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NY Algebra II CCSS Regents Exam Questions from Spring 2015 to January 2017 Sorted at Random

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1. On the axes below, graph one cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).

2. A recursive formula for the sequence 18,9,4.5,\ldots is

   1) $g_1 = 18$
   
   $g_n = \frac{1}{2}g_{n-1}$

   2) $g_n = 18\left(\frac{1}{2}\right)^{n-1}$

   3) $g_1 = 18$
   
   $g_n = 2g_{n-1}$

   4) $g_n = 18(2)^{n-1}$

3. The expression $\left(\frac{m^2}{\frac{1}{3}}\right)^{\frac{1}{2}}$ is equivalent to

   1) $-\sqrt[6]{m^5}$

   2) $\frac{1}{\sqrt[6]{m^5}}$

   3) $-m^{\frac{5}{3}}\sqrt{m}$

   4) $\frac{1}{m^{\frac{5}{3}}\sqrt{m}}$

4. Write an explicit formula for $a_n$, the $n$th term of the recursively defined sequence below.

   $a_1 = x + 1$

   $a_n = x(a_{n-1})$

   For what values of $x$ would $a_n = 0$ when $n > 1$?

5. The focal length, $F$, of a camera’s lens is related to the distance of the object from the lens, $J$, and the distance to the image area in the camera, $W$, by the formula below.

   \[
   \frac{1}{J} + \frac{1}{W} = \frac{1}{F}
   \]

   When this equation is solved for $J$ in terms of $F$ and $W$, $J$ equals

   1) $F - W$

   2) $\frac{FW}{F - W}$

   3) $\frac{FW}{W - F}$

   4) $\frac{1}{F} - \frac{1}{W}$
6. If \(a, b,\) and \(c\) are all positive real numbers, which graph could represent the sketch of the graph of 
\[ p(x) = -a(x + b)\left(x^2 - 2cx + c^2\right) \]?

7. Factored completely, \(m^5 + m^3 - 6m\) is equivalent to
1) \((m + 3)(m - 2)\)
2) \((m^2 + 3m)(m^2 - 2)\)
3) \(m(m^4 + m^2 - 6)\)
4) \(m(m^2 + 3)(m^2 - 2)\)

8. Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

9. If \(g(c) = 1 - c^2\) and \(m(c) = c + 1\), then which statement is not true?
1) \(g(c) \cdot m(c) = 1 + c - c^2 - c^3\)
2) \(g(c) + m(c) = 2 + c - c^2\)
3) \(m(c) - g(c) = c + c^2\)
4) \(\frac{m(c)}{g(c)} = \frac{-1}{1 - c}\)

10. One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the nearest day, the amount of time needed before the amount of I–131 in the patient’s body is approximately 7 milligrams.

11. If \(\sin^2(32^\circ) + \cos^2(M) = 1\), then \(M\) equals
1) \(32^\circ\)
2) \(58^\circ\)
3) \(68^\circ\)
4) \(72^\circ\)
12 What is the solution set of the equation
\[ \frac{3x + 25}{x + 7} - 5 = \frac{3}{x} \]
1) \[ \left\{ \frac{3}{2}, 7 \right\} \]
2) \[ \left\{ \frac{7}{2}, -3 \right\} \]
3) \[ \left\{ -\frac{3}{2}, 7 \right\} \]
4) \[ \left\{ -\frac{7}{2}, -3 \right\} \]

13 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton’s Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

\[ T = T_a + (T_o - T_a)e^{-kt} \]

- \( T_a \) = the temperature surrounding the object
- \( T_o \) = the initial temperature of the object
- \( t \) = the time in hours
- \( T \) = the temperature of the object after \( t \) hours
- \( k \) = decay constant

The turkey reaches the temperature of approximately 100°F after 2 hours. Find the value of \( k \), to the nearest thousandth, and write an equation to determine the temperature of the turkey after \( t \) hours. Determine the Fahrenheit temperature of the turkey, to the nearest degree, at 3 p.m.

