1. If \( a, b, \) and \( c \) are all positive real numbers, which graph could represent the sketch of the graph of \( p(x) = -a(x + b)\left(x^2 - 2cx + c^2\right) \)?

2. Which equation represents a parabola with a focus of \((0,4)\) and a directrix of \(y = 2\)?
   1) \( y = x^2 + 3 \)
   2) \( y = -x^2 + 1 \)
   3) \( y = \frac{x^2}{2} + 3 \)
   4) \( y = \frac{x^2}{4} + 3 \)

3. If the terminal side of angle \( \theta \), in standard position, passes through point \((-4,3)\), what is the numerical value of \( \sin \theta \)?
   1) \( \frac{3}{5} \)
   2) \( \frac{4}{5} \)
   3) \( -\frac{3}{5} \)
   4) \( -\frac{4}{5} \)

4. A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, \( B(t) \), can be represented by the function \( B(t) = 750(1.16)^t \), where the \( t \) represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function
   1) \( B(t) = 750(1.012)^{12t} \)
   2) \( B(t) = 750(1.012)^{12t} \)
   3) \( B(t) = 750(1.16)^{\frac{t}{12}} \)
   4) \( B(t) = 750(1.16)^{\frac{t}{12}} \)

5. Use the properties of rational exponents to determine the value of \( y \) for the equation:
   \[ \sqrt[3]{x^8} = x^y, \; x > 1 \]
   \[ \left(x^4\right)^{\frac{1}{3}} = x^y, \; x > 1 \]
6. Write \((5+2yi)(4-3i) - (5-2yi)(4-3i)\) in \(a+bi\) form, where \(y\) is a real number.

7. Use an appropriate procedure to show that \(x-4\) is a factor of the function \(f(x) = 2x^3 - 5x^2 - 11x - 4\). Explain your answer.

8. Solve algebraically for all values of \(x\):
\[\sqrt{x-5} + x = 7\]

9. Monthly mortgage payments can be found using the formula below:
\[M = \frac{P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n}{\left( 1 + \frac{r}{12} \right)^n - 1}\]

\(M\) = monthly payment  
\(P\) = amount borrowed  
\(r\) = annual interest rate  
\(n\) = number of monthly payments

The Banks family would like to borrow $120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the fewest number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than $720.

10. Solve the following system of equations algebraically for all values of \(x, y,\) and \(z\):
\[
\begin{align*}
    x + 3y + 5z & = 45 \\
    6x - 3y + 2z & = -10 \\
    -2x + 3y + 8z & = 72
\end{align*}
\]

11. Write an explicit formula for \(a_n\), the \(n\)th term of the recursively defined sequence below.
\[a_1 = x + 1\]
\[a_n = x(a_{n-1})\]
For what values of \(x\) would \(a_n = 0\) when \(n > 1\)?

12. Stephen’s Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products \(A, B,\) and the new product. Nine out of fifty participants preferred Stephen’s new cola to products \(A\) and \(B\). The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen’s new product, each of sample size 50, simulated 100 times.

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.
13 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on both issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

14 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form \( f(t) = A \cos(Bt) \), where \( A \) and \( B \) are real numbers, that models the water level, \( f(t) \), in inches above or below the average Carter Beach sea level, as a function of the time measured in \( t \) hours since 8:30 a.m. On the grid below, graph one cycle of this function.

People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.
15 What is the solution set of the equation \( \frac{3x + 25}{x + 7} - 5 = \frac{3}{x} \)?

1) \( \left\{ \frac{3}{2}, 7 \right\} \)
2) \( \left\{ \frac{7}{2}, -3 \right\} \)
3) \( \left\{ -\frac{3}{2}, 7 \right\} \)
4) \( \left\{ -\frac{7}{2}, -3 \right\} \)

16 Functions \( f, g, \) and \( h \) are given below.

\[ f(x) = \sin(2x) \]
\[ g(x) = f(x) + 1 \]

17 The expression \( \frac{6x^3 + 17x^2 + 10x + 2}{2x + 3} \) equals

1) \( 3x^2 + 4x - 1 + \frac{5}{2x + 3} \)
2) \( 6x^2 + 8x - 2 + \frac{5}{2x + 3} \)
3) \( 6x^2 - x + 13 - \frac{37}{2x + 3} \)
4) \( 3x^2 + 13x + \frac{49}{2} + \frac{151}{2x + 3} \)

18 The solutions to the equation \( -\frac{1}{2} x^2 = -6x + 20 \) are

1) \( -6 \pm 2i \)
2) \( -6 \pm 2\sqrt{19} \)
3) \( 6 \pm 2i \)
4) \( 6 \pm 2\sqrt{19} \)

19 What is the completely factored form of \( k^4 - 4k^3 + 8k^2 - 32k + 12k^2 - 48 \)?

1) \( (k - 2)(k - 2)(k + 3)(k + 4) \)
2) \( (k - 2)(k + 2)(k + 6)(k + 2) \)
3) \( (k + 2)(k - 2)(k + 3)(k + 4) \)
4) \( (k + 2)(k - 2)(k + 6)(k + 2) \)

20 Which statement is incorrect for the graph of the function \( y = -3 \cos \left( \frac{\pi}{3} (x - 4) \right) + 7 \)?

1) The period is 6.
2) The amplitude is 3.
3) The range is \([4, 10]\).
4) The midline is \( y = -4 \).

21 Algebraically determine the values of \( x \) that satisfy the system of equations below.

\[ y = -2x + 1 \]
\[ y = -2x^2 + 3x + 1 \]
22. The results of a poll of 200 students are shown in the table below:

<table>
<thead>
<tr>
<th>Preferred Music Style</th>
<th>Techno</th>
<th>Rap</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>54</td>
<td>25</td>
<td>27</td>
</tr>
<tr>
<td>Male</td>
<td>36</td>
<td>40</td>
<td>18</td>
</tr>
</tbody>
</table>

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

23. For the function \( f(x) = (x - 3)^3 + 1 \), find \( f^{-1}(x) \).

24. Given: \( h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2 \)

\[ k(x) = -|0.7x| + 5 \]

State the solutions to the equation \( h(x) = k(x) \), rounded to the nearest hundredth.

25. Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

26. Rewrite the expression \[ \left(4x^2 + 5x\right)^2 - 5\left(4x^2 + 5x\right) - 6 \] as a product of four linear factors.

27. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton’s Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

\[ T = T_a + (T_0 - T_a)e^{-kt} \]

\( T_a \) = the temperature surrounding the object
\( T_0 \) = the initial temperature of the object
\( t \) = the time in hours
\( T \) = the temperature of the object after \( t \) hours
\( k \) = decay constant

The turkey reaches the temperature of approximately 100°F after 2 hours. Find the value of \( k \), to the nearest thousandth, and write an equation to determine the temperature of the turkey after \( t \) hours. Determine the Fahrenheit temperature of the turkey, to the nearest degree, at 3 p.m.
28 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups’ final grades is shown below:

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>80.16</td>
<td>83.8</td>
</tr>
<tr>
<td>( S_x )</td>
<td>6.9</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students’ final grades were rerandomized 500 times. The results are shown below.

Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

29 Given \( z(x) = 6x^3 + bx^2 - 52x + 15 \), \( z(2) = 35 \), and \( z(-5) = 0 \), algebraically determine all the zeros of \( z(x) \).

