, each flavor can be assigned a number, 1-6. Use the simulation to see the number of times the same number is rolled 4 times in a row.

PTS: 2 REF: 081728aii NAT: S.IC.A.2 TOP: Analysis of Data

29 ANS:
\[a_1 = 4 \quad a_8 = 639\]
\[a_n = 2a_{n-1} + 1\]

PTS: 2 REF: 081729aii NAT: F.LE.A.2 TOP: Sequences

30 ANS:
\[8.75 = 1.25x^{49} \quad 4\]
\[7 = x^{49}\]
\[x = 49^{\sqrt{7}} \approx 1.04\]

PTS: 2 REF: 081730aii NAT: F.LE.A.4 TOP: Exponential Growth

31 ANS:
\[j(-x) = (-x)^4 - 3(-x)^2 - 4 = x^4 - 3x^2 - 4\]
Since \(j(x) = j(-x)\), the function is even.

PTS: 2 REF: 081731aii NAT: F.BF.B.3 TOP: Even and Odd Functions

32 ANS:

PTS: 2 REF: 081732aii NAT: A.APR.B.3 TOP: Zeros of Polynomials

KEY: AII
33 ANS:

\[
\frac{3p}{p-5} = \frac{p+2}{p+3}
\]

\[3p^2 + 9p = p^2 - 3p - 10\]

\[2p^2 + 12p + 10 = 0\]

\[p^2 + 6p + 5 = 0\]

\[(p + 5)(p + 1) = 0\]

\[p = -5, -1\]

PTS: 4 REF: 081733aii NAT: A.REI.A.2 TOP: Solving Rationals
KEY: rational solutions

34 ANS:

\[
\frac{6.25 - 2.25}{21 - 5} = \frac{4}{16} = .25 \text{ fine per day. } 2.25 - 5(.25) = \$1 \text{ replacement fee. } a_n = 1.25 + (n - 1)(.25). \quad a_{60} = \$16
\]

PTS: 4 REF: 081734aii NAT: F.LE.A.2 TOP: Sequences

35 ANS:

![Graph](image)

Part a sketch is shifted \(\frac{\pi}{3}\) units right.

PTS: 4 REF: 081735aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions
KEY: graph
36 ANS:

\[ y = 4.168(3.981)^{x} \]
\[ 100 = 4.168(3.981)^{x} \]

\[ \log \frac{100}{4.168} = \log(3.981)^{x} \]
\[ \log \frac{100}{4.168} = x \log(3.981) \]

\[ \frac{\log 100}{\log(3.981)} = x \]

\[ x \approx 2.25 \]

PTS: 4 REF: 081736aii NAT: S.ID.B.6 TOP: Regression KEY: exponential AII

37 ANS:

At 1.95 years, the value of the car equals the loan balance. Zach can cancel the policy after 6 years.