14 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Text Messages per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>15 4 37 68</td>
</tr>
<tr>
<td>11–50</td>
<td>6 25 87</td>
</tr>
<tr>
<td>23–60</td>
<td>25 47 157</td>
</tr>
</tbody>
</table>

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?
1) \[ \frac{157}{229} \]
2) \[ \frac{157}{312} \]
3) \[ \frac{157}{384} \]
4) \[ \frac{157}{456} \]

15 For the function \( f(x) = (x - 3)^3 + 1 \), find \( f^{-1}(x) \).

16 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, \( \theta \), intercepts the circle in Quadrant II at point \( C \). The y-coordinate of point \( C \) is 8. What is the value of \( \cos \theta \)?
1) \[ \frac{3}{5} \]
2) \[ \frac{3}{4} \]
3) \[ \frac{3}{5} \]
4) \[ \frac{4}{5} \]
17 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer. Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose. Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

18 The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation 
\[ s = \sqrt{t - 2t + 6} \], where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \). How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

19 Using the unit circle below, explain why \( \csc \theta = \frac{1}{y} \).
20 Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth’s wait time unusual? Justify your answer.

21 The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity $I_0$ to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, $I$, and the decibel rating, $d$, of this sound is found using $d = 10 \log \frac{I}{I_0}$.

The threshold sound audible to the average person is $1.0 \times 10^{-12}$ W/m$^2$ (watts per square meter). Consider the following sound level classifications:

<table>
<thead>
<tr>
<th>Classification</th>
<th>Decibel Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderate</td>
<td>45-69 dB</td>
</tr>
<tr>
<td>Loud</td>
<td>70-89 dB</td>
</tr>
<tr>
<td>Very loud</td>
<td>90-109 dB</td>
</tr>
<tr>
<td>Deafening</td>
<td>&gt;110 dB</td>
</tr>
</tbody>
</table>

How would a sound with intensity $6.3 \times 10^{-3}$ W/m$^2$ be classified?
1) moderate
2) loud
3) very loud
4) deafening

22 The guidance department has reported that of the senior class, 2.3% are members of key club, $K$, 8.6% are enrolled in AP Physics, $P$, and 1.9% are in both. Determine the probability of $P$ given $K$, to the nearest tenth of a percent. The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

23 The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are
1) $-6 \pm 2i$
2) $-6 \pm 2\sqrt{19}$
3) $6 \pm 2i$
4) $6 \pm 2\sqrt{19}$

24 Given the equal terms $\sqrt[3]{x^5}$ and $\frac{5}{6}$, determine and state $y$, in terms of $x$.

25 A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave decreasing, only?
1) $(0, 200)$
2) $(100, 300)$
3) $(200, 400)$
4) $(300, 400)$

26 Algebraically determine the values of $x$ that satisfy the system of equations below.

\[
y = -2x + 1
\]
\[
y = -2x^2 + 3x + 1
\]
27 When \( g(x) = \frac{2}{x+2} \) and \( h(x) = \log(x+1) + 3 \) are graphed on the same set of axes, which coordinates best approximate their point of intersection?

1) \((-0.9, 1.8)\)
2) \((-0.9, 1.9)\)
3) \((1.4, 3.3)\)
4) \((1.4, 3.4)\)

28 Given: \( h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2 \)

\[ k(x) = -|0.7x| + 5 \]

State the solutions to the equation \( h(x) = k(x) \), rounded to the nearest hundredth.

29 Given \( i \) is the imaginary unit, \((2 - yi)^2\) in simplest form is

1) \( y^2 - 4yi + 4 \)
2) \(-y^2 - 4yi + 4\)
3) \(-y^2 + 4\)
4) \( y^2 + 4\)

30 The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>250,000</td>
</tr>
<tr>
<td>2011</td>
<td>250,937</td>
</tr>
<tr>
<td>2012</td>
<td>251,878</td>
</tr>
<tr>
<td>2013</td>
<td>252,822</td>
</tr>
</tbody>
</table>

How can this sequence be recursively modeled?

1) \( j_n = 250,000(1.00375)^{n-1} \)
2) \( j_n = 250,000 + 937^{(n-1)} \)
3) \( j_1 = 250,000 \)
4) \( j_n = 1.00375j_{n-1} \)

31 Which function shown below has a greater average rate of change on the interval \([-2, 4]\)? Justify your answer.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0.3125</td>
</tr>
<tr>
<td>-3</td>
<td>0.625</td>
</tr>
<tr>
<td>-2</td>
<td>1.25</td>
</tr>
<tr>
<td>-1</td>
<td>2.5</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>320</td>
</tr>
</tbody>
</table>

\( g(x) = 4x^3 - 5x^2 + 3 \)

32 When \( g(x) \) is divided by \( x + 4 \), the remainder is 0.

Given \( g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8 \), which conclusion about \( g(x) \) is true?

1) \( g(4) = 0 \)
2) \( g(-4) = 0 \)
3) \( x - 4 \) is a factor of \( g(x) \).
4) No conclusion can be made regarding \( g(x) \).