30 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the nearest ten thousandth, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

31 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the nearest tenth. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.
1 When $b > 0$ and $d$ is a positive integer, the expression $(3b)^\frac{2}{d}$ is equivalent to
1) $\frac{1}{\left(\sqrt[4]{3b}\right)^2}$
2) $\left(\sqrt[3]{3b}\right)^d$
3) $\frac{1}{\sqrt[3]{3b^d}}$
4) $\left(\sqrt[4]{3b}\right)^2$

2 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, $T$, are left in the semester?
1) $\frac{255 + 93T}{3T} = 90$
2) $\frac{255 + 90T}{3T} = 93$
3) $\frac{255 + 93T}{T+3} = 90$
4) $\frac{255 + 90T}{T+3} = 93$

3 Given $i$ is the imaginary unit, $(2 - yi)^2$ in simplest form is
1) $y^2 - 4yi + 4$
2) $-y^2 - 4yi + 4$
3) $-y^2 + 4$
4) $y^2 + 4$

4 Which graph has the following characteristics?
• three real zeros
• as $x \to -\infty$, $f(x) \to -\infty$
• as $x \to \infty$, $f(x) \to \infty$

5 The solution set for the equation $\sqrt{56-x} = x$ is
1) $\{-8, 7\}$
2) $\{-7, 8\}$
3) $\{7\}$
4) $\{}$
6 The zeros for \( f(x) = x^4 - 4x^3 - 9x^2 + 36x \) are
1) \( \{0, \pm 3, 4\} \)
2) \( \{0, 3, 4\} \)
3) \( \{0, \pm 3, -4\} \)
4) \( \{0, 3, -4\} \)

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Given the results of her coin flips and of her computer simulation, which statement is most accurate?
1) 73 of the computer's next 100 coin flips will be heads.
2) 50 of her next 100 coin flips will be heads.
3) Her coin is not fair.
4) Her coin is fair.

8 If \( g(c) = 1 - c^2 \) and \( m(c) = c + 1 \), then which statement is not true?
1) \( g(c) \cdot m(c) = 1 + c - c^2 - c^3 \)
2) \( g(c) + m(c) = 2 + c - c^2 \)
3) \( m(c) - g(c) = c + c^2 \)
4) \( \frac{m(c)}{g(c)} = \frac{-1}{1 - c} \)

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the nearest whole percent, is
1) 6
2) 48
3) 68
4) 95

10 The formula below can be used to model which scenario?

\[ a_1 = 3000 \]
\[ a_n = 0.80a_{n-1} \]
1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of $3000, and each year it grows by 80%.
4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are
1) independent
2) dependent
3) mutually exclusive
4) complements

12 A solution of the equation \( 2x^2 + 3x + 2 = 0 \) is
1) \( -\frac{3}{4} + \frac{1}{4}i\sqrt{7} \)
2) \( -\frac{3}{4} + \frac{1}{4}i \)
3) \( -\frac{3}{4} + \frac{1}{4}\sqrt{7} \)
4) \( \frac{1}{2} \)
13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, \(H\), in feet, above the ground of one of the six-person cars can be modeled by
\[
H(t) = 70 \sin \left( \frac{2\pi}{7} (t - 1.75) \right) + 80,
\]
where \(t\) is time, in minutes. Using \(H(t)\) for one full rotation, this car's minimum height, in feet, is
1) 150
2) 70
3) 10
4) 0

14 The expression \(\frac{4x^3 + 5x + 10}{2x + 3}\) is equivalent to
1) \(2x^2 + 3x - 7 + \frac{31}{2x + 3}\)
2) \(2x^2 - 3x + 7 - \frac{11}{2x + 3}\)
3) \(2x^2 + 2.5x + 5 + \frac{15}{2x + 3}\)
4) \(2x^2 - 2.5x - 5 - \frac{20}{2x + 3}\)

15 Which function represents exponential decay?
1) \(y = 2^{0.3t}\)
2) \(y = 1.2^{3t}\)
3) \(y = \left( \frac{1}{2} \right)^{-t}\)
4) \(y = 5^{-t}\)

16 Given \(f^{-1}(x) = -\frac{3}{4}x + 2\), which equation represents \(f(x)\)?
1) \(f(x) = \frac{4}{3}x - \frac{8}{3}\)
2) \(f(x) = \frac{4}{3}x + \frac{8}{3}\)
3) \(f(x) = \frac{3}{4}x - 2\)
4) \(f(x) = -\frac{3}{4}x + 2\)

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, \(\theta\), intercepts the circle in Quadrant II at point \(C\). The \(y\)-coordinate of point \(C\) is 8. What is the value of \(\cos \theta\)?
1) \(-\frac{3}{5}\)
2) \(-\frac{3}{4}\)
3) \(\frac{3}{5}\)
4) \(\frac{4}{5}\)

18 Which statement about the graph of \(c(x) = \log_{10}x\) is false?
1) The asymptote has equation \(y = 0\).
2) The graph has no \(y\)-intercept.
3) The domain is the set of positive reals.
4) The range is the set of all real numbers.

19 The equation \(4x^2 - 24x + 4y^2 + 72y = 76\) is equivalent to
1) \(4(x - 3)^2 + 4(y + 9)^2 = 76\)
2) \(4(x - 3)^2 + 4(y + 9)^2 = 121\)
3) \(4(x - 3)^2 + 4(y + 9)^2 = 166\)
4) \(4(x - 3)^2 + 4(y + 9)^2 = 436\)
20 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.

Which statement about this function is incorrect?
1) The degree of the polynomial is even.
2) There is a positive leading coefficient.
3) At two pH values, there is a relative maximum value.
4) There are two intervals where the function is decreasing.

21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]
1) $(1.0525)^m$
2) $\left(1.0525\right)^{\frac{12}{m}}$
3) $(1.00427)^m$
4) $(1.00427)^{\frac{m}{12}}$

22 Which value, to the nearest tenth, is not a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$?
1) $-3.9$
2) $-1.1$
3) $2.1$
4) $4.7$

23 The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:
250,000  250,937  251,878  252,822
How can this sequence be recursively modeled?
1) $j_n = 250,000(1.00375)^{n-1}$
2) $j_n = 250,000 + 937^{(n-1)}$
3) $j_1 = 250,000$
4) $j_1 = 250,000$
   
   $j_n = 1.00375j_{n-1}$
   $j_n = j_{n-1} + 937$

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Which equation best represents the value of the voltage as it flows through the electric wires, where $t$ is time in seconds?
1) $V = 120\sin(t)$
2) $V = 120\sin(60t)$
3) $V = 120\sin(60\pi t)$
4) $V = 120\sin(120\pi t)$

25 Solve for $x$: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

26 Describe how a controlled experiment can be created to examine the effect of ingredient $X$ in a toothpaste.

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.
28 On the axes below, graph one cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

31 Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the nearest percent.

33 Solve the system of equations shown below algebraically.

\begin{align*}
(x - 3)^2 + (y + 2)^2 &= 16 \\
2x + 2y &= 10
\end{align*}

34 Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, $S_n$, for Alexa's total earnings over $n$ years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the nearest cent.

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the nearest hundredth. Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.
36 Which function shown below has a greater average rate of change on the interval \([-2,4]\)? Justify your answer.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0.3125</td>
</tr>
<tr>
<td>-3</td>
<td>0.625</td>
</tr>
<tr>
<td>-2</td>
<td>1.25</td>
</tr>
<tr>
<td>-1</td>
<td>2.5</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
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<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
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</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>320</td>
</tr>
</tbody>
</table>

g(x) = 4x^3 - 5x^2 + 3

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function \(N(t) = N_0(e^{-rt})\), where \(N(t)\) is the amount left in the body, \(N_0\) is the initial dosage, \(r\) is the decay rate, and \(t\) is time in hours. Patient A, \(A(t)\), is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, \(B(t)\), is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, \(A(t)\) and \(B(t)\), to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

To the nearest hour, \(t\), when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A? The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the nearest tenth of an hour, how long patient A will have to wait to take another 800 milligram dose of the drug.
1 Which equation has \(1 - i\) as a solution?
   1) \(x^2 + 2x - 2 = 0\)
   2) \(x^2 + 2x + 2 = 0\)
   3) \(x^2 - 2x - 2 = 0\)
   4) \(x^2 - 2x + 2 = 0\)

2 Which statement(s) about statistical studies is true?
   I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
   II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
   III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
   IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don’t like math.
   1) I, only
   2) II, only
   3) I and III
   4) III and IV

3 To the nearest tenth, the value of \(x\) that satisfies \(2^x = -2x + 11\) is
   1) 2.5
   2) 2.6
   3) 5.8
   4) 5.9

4 The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?
   1) 0.3803
   2) 0.4612
   3) 0.8415
   4) 0.9612

5 Which factorization is incorrect?
   1) \(4k^2 - 49 = (2k + 7)(2k - 7)\)
   2) \(a^3 - 8b^3 = (a - 2b)(a^2 + 2ab + 4b^2)\)
   3) \(m^3 + 3m^2 - 4m + 12 = (m - 2)^2(m + 3)\)
   4) \(t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t + 1)(t + 2)(t + 3)\)

6 Sally’s high school is planning their spring musical. The revenue, \(R\), generated can be determined by the function \(R(t) = -33t^2 + 360t\), where \(t\) represents the price of a ticket. The production cost, \(C\), of the musical is represented by the function \(C(t) = 700 + 5t\). What is the highest ticket price, to the nearest dollar, they can charge in order to not lose money on the event?
   1) \(t = 3\)
   2) \(t = 5\)
   3) \(t = 8\)
   4) \(t = 11\)
7 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Text Messages per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0–10</td>
</tr>
<tr>
<td>15–18</td>
<td>4</td>
</tr>
<tr>
<td>19–22</td>
<td>6</td>
</tr>
<tr>
<td>23–60</td>
<td>25</td>
</tr>
</tbody>
</table>

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

1) \( \frac{157}{229} \)
2) \( \frac{157}{312} \)
3) \( \frac{157}{384} \)
4) \( \frac{157}{456} \)

8 A recursive formula for the sequence 18, 9, 4.5, ... is

1) \( g_1 = 18 \)
   \( g_n = \frac{1}{2} g_{n-1} \)

2) \( g_n = 18 \left( \frac{1}{2} \right)^{n-1} \)
3) \( g_1 = 18 \)
   \( g_n = 2g_{n-1} \)
4) \( g_n = 18(2)^{n-1} \)

9 Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1) \( \sum_{n=1}^{6} 8(1.10)^{n-1} \)
2) \( \sum_{n=1}^{6} 8(1.10)^n \)
3) \( \frac{8 - 8(1.10)^6}{0.90} \)
4) \( \frac{8 - 8(0.10)^n}{1.10} \)

10 A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave decreasing, only?

1) \((0, 200)\)
2) \((100, 300)\)
3) \((200, 400)\)
4) \((300, 400)\)

11 The expression \( \frac{x^3 + 2x^2 + x + 6}{x + 2} \) is equivalent to

1) \( x^2 + 3 \)
2) \( x^2 + 1 + \frac{4}{x + 2} \)
3) \( 2x^2 + x + 6 \)
4) \( 2x^2 + 1 + \frac{4}{x + 2} \)
12 A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the diagram below.

Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to
1) 0.01
2) 0.03
3) 0.06
4) 0.12

13 An equation to represent the value of a car after $t$ months of ownership is $v = 32,000(0.81)^{\frac{t}{12}}$. Which statement is not correct?
1) The car lost approximately 19% of its value each month.
2) The car maintained approximately 98% of its value each month.
3) The value of the car when it was purchased was $32,000.
4) The value of the car 1 year after it was purchased was $25,920.

14 Which equation represents an odd function?
1) $y = \sin x$
2) $y = \cos x$
3) $y = (x + 1)^3$
4) $y = e^{5x}$

15 The completely factored form of $2d^4 + 6d^3 - 18d^2 - 54d$ is
1) $2d(d^3 - 9)(d + 3)$
2) $2d(d^2 + 9)(d + 3)$
3) $2d(d + 3)^2(d - 3)$
4) $2d(d - 3)^2(d + 3)$
16 Which diagram shows an angle rotation of 1 radian on the unit circle?

1)

2)

3)

4)

17 The focal length, $F$, of a camera’s lens is related to the distance of the object from the lens, $J$, and the distance to the image area in the camera, $W$, by the formula below.

$$\frac{1}{J} + \frac{1}{W} = \frac{1}{F}$$

When this equation is solved for $J$ in terms of $F$ and $W$, $J$ equals

1) $\frac{FW}{F - W}$
2) $\frac{FW}{W - F}$
3) $\frac{FW}{W} - F$
4) $\frac{1}{F} - \frac{1}{W}$

18 The sequence $a_1 = 6, a_n = 3a_{n-1}$ can also be written as

1) $a_n = 6 \cdot 3^n$
2) $a_n = 6 \cdot 3^{n+1}$
3) $a_n = 2 \cdot 3^n$
4) $a_n = 2 \cdot 3^{n+1}$
19 Which equation represents the set of points equidistant from line \( \ell \) and point \( R \) shown on the graph below?

\[
\begin{align*}
1) & \quad y = -\frac{1}{8} (x + 2)^2 + 1 \\
2) & \quad y = -\frac{1}{8} (x + 2)^2 - 1 \\
3) & \quad y = -\frac{1}{8} (x - 2)^2 + 1 \\
4) & \quad y = -\frac{1}{8} (x - 2)^2 - 1
\end{align*}
\]

20 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

\[
\begin{align*}
I & \quad (m + p)^2 = m^2 + 2mp + p^2 \\
II & \quad (x + y)^3 = x^3 + 3xy + y^3 \\
III & \quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2
\end{align*}
\]

1) I, only \\
2) I and II \\
3) II and III \\
4) I and III

21 The graph of \( p(x) \) is shown below.

What is the remainder when \( p(x) \) is divided by \( x + 4 \)?