33 Use the properties of rational exponents to determine the value of \( y \) for the equation:

\[ \frac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y, \ x > 1 \]
34 If the terminal side of angle $\theta$, in standard position, passes through point $(-4, 3)$, what is the numerical value of $\sin \theta$?

1) $\frac{3}{5}$
2) $\frac{4}{5}$
3) $-\frac{3}{5}$
4) $-\frac{4}{5}$

35 The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of $3000, and each year it grows by 80%.
4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

36 Write $(5 + 2yi)(4 - 3i) - (5 - 2yi)(4 - 3i)$ in $a + bi$ form, where $y$ is a real number.

37 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

38 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, $T$, are left in the semester?

1) $\frac{255 + 93T}{3T} = 90$
2) $\frac{255 + 90T}{3T} = 93$
3) $\frac{255 + 93T}{T+3} = 90$
4) $\frac{255 + 90T}{T+3} = 93$

39 Which equation represents the set of points equidistant from line $\ell$ and point $R$ shown on the graph below?

1) $y = -\frac{1}{8}(x + 2)^2 + 1$
2) $y = -\frac{1}{8}(x + 2)^2 - 1$
3) $y = -\frac{1}{8}(x - 2)^2 + 1$
4) $y = -\frac{1}{8}(x - 2)^2 - 1$
40 A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If \( t \) represents the time, in weeks, and \( P(t) \) is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?
1) 56
2) 152
3) 3688
4) 81,920

41 Which statement about statistical analysis is false?
1) Experiments can suggest patterns and relationships in data.
2) Experiments can determine cause and effect relationships.
3) Observational studies can determine cause and effect relationships.
4) Observational studies can suggest patterns and relationships in data.

42 The expression \( \frac{6x^3 + 17x^2 + 10x + 2}{2x + 3} \) equals
1) \( 3x^2 + 4x - 1 + \frac{5}{2x + 3} \)
2) \( 6x^2 + 8x - 2 + \frac{5}{2x + 3} \)
3) \( 6x^2 - x + 13 - \frac{37}{2x + 3} \)
4) \( 3x^2 + 13x + \frac{49}{2} + \frac{151}{2x + 3} \)

43 Which statement about the graph of \( c(x) = \log_6 x \) is false?
1) The asymptote has equation \( y = 0 \).
2) The graph has no \( y \)-intercept.
3) The domain is the set of positive reals.
4) The range is the set of all real numbers.

44 An equation to represent the value of a car after \( t \) months of ownership is \( v = 32,000(0.81)^{\frac{t}{12}} \). Which statement is not correct?
1) The car lost approximately 19% of its value each month.
2) The car maintained approximately 98% of its value each month.
3) The value of the car when it was purchased was $32,000.
4) The value of the car 1 year after it was purchased was $25,920.

45 A payday loan company makes loans between $100 and $1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a $300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?
1) \( 300(0.30)^{\frac{365}{14}} \)
2) \( 300(1.30)^{\frac{365}{14}} \)
3) \( 300(0.30)^{\frac{14}{365}} \)
4) \( 300(1.30)^{\frac{14}{365}} \)

46 To the nearest tenth, the value of \( x \) that satisfies \( 2^x = -2x + 11 \) is
1) 2.5
2) 2.6
3) 5.8
4) 5.9
47 Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

<table>
<thead>
<tr>
<th>Group 1 (seconds)</th>
<th>Group 2 (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.4</td>
<td>23.3</td>
</tr>
<tr>
<td>18.1</td>
<td>18.8</td>
</tr>
<tr>
<td>18.2</td>
<td>22.1</td>
</tr>
<tr>
<td>19.6</td>
<td>12.7</td>
</tr>
<tr>
<td>18.6</td>
<td>16.9</td>
</tr>
<tr>
<td>16.2</td>
<td>24.4</td>
</tr>
<tr>
<td>16.1</td>
<td>21.2</td>
</tr>
<tr>
<td>15.3</td>
<td>21.2</td>
</tr>
<tr>
<td>17.8</td>
<td>16.3</td>
</tr>
<tr>
<td>19.7</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Mean = 17.7 Mean = 19.1

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva’s hypothesis may be incorrect. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.

Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.
48 Pedro and Bobby each own an ant farm. Pedro starts with 100 ants and says his farm is growing exponentially at a rate of 15% per month. Bobby starts with 350 ants and says his farm is steadily decreasing by 5 ants per month. Assuming both boys are accurate in describing the population of their ant farms, after how many months will they both have approximately the same number of ants?
1) 7
2) 8
3) 13
4) 36

49 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

\[ P_n \] = present amount borrowed
\[ n \] = number of monthly pay periods
\[ PMT \] = monthly payment
\[ i \] = interest rate per month

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

50 In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects 95% of the Critical Reading scores?
1) 430 ± 115
2) 430 ± 230
3) 496 ± 115
4) 496 ± 230

51 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the nearest tenth. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.

52 Algebraically determine the values of \( h \) and \( k \) to correctly complete the identity stated below.

\[ 2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k \]

53 Which statement(s) about statistical studies is true?
I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don’t like math.
1) I, only
2) II, only
3) I and III
4) III and IV
54 Solve the equation $\sqrt{2x - 7} + x = 5$ algebraically, and justify the solution set.

55 Stephen’s Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products $A$, $B$, and the new product. Nine out of fifty participants preferred Stephen’s new cola to products $A$ and $B$. The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen’s new product, each of sample size 50, simulated 100 times.

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.

56 Express $(1 - i)^3$ in $a + bi$ form.

57 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Which equation best represents the value of the voltage as it flows through the electric wires, where $t$ is time in seconds?

1) $V = 120 \sin(t)$
2) $V = 120 \sin(60t)$
3) $V = 120 \sin(60\pi t)$
4) $V = 120 \sin(120\pi t)$

58 The price of a postage stamp in the years since the end of World War I is shown in the scatterplot below.

The equation that best models the price, in cents, of a postage stamp based on these data is

1) $y = 0.59x - 14.82$
2) $y = 1.04(1.43)^x$
3) $y = 1.43(1.04)^x$
4) $y = 24 \sin(14x) + 25$
59 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function \( N(t) = N_0(e^{-rt}) \), where \( N(t) \) is the amount left in the body, \( N_0 \) is the initial dosage, \( r \) is the decay rate, and \( t \) is time in hours. Patient \( A \), \( A(t) \), is given 800 milligrams of a drug with a decay rate of 0.347. Patient \( B \), \( B(t) \), is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, \( A(t) \) and \( B(t) \), to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

To the nearest hour, \( t \), when does the amount of the given drug remaining in patient \( B \) begin to exceed the amount of the given drug remaining in patient \( A \)? The doctor will allow patient \( A \) to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the nearest tenth of an hour, how long patient \( A \) will have to wait to take another 800 milligram dose of the drug.

60 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is \( \frac{974}{1376} \), what is the probability that a student participates in both sports and music?

61 The sequence \( a_1 = 6, a_n = 3a_{n-1} \) can also be written as
1) \( a_n = 6 \cdot 3^n \)
2) \( a_n = 6 \cdot 3^{n-1} \)
3) \( a_n = 2 \cdot 3^n \)
4) \( a_n = 2 \cdot 3^{n+1} \)

62 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario. State one conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

63 The solution set for the equation \( \sqrt{56-x} = x \) is
1) \{8, 7\}
2) \{-7, 8\}
3) \{7\}
4) \{\}
64 Sally’s high school is planning their spring musical. The revenue, \( R \), generated can be determined by the function \( R(t) = -33t^2 + 360t \), where \( t \) represents the price of a ticket. The production cost, \( C \), of the musical is represented by the function \( C(t) = 700 + 5t \). What is the highest ticket price, to the nearest dollar, they can charge in order to not lose money on the event?

1) \( t = 3 \)
2) \( t = 5 \)
3) \( t = 8 \)
4) \( t = 11 \)

65 A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, \( B(t) \), can be represented by the function \( B(t) = 750(1.16)^t \), where the \( t \) represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

1) \( B(t) = 750(1.012)^t \)
2) \( B(t) = 750(1.012)^{12t} \)
3) \( B(t) = 750(1.16)^{12t} \)
4) \( B(t) = 750(1.16)^{\frac{t}{12}} \)

66 The eighth and tenth terms of a sequence are 64 and 100. If the sequence is either arithmetic or geometric, the ninth term can not be

1) -82
2) -80
3) 80
4) 82

67 The directrix of the parabola \( 12(y + 3) = (x - 4)^2 \) has the equation \( y = -6 \). Find the coordinates of the focus of the parabola.