1) \( x - 4 \) \\
2) \(-4\) \\
3) \( 0 \) \\
4) \( 4 \)

22 A payday loan company makes loans between $100 and $1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a $300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

\[
\begin{align*}
1) & \quad 300(.30)^{\frac{14}{365}} \\
2) & \quad 300(1.30)^{\frac{14}{365}} \\
3) & \quad 300(.30)^{\frac{14}{14}} \\
4) & \quad 300(1.30)^{\frac{14}{14}}
\end{align*}
\]
23 Which value is not contained in the solution of the system shown below?
\[ a + 5b - c = -20 \]
\[ 4a - 5b + 4c = 19 \]
\[ -a - 5b - 5c = 2 \]
1) \(-2\)
2) 2
3) 3
4) \(-3\)

24 In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State \( t \) years after 2010?
1) \( P_t = 19,378,000(1.5)^t \)
2) \( P_0 = 19,378,000 \quad P_t = 19,378,000 + 1.015P_{t-1} \)
3) \( P_t = 19,378,000(1.015)^t \)
4) \( P_0 = 19,378,000 \quad P_t = 1.015P_{t-1} \)

25 The volume of air in a person’s lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

26 Explain how \( \left( \frac{1}{3^5} \right)^2 \) can be written as the equivalent radical expression \( \frac{1}{\sqrt[5]{9}} \).

27 Simplify \( xi(i-7i)^2 \), where \( i \) is the imaginary unit.

28 Using the identity \( \sin^2 \theta + \cos^2 \theta = 1 \), find the value of \( \tan \theta \), to the nearest hundredth, if \( \cos \theta \) is –0.7 and \( \theta \) is in Quadrant II.

29 Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth’s wait time unusual? Justify your answer.

30 The \( x \)-value of which function’s \( x \)-intercept is larger, \( f \) or \( h \)? Justify your answer.

\[ f(x) = \log(x - 4) \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( h(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1)</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>(-2)</td>
</tr>
</tbody>
</table>
31 The distance needed to stop a car after applying the brakes varies directly with the square of the car’s speed. The table below shows stopping distances for various speeds.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>6.25</td>
<td>25</td>
<td>56.25</td>
<td>100</td>
<td>156.25</td>
<td>225</td>
<td>306.25</td>
</tr>
</tbody>
</table>

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

32 Given events $A$ and $B$, such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether $A$ and $B$ are independent or dependent.

33 Find algebraically the zeros for $p(x) = x^3 + x^2 - 4x - 4$. On the set of axes below, graph $y = p(x)$.

34 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the nearest day, the amount of time needed before the amount of I–131 in the patient’s body is approximately 7 milligrams.

35 Solve the equation $\sqrt{2x - 7} + x = 5$ algebraically, and justify the solution set.
Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

<table>
<thead>
<tr>
<th>Group 1 (seconds)</th>
<th>Group 2 (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.4</td>
<td>23.3</td>
</tr>
<tr>
<td>18.1</td>
<td>18.8</td>
</tr>
<tr>
<td>18.2</td>
<td>22.1</td>
</tr>
<tr>
<td>19.6</td>
<td>12.7</td>
</tr>
<tr>
<td>18.6</td>
<td>16.9</td>
</tr>
<tr>
<td>16.2</td>
<td>24.4</td>
</tr>
<tr>
<td>16.1</td>
<td>21.2</td>
</tr>
<tr>
<td>15.3</td>
<td>21.2</td>
</tr>
<tr>
<td>17.8</td>
<td>16.3</td>
</tr>
<tr>
<td>19.7</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Mean = 17.7 Mean = 19.1

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva’s hypothesis may be incorrect. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.

Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.
37 Seth’s parents gave him $5000 to invest for his 16th birthday. He is considering two investment options. Option A will pay him 4.5% interest compounded annually. Option B will pay him 4.6% compounded quarterly. Write a function of option A and option B that calculates the value of each account after $n$ years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option B will earn than option A to the nearest cent. Algebraically determine, to the nearest tenth of a year, how long it would take for option B to double Seth’s initial investment.
2015 Algebra II Common Core State Standards Sample Items
Answer Section

1 ANS: 1
The zeros of the polynomial are at \(-b\) and \(c\). The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as \(x\) goes to negative infinity, \(f(x)\) goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

PTS: 2 REF: spr1501a1i1 NAT: A.APR.B.3 TOP: Zeros of Polynomials
KEY: AII

2 ANS: 4

A parabola with a focus of (0,4) and a directrix of \(y = 2\) is sketched as follows:

By inspection, it is determined that the vertex of the parabola is (0,3). It is also evident that the distance, \(p\), between the vertex and the focus is 1. It is possible to use the formula \((x - h)^2 = 4p(y - k)\) to derive the equation of the parabola as follows: \((x - 0)^2 = 4(1)(y - 3)\)

\[
x^2 = 4y - 12
\]
\[
x^2 + 12 = 4y
\]
\[
\frac{x^2}{4} + 3 = y
\]
or A point \((x,y)\) on the parabola must be the same distance from the focus as it is from the directrix. For any such point \((x,y)\), the distance to the focus is \(\sqrt{(x - 0)^2 + (y - 4)^2}\) and the distance to the directrix is \(y - 2\). Setting this equal leads to: \(x^2 + y^2 - 8y + 16 = y^2 - 4y + 4\)

\[
x^2 + 16 = 4y + 4
\]
\[
\frac{x^2}{4} + 3 = y
\]

PTS: 2 REF: spr1502a1i1 NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

3 ANS: 1
A reference triangle can be sketched using the coordinates \((-4,3)\) in the second quadrant to find the value of \(\sin \theta\).

PTS: 2 REF: spr1503a1i1 NAT: F.TF.A.2 TOP: Determining Trigonometric Functions
4 ANS: 2

\[ B(t) = 750 \left( 1.16 \right)^{12t} \approx 750(1.012)^{12t} \]  
\[ B(t) = 750 \left( 1 + \frac{0.16}{12} \right)^{12t} \]

is wrong, because the growth is an annual rate that is not compounded monthly.

PTS: 2  REF: spr1504aii  NAT: A.SSE.B.3  TOP: Modeling Exponential Functions

KEY: AII

5 ANS:

\[ x^{\frac{8}{3}} = x^y \]
\[ x^{\frac{4}{3}} = x^y \]
\[ \frac{4}{3} = y \]

PTS: 2  REF: spr1505aii  NAT: N.RN.A.2  TOP: Radicals and Rational Exponents

6 ANS:

\[ (4 - 3i)(5 + 2yi - 5 + 2yi) \]
\[ = (4 - 3i)(4yi) \]
\[ = 16yi - 12yi^2 \]
\[ = 12y + 16yi \]

PTS: 2  REF: spr1506aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

7 ANS:

\[ f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 128 - 80 - 44 - 4 = 0 \]

Any method that demonstrates 4 is a zero of \( f(x) \) confirms that \( x - 4 \) is a factor, as suggested by the Remainder Theorem.