68 Seth’s parents gave him $5000 to invest for his 16th birthday. He is considering two investment options. Option \( A \) will pay him 4.5% interest compounded annually. Option \( B \) will pay him 4.6% compounded quarterly. Write a function of option \( A \) and option \( B \) that calculates the value of each account after \( n \) years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option \( B \) will earn than option \( A \) to the nearest cent. Algebraically determine, to the nearest tenth of a year, how long it would take for option \( B \) to double Seth’s initial investment.

69 The zeros for \( f(x) = x^4 - 4x^3 - 9x^2 + 36x \) are

1) \( \{0, \pm 3, 4\} \)
2) \( \{0, 3, 4\} \)
3) \( \{0, 3, -4\} \)
4) \( \{0, 3, -4\} \)

70 Solve the following system of equations algebraically for all values of \( x, y, \) and \( z \):

\[
\begin{align*}
x + 3y + 5z &= 45 \\
6x - 3y + 2z &= -10 \\
-2x + 3y + 8z &= 72
\end{align*}
\]
71. A cardboard box manufacturing company is building boxes with length represented by \( x + 1 \), width by \( 5 - x \), and height by \( x - 1 \). The volume of the box is modeled by the function below.

Over which interval is the volume of the box changing at the fastest average rate?
1) \([1, 2]\)
2) \([1, 3.5]\)
3) \([1, 5]\)
4) \([0, 3.5]\)

72. If \( p(x) = ab^x \) and \( r(x) = cd^x \), then \( p(x) \cdot r(x) \) equals
1) \( ac(b + d)^x \)
2) \( ac(b + d)^{2x} \)
3) \( ac(bd)^x \)
4) \( ac(bd)^{2x} \)

73. Rewrite the expression \( \left(4x^2 + 5x\right)^2 - 5\left(4x^2 + 5x\right) - 6 \) as a product of four linear factors.

74. Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups’ final grades is shown below:

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>80.16</td>
<td>83.8</td>
</tr>
<tr>
<td>( S_x )</td>
<td>6.9</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students’ final grades were rerandomized 500 times. The results are shown below.

Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.
75 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on both issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

76 The expression $\frac{4x^3 + 5x + 10}{2x + 3}$ is equivalent to
1) $2x^2 + 3x - 7 + \frac{31}{2x + 3}$
2) $2x^2 - 3x + 7 - \frac{11}{2x + 3}$
3) $2x^2 + 2.5x + 5 + \frac{15}{2x + 3}$
4) $2x^2 - 2.5x - 5 - \frac{20}{2x + 3}$

77 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are
1) independent
2) dependent
3) mutually exclusive
4) complements

78 Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, $S_n$, for Alexa's total earnings over $n$ years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the nearest cent.

79 A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the diagram below.

Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to
1) 0.01
2) 0.03
3) 0.06
4) 0.12
80 Which statement regarding the graphs of the functions below is *untrue*?

\[ f(x) = 3 \sin 2x, \text{ from } -\pi < x < \pi \]
\[ g(x) = (x - 0.5)(x + 4)(x - 2) \]
\[ h(x) = \log_2 x \]
\[ j(x) = -|4x - 2| + 3 \]

1) \( f(x) \) and \( j(x) \) have a maximum \( y \)-value of 3.
2) \( f(x), h(x), \) and \( j(x) \) have one \( y \)-intercept.
3) \( g(x) \) and \( j(x) \) have the same end behavior as \( x \to -\infty \).
4) \( g(x), h(x), \) and \( j(x) \) have rational zeros.

81 Which equation has \( 1 - i \) as a solution?

1) \( x^2 + 2x - 2 = 0 \)
2) \( x^2 + 2x + 2 = 0 \)
3) \( x^2 - 2x - 2 = 0 \)
4) \( x^2 - 2x + 2 = 0 \)

82 The solution to the equation \( 18x^2 - 24x + 87 = 0 \) is

1) \( \frac{2}{3} \pm 6i \sqrt{158} \)
2) \( \frac{2}{3} \pm \frac{1}{6} i \sqrt{158} \)
3) \( \frac{2}{3} \pm 6i \sqrt{158} \)
4) \( \frac{2}{3} \pm \frac{1}{6} i \sqrt{158} \)

83 Which statement is *incorrect* for the graph of the function \( y = -3 \cos \left[ \frac{\pi}{3} (x - 4) \right] + 7 \)?

1) The period is 6.
2) The amplitude is 3.
3) The range is \([4,10]\).
4) The midline is \( y = -4 \).

84 What is the solution to the system of equations \( y = 3x - 2 \) and \( y = g(x) \) where \( g(x) \) is defined by the function below?

\[
\begin{align*}
1) & \quad \{(0, -2)\} \\
2) & \quad \{(0, -2), (1, 6)\} \\
3) & \quad \{(1, 6)\} \\
4) & \quad \{(1, 1), (6, 16)\}
\end{align*}
\]
85 Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of $B$ dollars after $m$ months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after $m$ months.

<table>
<thead>
<tr>
<th>m</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000.00</td>
</tr>
<tr>
<td>10</td>
<td>1172.00</td>
</tr>
<tr>
<td>19</td>
<td>1352.00</td>
</tr>
<tr>
<td>36</td>
<td>1770.80</td>
</tr>
<tr>
<td>60</td>
<td>2591.90</td>
</tr>
<tr>
<td>69</td>
<td>2990.00</td>
</tr>
<tr>
<td>72</td>
<td>3135.80</td>
</tr>
<tr>
<td>73</td>
<td>3186.00</td>
</tr>
</tbody>
</table>

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?
1) month 10 to month 60
2) month 19 to month 69
3) month 36 to month 72
4) month 60 to month 73

86 When $b > 0$ and $d$ is a positive integer, the expression $(3b)^{2d}$ is equivalent to
1) $\frac{1}{(\sqrt[2]{3b})^{2d}}$
2) $(\sqrt[2]{3b})^{-d}$
3) $\frac{1}{\sqrt[2]{3b^{d}}}$
4) $(\sqrt[2]{3b})^{-2}$

87 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Given the results of her coin flips and of her computer simulation, which statement is most accurate?
1) 73 of the computer's next 100 coin flips will be heads.
2) 50 of her next 100 coin flips will be heads.
3) Her coin is not fair.
4) Her coin is fair.

88 Using the identity $\sin^2 \theta + \cos^2 \theta = 1$, find the value of $\tan \theta$, to the nearest hundredth, if $\cos \theta$ is –0.7 and $\theta$ is in Quadrant II.

89 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the nearest whole percent, is
1) 6
2) 48
3) 68
4) 95
Gabriel performed an experiment to see if planting 13 tomato plants in black plastic mulch leads to larger tomatoes than if 13 plants are planted without mulch. He observed that the average weight of the tomatoes from tomato plants grown in black plastic mulch was 5 ounces greater than those from the plants planted without mulch. To determine if the observed difference is statistically significant, he rerandomized the tomato groups 100 times to study these random differences in the mean weights. The output of his simulation is summarized in the dotplot below.

Given these results, what is an appropriate inference that can be drawn?

1) There was no effect observed between the two groups.

2) There was an effect observed that could be due to the random assignment of plants to the groups.

3) There is strong evidence to support the hypothesis that tomatoes from plants planted in black plastic mulch are larger than those planted without mulch.

4) There is strong evidence to support the hypothesis that tomatoes from plants planted without mulch are larger than those planted in black plastic mulch.

In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State \( t \) years after 2010?

1) \( P_t = 19,378,000(1.5)^t \)

2) \( P_0 = 19,378,000 \)

\[ P_t = 19,378,000 + 1.015P_{t-1} \]

3) \( P_t = 19,378,000(1.015)^{t-1} \)

4) \( P_0 = 19,378,000 \)

\[ P_t = 1.015P_{t-1} \]

Given \( f^{-1}(x) = \frac{3}{4}x + 2 \), which equation represents \( f(x) \)?

1) \( f(x) = \frac{4}{3}x - \frac{8}{3} \)

2) \( f(x) = -\frac{4}{3}x + \frac{8}{3} \)

3) \( f(x) = \frac{3}{4}x - 2 \)

4) \( f(x) = -\frac{3}{4}x + 2 \)
93 The equation \(4x^2 - 24x + 4y^2 + 72y = 76\) is equivalent to

1) \(4(x - 3)^2 + 4(y + 9)^2 = 76\)
2) \(4(x - 3)^2 + 4(y + 9)^2 = 121\)
3) \(4(x - 3)^2 + 4(y + 9)^2 = 166\)
4) \(4(x - 3)^2 + 4(y + 9)^2 = 436\)

94 Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1) \(8(1.10)^n - 1\)
2) \(8(1.10)^n\)
3) \(8 - 8(1.10)^6 / 0.90\)
4) \(8 - 8(0.10)^n / 1.10\)

95 What is the completely factored form of \(k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48\)?

1) \((k - 2)(k - 2)(k + 3)(k + 4)\)
2) \((k - 2)(k - 2)(k + 6)(k + 2)\)
3) \((k + 2)(k - 2)(k + 3)(k + 4)\)
4) \((k + 2)(k - 2)(k + 6)(k + 2)\)

96 Given \(z(x) = 6x^3 + bx^2 - 52x + 15\), \(z(2) = 35\), and \(z(-5) = 0\), algebraically determine all the zeros of \(z(x)\).