PTS: 2  REF: spr1507aii  NAT: A.APR.B.2  TOP: Remainder Theorem
8 ANS:

\[ \sqrt{x - 5} = -x + 7 \]

\[ \sqrt{x - 5} = -9 + 7 = -2 \] is extraneous.

\[ x - 5 = x^2 - 14x + 49 \]

\[ 0 = x^2 - 15x + 54 \]

\[ 0 = (x - 6)(x - 9) \]

\[ x = 6, 9 \]

PTS: 2  REF: spr1508aii  NAT: A.REI.A.2  TOP: Solving Radicals

KEY: extraneous solutions

9 ANS:

\[ 720 = \frac{120000 \left( \frac{0.048}{12} \right) \left( 1 + \frac{0.048}{12} \right)^n}{\left( 1 + \frac{0.048}{12} \right)^n - 1} \]

\[ \frac{275.2}{12} \approx 23 \text{ years} \]

\[ 720(1.004)^n - 720 = 480(1.004)^n \]

\[ 240(1.004)^n = 720 \]

\[ 1.004^n = 3 \]

\[ n \log 1.004 = \log 3 \]

\[ n \approx 275.2 \text{ months} \]

PTS: 4  REF: spr1509aii  NAT: A.CED.A.1  TOP: Exponential Growth
10 ANS:

\[
\begin{align*}
6x - 3y + 2z &= -10 \\
x + 3y + 5z &= 45 \\
4x + 10z &= 62 \\
4x + 4(7) &= 20 \\
-2x + 3y + 8z &= 72 \\
x + 3y + 2z &= -10 \\
4x + 4z &= 20 \\
7x + 7z &= 35 \\
6z &= 42 \\
x &= -2 \\
4x + 4z &= 20 \\
z &= 7
\end{align*}
\]

\[
6(-2) - 3y + 2(7) = -10
\]

\[
y = 4
\]

PTS: 4 REF: spr1510aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

11 ANS:

\[
a_n = x^{n-1}(x + 1) \]

\[
x^{n-1} = 0 \quad x + 1 = 0
\]

\[
x = 0 \quad x = -1
\]

PTS: 4 REF: spr1511aii NAT: F.LE.A.2 TOP: Sequences

12 ANS:

Yes. The margin of error from this simulation indicates that 95% of the observations fall within \( \pm 0.12 \) of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to be 0.06 in the dotplot, by 2, or applying the estimated standard error formula,

\[
\sqrt{\frac{p(1-p)}{n}} \quad \text{or} \quad \sqrt{\frac{(0.25)(0.75)}{50}}
\]

and multiplying by 2. The interval 0.25 \( \pm 0.12 \) includes plausible values for the true proportion of people who prefer Stephen’s new product. The company has evidence that the population proportion could be at least 25%. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 (9 out of 50) or less several times, even when the population proportion is 0.25, due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

PTS: 4 REF: spr1512aii NAT: S.IC.B.4 TOP: Analysis of Data
This scenario can be modeled with a Venn Diagram:

Since 

\[ P(S \cup I) = 0.2, \quad P(S \cup I) = 0.8. \]

Then, 

\[ P(S \cap I) = P(S) + P(I) - P(S \cup I) \]

If \( S \) and \( I \) are independent, then the

\[ = 0.5 + 0.7 - 0.8 \]

\[ = 0.4 \]

Product Rule must be satisfied. However, \((0.5)(0.7) \neq 0.4\). Therefore, salary and insurance have not been treated independently.

---

The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of \(-12\) and a maximum of 12. The value of \( A \) is \(-12\) since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter \( B \). By experimentation with technology or using the relation 

\[ P = \frac{2\pi}{B} \]

(where \( P \) is the period), it is determined that \( B = \frac{2\pi}{13} \).

\[ f(t) = -12 \cos \left( \frac{2\pi}{13} t \right) \]

In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from \( t = 13 \) to \( t = 19.5 \) (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.
\[ x(x + 7) \left( \frac{3x + 25}{x + 7} - \frac{3}{x} \right) = x(3x + 25) - 5x(x + 7) = 3(x + 7) \]

\[ 3x^2 + 25x - 5x^2 - 35x = 3x + 21 \]

\[ 2x^2 + 13x + 21 = 0 \]

\[ (2x + 7)(x + 3) = 0 \]

\[ x = \frac{7}{2}, -3 \]

PTS: 2  REF: fall1501aii  NAT: A.REI.A.2  TOP: Solving Rationals
KEY: rational solutions

16  ANS: 3

\[ f(x) = -f(x), \text{ so } f(x) \text{ is odd. } g(-x) \neq g(x), \text{ so } g(x) \text{ is not even. } g(-x) \neq -g(x), \text{ so } g(x) \text{ is not odd. } h(-x) = h(x), \text{ so } h(x) \text{ is even.} \]

PTS: 2  REF: fall1502aii  NAT: F.BF.B.3  TOP: Even and Odd Functions

17  ANS: 1

\[ \frac{3x^2 + 4x - 1}{2x + 3} = \frac{3x^2 + 4x - 1}{2x + 3} = \frac{6x^2 + 9x^2}{2x + 3} = \frac{8x^2 + 16x}{2x + 3} = \frac{-2x + 2}{2x - 3} = \frac{-2x - 3}{5} \]

PTS: 2  REF: fall1503aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: remainder
18 ANS: 3

\[-2 \left( \frac{1}{2} x^2 = -6x + 20 \right)\]

\[x^2 - 12x = 40\]

\[x^2 - 12x + 36 = 40 + 36\]

\[(x - 6)^2 = 4\]

\[x - 6 = \pm 2i\]

\[x = 6 \pm 2i\]

PTS: 2  REF: fall1504aii  NAT: A.REI.B.4  TOP: Solving Quadratics

KEY: complex solutions | completing the square

19 ANS: 4

\[k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48\]

\[k^2(k^2 - 4) + 8k(k^2 - 4) + 12(k^2 - 4)\]

\[(k^2 - 4)(k^2 + 8k + 12)\]

\[(k + 2)(k - 2)(k + 6)(k + 2)\]

PTS: 2  REF: fall1505aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials

KEY: factoring by grouping

20 ANS: 4

As the range is \([4,10]\), the midline is \(y = \frac{4 + 10}{2} = 7\).

PTS: 2  REF: fall1506aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions

KEY: mixed
21 ANS:

\[-2x + 1 = -2x^2 + 3x + 1\]
\[2x^2 - 5x = 0\]
\[x(2x - 5) = 0\]
\[x = 0, \frac{5}{2}\]

PTS: 2 REF: fall1507aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems
KEY: AII

22 ANS:

Based on these data, the two events do not appear to be independent. \(P(F) = \frac{106}{200} = 0.53\), while \(P(F|T) = \frac{54}{90} = 0.6\), \(P(F|R) = \frac{25}{65} = 0.39\), and \(P(F|C) = \frac{27}{45} = 0.6\). The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.

PTS: 2 REF: fall1508aii NAT: S.CP.A.4 TOP: Conditional Probability

23 ANS:

\[x = (y - 3)^3 + 1\]
\[x - 1 = (y - 3)^3\]
\[\sqrt[3]{x - 1} = y - 3\]
\[\sqrt[3]{x - 1} + 3 = y\]
\[f^{-1}(x) = \sqrt[3]{x - 1} + 3\]

PTS: 2 REF: fall1509aii NAT: F.BF.B.4 TOP: Inverse of Functions
KEY: equations
24 ANS:

Let $x$ equal the first integer and $x + 1$ equal the next. $(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$. $2x + 1$ is an odd integer.

25 ANS:

The expression is of the form $y^2 - 5y - 6$ or $(y - 6)(y + 1)$. Let $y = 4x^2 + 5x:

$$\left(4x^2 + 5x - 6\right)\left(4x^2 + 5x + 1\right)$$

$(4x - 3)(x + 2)(4x + 1)(x + 1)$

26 ANS:

100 = 325 + (68 - 325)e^{-2k} \quad T = 325 - 257e^{-0.066t}

$-225 = -257e^{-2k} \quad T = 325 - 257e^{-0.066(7)} \approx 163$

$$k = \frac{\ln\left(\frac{-225}{-257}\right)}{-2}$$

$k \approx 0.066$
28 ANS: The mean difference between the students’ final grades in group 1 and group 2 is –3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription. One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students’ final grades many (500) times. If the observed difference –3.64 is the result of the assignment of students to groups alone, then a difference of –3.64 or less should be observed fairly regularly in the simulation output. However, a difference of –3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups’ mean final grades.

29 ANS:

\[
0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 \quad z(x) = 6x^3 + 19x^2 - 52x + 15
\]

\[
0 = -750 + 25b + 260 + 15
\]

\[
475 = 25b
\]

\[
19 = b
\]

\[
\begin{array}{c|cccc}
-5 & 6 & 19 & -52 & 15 \\
-30 & 55 & 15 \\
6 & -11 & 3 & 0
\end{array}
\]

\[6x^2 - 11x + 3 = 0\]

\[(2x - 3)(3x - 1) = 0\]

\[x = \frac{3}{2}, \frac{1}{3}, -5\]

30 ANS:

\[
\text{normcdf}(510, 540, 480, 24) = 0.0994 \quad z = \frac{510 - 480}{24} = 1.25 \quad 1.25 = \frac{x - 510}{20} \quad 2.5 = \frac{x - 510}{20} \quad 535-560
\]

\[
z = \frac{540 - 480}{24} = 2.5 \quad x = 535 \quad x = 560
\]

31 ANS:

\[
A(t) = 100(0.5)^{\frac{t}{63}}, \text{ where } t \text{ is time in years, and } A(t) \text{ is the amount of titanium-44 left after } t \text{ years.}
\]

\[
\frac{A(10) - A(0)}{10 - 0} = \frac{89.58132 - 100}{10} = -1.041868 \quad \text{The estimated mass at } t = 40 \text{ is } 100 - 40(-1.041868) \approx 58.3. \quad \text{The actual mass is } A(40) = 100(0.5)^{\frac{40}{63}} \approx 64.3976. \quad \text{The estimated mass is less than the actual mass.}
\]
0616AII Common Core State Standards
Answer Section

1 ANS: 4 PTS: 2 REF: 061601a2i NAT: N.RN.A.2
TOP: Radicals and Rational Exponents

2 ANS: 3 PTS: 2 REF: 061602a2i NAT: A.CED.A.1
TOP: Modeling Rationals

3 ANS: 2

\[(2 - yi)(2 - yi) = 4 - 4yi + y^2i^2 = -y^2 - 4yi + 4\]

PTS: 2 REF: 061603a2i NAT: N.CN.A.2 TOP: Operations with Complex Numbers

4 ANS: 3

The graph shows three real zeros, and has end behavior matching the given end behavior.

PTS: 2 REF: 061604a2i NAT: A.APR.B.3 TOP: Zeros of Polynomials
KEY: AII

5 ANS: 3

\[\sqrt{56-x} = x\quad -8\text{ is extraneous.}\]

\[56 - x = x^2\]

\[0 = x^2 + x - 56\]

\[0 = (x + 8)(x - 7)\]

\[x = 7\]

PTS: 2 REF: 061605a2i NAT: A.REI.A.2 TOP: Solving Radicals
KEY: extraneous solutions
6 ANS: 1

\[x^4 - 4x^3 - 9x^2 + 36x = 0\]
\[x^3(x - 4) - 9x(x - 4) = 0\]
\[(x^3 - 9x)(x - 4) = 0\]
\[x(x^2 - 9)(x - 4) = 0\]
\[x(x + 3)(x - 3)(x - 4) = 0\]
\[x = 0, \pm 3, 4\]

PTS: 2 REF: 061606aii NAT: A.APR.B.3 TOP: Zeros of Polynomials

KEY: AII

7 ANS: 3 PTS: 2 REF: 061607aii NAT: S.IC.A.2 TOP: Analysis of Data

8 ANS: 4

\[
\frac{m(c)}{g(c)} = \frac{c + 1}{1 - c^2} = \frac{c + 1}{(1 + c)(1 - c)} = \frac{1}{1 - c}
\]

PTS: 2 REF: 061608aii NAT: F.BF.A.1 TOP: Operations with Functions

9 ANS: 2

\[x + 2\sigma\] represents approximately 48% of the data.

PTS: 2 REF: 061609aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: percent

10 ANS: 4

The scenario represents a decreasing geometric sequence with a common ratio of 0.80.

PTS: 2 REF: 061610aii NAT: F.BF.A.2 TOP: Sequences

11 ANS: 1

The probability of rain equals the probability of rain, given that Sean pitches.

PTS: 2 REF: 061611aii NAT: S.CP.A.3 TOP: Conditional Probability
12 ANS: 1
\[
x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)} = \frac{-3 \pm \sqrt{-7}}{4} = \frac{-3}{4} \pm \frac{i\sqrt{7}}{4}
\]

PTS: 2  REF: 061612aii  NAT: A.REI.B.4  TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

13 ANS: 3

\[
H(t) \text{ is at a minimum at } 70(-1) + 80 = 10
\]

PTS: 2  REF: 061613aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions
KEY: maximum/minimum

14 ANS: 2

\[
2x^2 - 3x + 7
\]
\[
2x + 3 (4x^3 + 0x^2 + 5x + 10)
\]
\[
4x^3 + 6x^2
\]
\[
-6x^2 + 5x
\]
\[
-6x^2 - 9x
\]
\[
14x + 10
\]
\[
14x + 21
\]
\[
-11
\]

PTS: 2  REF: 061614aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: remainder

15 ANS: 4

\[
y = 5^{-t} = \left(\frac{1}{5}\right)^t
\]

PTS: 2  REF: 061615aii  NAT: F.IF.C.8  TOP: Modeling Exponential Functions
16 ANS: 2
\[ x = \frac{-3}{4}y + 2 \]
\[-4x = 3y - 8 \]
\[-4x + 8 = 3y \]
\[ \frac{4}{3}x + \frac{8}{3} = y \]

PTS: 2 REF: 061616a1i NAT: F.BF.B.4 TOP: Inverse of Functions

17 ANS: 1

PTS: 2 REF: 061617a1i NAT: F.TF.A.2 TOP: Determining Trigonometric Functions

18 ANS: 1

PTS: 2 REF: 061618a1i NAT: F.IF.C.7 TOP: Graphing Exponential and Logarithmic Functions

19 ANS: 4
\[ 4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324 \]
\[ 4(x - 3)^2 + 4(y + 9)^2 = 436 \]

PTS: 2 REF: 061619a1i NAT: A.SSE.A.2 TOP: Equations of Conics

20 ANS: 2 PTS: 2 REF: 061620a1i NAT: F.IF.B.4 TOP: Graphing Polynomial Functions
21 ANS: 3
\[ \frac{1}{12} \approx 1.00427 \]