97 Which factorization is incorrect?

1) \(4k^2 - 49 = (2k + 7)(2k - 7)\)
2) \(a^3 - 8b^3 = (a - 2b)(a^2 + 2ab + 4b^2)\)
3) \(m^3 + 3m^2 - 4m + 12 = (m - 2)^2 (m + 3)\)
4) \(t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t + 1)(t + 2)(t + 3)\)

98 Functions \(f, g,\) and \(h\) are given below.

\[f(x) = \sin(2x)\]
\[g(x) = f(x) + 1\]

Which statement is true about functions \(f, g,\) and \(h\)?

1) \(f(x)\) and \(g(x)\) are odd, \(h(x)\) is even.
2) \(f(x)\) and \(g(x)\) are even, \(h(x)\) is odd.
3) \(f(x)\) is odd, \(g(x)\) is neither, \(h(x)\) is even.
4) \(f(x)\) is even, \(g(x)\) is neither, \(h(x)\) is odd.
The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form \( f(t) = A \cos(Bt) \), where \( A \) and \( B \) are real numbers, that models the water level, \( f(t) \), in inches above or below the average Carter Beach sea level, as a function of the time measured in \( t \) hours since 8:30 a.m. On the grid below, graph one cycle of this function.

People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the nearest ten thousandth, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

Which graph represents a cosine function with no horizontal shift, an amplitude of 2, and a period of \( \frac{2\pi}{3} \)?

Solve algebraically for all values of \( x \):
\[ \sqrt{x - 5} + x = 7 \]
103. The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?

1) 0.3803
2) 0.4612
3) 0.8415
4) 0.9612

104. The function \( M(t) \) represents the mass of radium over time, \( t \), in years.

\[
M(t) = 100e^{\frac{\ln \frac{1}{2}}{1590} t}
\]

Determine if the function \( M(t) \) represents growth or decay. Explain your reasoning.

105. What is the inverse of the function \( y = \log_3 x \)?

1) \( y = x^3 \)
2) \( y = \log_3 3 \)
3) \( y = 3^x \)
4) \( x = 3^y \)

106. The completely factored form of \( 2d^4 + 6d^3 - 18d^2 - 54d \) is

1) \( 2d(d^2 - 9)(d + 3) \)
2) \( 2d(d^2 + 9)(d + 3) \)
3) \( 2d(d + 3)^2(d - 3) \)
4) \( 2d(d - 3)^2(d + 3) \)

107. Which diagram shows an angle rotation of 1 radian on the unit circle?

1) \[
\begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3} \\
\text{Diagram 4}
\end{array}
\]

108. Algebraically prove that \( \frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8} \), where \( x \neq -2 \).
109 Use an appropriate procedure to show that \( x - 4 \) is a factor of the function \( f(x) = 2x^3 - 5x^2 - 11x - 4 \). Explain your answer.

110 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, \( H \), in feet, above the ground of one of the six-person cars can be modeled by

\[
H(t) = 70 \sin \left( \frac{2\pi}{7} (t - 1.75) \right) + 80, \quad \text{where } t \text{ is time, in minutes.}
\]

Using \( H(t) \) for one full rotation, this car's minimum height, in feet, is

1) 150  
2) 70  
3) 10  
4) 0

111 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let \( m \) represent months.]

1) \((1.0525)^m\)  
2) \((1.0525)^{12/m}\)  
3) \((1.00427)^m\)  
4) \((1.00427)^{m/12}\)

112 Given \( f(x) = 3x^2 + 7x - 20 \) and \( g(x) = x - 2 \), state the quotient and remainder of \( \frac{f(x)}{g(x)} \), in the form \( q(x) + \frac{r(x)}{g(x)} \).

113 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.

Which statement about this function is incorrect?
1) The degree of the polynomial is even.  
2) There is a positive leading coefficient.  
3) At two pH values, there is a relative maximum value.  
4) There are two intervals where the function is decreasing.

114 Relative to the graph of \( y = 3 \sin x \), what is the shift of the graph of \( y = 3 \sin \left( x + \frac{\pi}{3} \right) \)?

1) \( \frac{\pi}{3} \) right  
2) \( \frac{\pi}{3} \) left  
3) \( \frac{\pi}{3} \) up  
4) \( \frac{\pi}{3} \) down
115 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

116 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the nearest percent.