PTS: 2  REF: 061621aii  NAT: F.BF.A.1  TOP: Modeling Exponential Functions
KEY: AII

22 ANS: 4

PTS: 2  REF: 061622aii  NAT: A.REI.D.11  TOP: Other Systems
KEY: AII

23 ANS: 3

PTS: 2  REF: 061623aii  NAT: F.LE.A.2  TOP: Sequences

24 ANS: 4
period = \( \frac{2\pi}{B} \)

\[ \frac{1}{60} = \frac{2\pi}{B} \]

\[ B = 120\pi \]

PTS: 2  REF: 061624aii  NAT: F.TF.B.5  TOP: Modeling Trigonometric Functions

25 ANS:
\[ \frac{1}{x} - \frac{1}{3} = -\frac{1}{3x} \]

\[ \frac{3-x}{3x} = -\frac{1}{3x} \]

\[ 3 - x = -1 \]

\[ x = 4 \]

PTS: 2  REF: 061625aii  NAT: A.REI.A.2  TOP: Solving Rationals
KEY: rational solutions
26 ANS: Randomly assign participants to two groups. One group uses the toothpaste with ingredient $X$ and the other group uses the toothpaste without ingredient $X$.

PTS: 2 REF: 061626aii NAT: S.IC.B.3 TOP: Analysis of Data

27 ANS: 

\[
\frac{2x^2 + 6x + 23}{x - 5} \frac{2x^3 - 4x^2 - 7x - 10}{2x^3 - 10x^2}
\]

Since there is a remainder, $x - 5$ is not a factor.

\[
\frac{6x^2 - 7x}{6x^2 - 30x}
\]

\[
\frac{23x - 10}{23x - 115}
\]

\[
105
\]

PTS: 2 REF: 061627aii NAT: A.APR.B.2 TOP: Remainder Theorem

28 ANS: 

PTS: 2 REF: 061628aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions

29 ANS: 

\[
P(S \cap M) = P(S) + P(M) - P(S \cup M) = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376}
\]

PTS: 2 REF: 061629aii NAT: S.CP.B.7 TOP: Theoretical Probability
The vertex of the parabola is (4,−3). The x-coordinate of the focus and the vertex is the same. Since the distance from the vertex to the directrix is 3, the distance from the vertex to the focus is 3, so the y-coordinate of the focus is 0. The coordinates of the focus are (4,0).

\[ \frac{x^3 + 9}{x^3 + 8} = \frac{x^3 + 8 + \frac{1}{x^3 + 8}}{x^3 + 8} \]

\[ \frac{x^3 + 9}{x^3 + 8} = \frac{x^3 + 9}{x^3 + 8} \]

\[ A = Pe^{rt} \]

\[ 135000 = 100000e^{5r} \]

\[ 1.35 = e^{5r} \]

\[ \ln 1.35 = \ln e^{5r} \]

\[ \ln 1.35 = 5r \]

\[ .06 \approx r \text{ or } 6\% \]
ANS:

\[ y = -x + 5 \quad y = -7 + 5 = -2 \]

\[ (x - 3)^2 + (-x + 5 + 2)^2 = 16 \quad y = -3 + 5 = 2 \]

\[ x^2 - 6x + 9 + x^2 - 14x + 49 = 16 \]

\[ 2x^2 - 20x + 42 = 0 \]

\[ x^2 - 10x + 21 = 0 \]

\[ (x - 7)(x - 3) = 0 \]

\[ x = 7, 3 \]

PTS: 4  REF: 061633a1i  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems

KEY: AII

34 ANS:

\[ S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04} \]

\[ S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04} = 660778.39 \]

PTS: 4  REF: 061634a1i  NAT: A.SSE.B.4  TOP: Series

35 ANS:

\[ 0.602 \pm 2 \cdot 0.066 = 0.47 - 0.73. \] Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

PTS: 4  REF: 061635a1i  NAT: S.IC.B.5  TOP: Analysis of Data

36 ANS:

\[ \frac{f(4) - f(-2)}{4 - (-2)} = \frac{80 - 1.25}{6} = 13.125 \quad g(x) \text{ has a greater rate of change} \]

\[ \frac{g(4) - g(-2)}{4 - (-2)} = \frac{179 - (-49)}{6} = 38 \]

PTS: 4  REF: 061636a1i  NAT: F.IF.B.6  TOP: Rate of Change

KEY: AII
\[ A(t) = 800e^{-0.347t} \]
\[ B(t) = 400e^{-0.231t} \]

\[ \begin{align*}
800e^{-0.347t} &= 400e^{-0.231t} \\
0.15 &= e^{-0.347t} \\
\ln 2e^{-0.347t} &= \ln e^{-0.231t} \\
\ln 0.15 &= \ln e^{-0.347t} \\
\ln 2 + \ln e^{-0.347t} &= \ln e^{-0.231t} \\
\ln 0.15 &= -0.347t \cdot \ln e \\
\ln 2 - 0.347t &= -0.231t \\
\ln 2 &= 0.116t \\
6 &\approx t
\end{align*} \]
0816AII Common Core State Standards
Answer Section

1 ANS: 4
If $1 - i$ is one solution, the other is $1 + i$. $(x - (1 - i))(x - (1 + i)) = 0$

$x^2 - x - ix - x + ix + (1 - i^2) = 0$

$x^2 - 2x + 2 = 0$

PTS: 2  REF: 081601aii  NAT: A.REI.B.4  TOP: Complex Conjugate Root Theorem

2 ANS: 1
II. Ninth graders drive to school less often; III. Students know little about adults; IV. Calculus students love math!

PTS: 2  REF: 081602aii  NAT: S.IC.B.3  TOP: Analysis of Data  KEY: bias

3 ANS: 2


4 ANS: 3

PTS: 2  REF: 081604aii  NAT: S.ID.A.4  TOP: Normal Distributions  KEY: probability

5 ANS: 3
$(m - 2)^2(m + 3) = (m^2 - 4m + 4)(m + 3) = m^3 + 3m^2 - 4m^2 - 12m + 4m + 12 = m^3 - m^2 - 8m + 12$

PTS: 2  REF: 081605aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials  KEY: factoring by grouping
6 ANS: 3

\[-33t^2 + 360t = 700 + 5t\]
\[-33t^2 + 355t - 700 = 0\]

\[t = \frac{-355 \pm \sqrt{355^2 - 4(-33)(-700)}}{2(-33)} \approx 3.8\]

KEY: AII

7 ANS: 1

\[
\frac{157}{25 + 47 + 157}
\]

PTS: 2 REF: 081607aaii NAT: S.CP.A.4 TOP: Conditional Probability

8 ANS: 1

(2) is not recursive

PTS: 2 REF: 081608aaii NAT: F.LE.A.2 TOP: Sequences

9 ANS: 1

PTS: 2 REF: 081609aaii NAT: A.SSE.B.4

TOP: Series KEY: sigma notation

10 ANS: 2

PTS: 2 REF: 081610aaii NAT: F.IF.B.4

TOP: Graphing Trigonometric Functions KEY: increasing/decreasing

11 ANS: 2

\[
x^2 + 0x + 1
= x + 2 \left( x^3 + 2x^2 + x + 6 \right)
= x^3 + 2x^2
\]
\[
= 0x^2 + x
\]
\[
= 0x^2 + 0x
\]
\[
= x + 6
\]
\[
= x + 2
\]
\[
= 4
\]

PTS: 2 REF: 081611aaii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: remainder

12 ANS: 2

\[
ME = \left( z \sqrt{\frac{p(1-p)}{n}} \right) = \left( 1.96 \sqrt{\frac{(0.55)(0.45)}{900}} \right) \approx 0.03
\]

PTS: 2 REF: 081612aaii NAT: S.IC.B.4 TOP: Analysis of Data
13 ANS: 1
The car lost approximately 19% of its value each year.

PTS: 2 REF: 081613aii NAT: F.LE.B.5 TOP: Modeling Exponential Functions

14 ANS: 1
The graph of $y = \sin x$ is unchanged when rotated 180º about the origin.

PTS: 2 REF: 081614aii NAT: F.BF.B.3 TOP: Even and Odd Functions

15 ANS: 3

\[
2d(d^3 + 3d^2 - 9d - 27) \\
2d(d^2 (d + 3) - 9(d + 3)) \\
2d(d^2 - 9)(d + 3) \\
2d(d + 3)(d - 3)(d + 3) \\
2d(d + 3)^2(d - 3)
\]

PTS: 2 REF: 081615aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: factoring by grouping

16 ANS: 1

PTS: 2 REF: 081616aii NAT: F.TF.A.1
TOP: Unit Circle

17 ANS: 3

\[
\frac{1}{J} = \frac{1}{F - \frac{1}{W}} \\
\frac{1}{J} = \frac{W - F}{FW} \\
J = \frac{FW}{W - F}
\]

PTS: 2 REF: 081617aii NAT: A.REI.A.2 TOP: Solving Rationals
KEY: rational solutions

18 ANS: 3

PTS: 2 REF: 081618aii NAT: F.LE.A.2
TOP: Sequences

19 ANS: 4
The vertex is $(2, -1)$ and $p = 2$. \( y = -\frac{1}{4(2)}(x - 2)^2 - 1 \)

PTS: 2 REF: 081619aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

20 ANS: 4
\((x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3\)

PTS: 2 REF: 081620aii NAT: A.APR.C.4 TOP: Polynomial Identities

21 ANS: 3
Since $x + 4$ is a factor of $p(x)$, there is no remainder.

PTS: 2 REF: 081621aii NAT: A.APR.B.2 TOP: Remainder Theorem
Combining (1) and (3): $−6c = −18$ Combining (1) and (2): $5a + 3c = −1$

$c = 3$

$5a + 3(3) = −1$

$2 − 5b − 15 = 2$

$5a = −10$

$b = −3$

$a = −2$

Amplitude, because the height of the graph shows the volume of the air.

Applying the commutative property, $\left(\frac{1}{3^\frac{1}{5}}\right)^2$ can be rewritten as $\left(3^2\right)^\frac{1}{5}$ or $9^\frac{1}{5}$. A fractional exponent can be rewritten as a radical with the denominator as the index, or $9^\frac{1}{5} = \sqrt[5]{9}$.

$x(-6i)^2 = x(36i^2) = 36xi^3 = −36xi$

$\sin^2 \theta + (−0.7)^2 = 1$ Since $\theta$ is in Quadrant II, $\sin \theta = \sqrt{.51}$ and $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{.51}}{−0.7} \approx −1.02$

$\sin^2 \theta = .51$

$\sin \theta = ±\sqrt{.51}$

Using a 95% level of confidence, $x ± 2$ standard deviations sets the usual wait time as 150-302 seconds. 360 seconds is unusual.
30 ANS: 

\[ 0 = \log_{10}(x - 4) \]

The x-intercept of \( h \) is (2,0). \( f \) has the larger value.

\[ 10^0 = x - 4 \]
\[ 1 = x - 4 \]
\[ x = 5 \]

PTS: 2 REF: 081630aii NAT: F.IF.C.9 TOP: Comparing Functions KEY: AII

31 ANS:

\[ \frac{156.25 - 56.25}{70 - 50} = \frac{150}{20} = 7.5 \]

Between 50-70 mph, each additional mph in speed requires 7.5 more feet to stop.

PTS: 2 REF: 081631aii NAT: F.IF.B.6 TOP: Rate of Change KEY: AII

32 ANS:

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

\[ A \text{ and } B \text{ are independent since } P(A \cap B) = P(A) \cdot P(B) \]

\[ 0.8 = 0.6 + 0.5 - P(A \cap B) \]
\[ 0.3 = 0.6 \cdot 0.5 \]

\[ P(A \cap B) = 0.3 \]

\[ 0.3 = 0.3 \]

PTS: 2 REF: 081632aii NAT: S.CP.A.2 TOP: Theoretical Probability

33 ANS:

\[ 0 = x^2(x + 1) - 4(x + 1) \]
\[ 0 = (x^2 - 4)(x + 1) \]
\[ 0 = (x + 2)(x - 2)(x + 1) \]
\[ x = -2, -1, 2 \]

PTS: 4 REF: 081633aii NAT: F.IF.C.7 TOP: Zeros of Polynomials
34 ANS:

\[ 7 = 20(0.5)^{\frac{t}{8.02}} \]

\[ \log 0.35 = \log 0.5^{\frac{t}{8.02}} \]

\[ \log 0.35 = t \log 0.5 \cdot \frac{1}{8.02} \]

\[ \frac{8.02 \log 0.35}{\log 0.5} = t \]

\[ t \approx 12 \]

PTS: 4  REF: 081634aii  NAT: F.LE.A.4  TOP: Exponential Decay

35 ANS:

\[ \left( \sqrt{2x - 7} \right)^2 = (5 - x)^2 \]

\[ \sqrt{2(4) - 7 + 4} = 5 \cdot \sqrt{2(8) - 7 + 8} = 5 \]

\[ 2x - 7 = 25 - 10x + x^2 \]

\[ \sqrt{1} = 1 \]

\[ \sqrt{9} \neq -3 \]

\[ 0 = x^2 - 12x + 32 \]

\[ 0 = (x - 8)(x - 4) \]

\[ x = 4, 8 \]

PTS: 4  REF: 081635aii  NAT: A.REI.A.2  TOP: Solving Radicals

KEY: extraneous solutions

36 ANS:

Some of the students who did not drink energy drinks read faster than those who did drink energy drinks.

\[ 17.7 - 19.1 = -1.4 \]

Differences of -1.4 and less occur \( \frac{25}{232} \) or about 10% of the time, so the difference is not unusual.

PTS: 4  REF: 081636aii  NAT: S.IC.B.5  TOP: Analysis of Data

37 ANS:

\[ A = 5000(1.045)^n \]

\[ 5000 \left( 1 + \frac{0.046}{4} \right)^{4n} - 5000(1.045)^6 \approx 6578.87 - 6511.30 \approx 67.57 \]

\[ 10000 = 5000 \left( 1 + \frac{0.046}{4} \right)^{4n} \]

\[ 2 = 1.0115^{4n} \]

\[ \log 2 = 4n \cdot \log 1.0115 \]

\[ n = \frac{\log 2}{4 \log 1.0115} \]

\[ n \approx 15.2 \]

PTS: 6  REF: 081637aii  NAT: A.CED.A.1  TOP: Exponential Growth