117 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

118 Which value, to the nearest tenth, is not a solution of \( p(x) = q(x) \) if \( p(x) = x^3 + 3x^2 - 3x - 1 \) and \( q(x) = 3x + 8 \)?
   1) -3.9
   2) -1.1
   3) 2.1
   4) 4.7

119 Solve for \( x \):
   \[
   \frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}
   \]

120 Find algebraically the zeros for \( p(x) = x^3 + x^2 - 4x - 4 \). On the set of axes below, graph \( y = p(x) \).

121 The expression \( \frac{x^3 + 2x^2 + x + 6}{x + 2} \) is equivalent to
   1) \( x^2 + 3 \)
   2) \( x^2 + 1 + \frac{4}{x + 2} \)
   3) \( 2x^2 + x + 6 \)
   4) \( 2x^2 + 1 + \frac{4}{x + 2} \)

122 Simplify \( xi(i - 7i)^2 \), where \( i \) is the imaginary unit.
123 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

<table>
<thead>
<tr>
<th>Comedy Series</th>
<th>Drama Series</th>
<th>Reality Series</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>Females</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>80</td>
<td>70</td>
<td>230</td>
</tr>
<tr>
<td>175</td>
<td>135</td>
<td>180</td>
<td>400</td>
</tr>
</tbody>
</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

124 Explain how \( \left( \frac{1}{3} \right)^2 \) can be written as the equivalent radical expression \( \sqrt[5]{9} \).

125 Solve the system of equations shown below algebraically.

\[
\begin{align*}
(x - 3)^2 + (y + 2)^2 &= 16 \\
2x + 2y &= 10
\end{align*}
\]

126 Which equation represents an odd function?

1) \( y = \sin x \)
2) \( y = \cos x \)
3) \( y = (x + 1)^3 \)
4) \( y = e^{5x} \)

127 Given events \( A \) and \( B \), such that \( P(A) = 0.6 \), \( P(B) = 0.5 \), and \( P(A \cup B) = 0.8 \), determine whether \( A \) and \( B \) are independent or dependent.

128 The \( x \)-value of which function’s \( x \)-intercept is larger, \( f \) or \( h \)? Justify your answer.

\( f(x) = \log(x - 4) \)

\[
\begin{array}{|c|c|}
\hline
x & h(x) \\
\hline
-1 & 6 \\
0 & 4 \\
1 & 2 \\
2 & 0 \\
3 & -2 \\
\hline
\end{array}
\]

129 What is the solution, if any, of the equation \( \frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12} \)?

1) -1
2) -5
3) all real numbers
4) no real solution
Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the nearest hundredth.

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% − 50% split. Explain what statistical evidence supports this concern.

Which equation represents a parabola with a focus of (0, 4) and a directrix of y = 2?

1) \( y = x^2 + 3 \)
2) \( y = -x^2 + 1 \)
3) \( y = \frac{x^2}{2} + 3 \)
4) \( y = \frac{x^2}{4} + 3 \)

Which graph has the following characteristics?

- three real zeros
- as \( x \to -\infty \), \( f(x) \to -\infty \)
- as \( x \to \infty \), \( f(x) \to \infty \)

1) 
2) 
3) 
4)
133 The distance needed to stop a car after applying the brakes varies directly with the square of the car’s speed. The table below shows stopping distances for various speeds.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>6.25</td>
<td>25</td>
<td>56.25</td>
<td>100</td>
<td>156.25</td>
<td>225</td>
<td>306.25</td>
</tr>
</tbody>
</table>

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

134 Which value is not contained in the solution of the system shown below?
\[ \begin{align*}
      a + 5b - c &= -20 \\
      4a - 5b + 4c &= 19 \\
      -a - 5b - 5c &= 2
\end{align*} \]
1) −2  
2) 2  
3) 3  
4) −3

135 The volume of air in a person’s lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

136 Which function represents exponential decay?
1) \( y = 2^{0.3t} \)  
2) \( y = 1.2^{3t} \)  
3) \( y = \left( \frac{1}{2} \right)^{-t} \)  
4) \( y = 5^{-t} \)

137 A solution of the equation \( 2x^2 + 3x + 2 = 0 \) is
1) \( -\frac{3}{4} + \frac{1}{4}i\sqrt{7} \)  
2) \( -\frac{3}{4} + \frac{1}{4}i \)  
3) \( -\frac{3}{4} + \frac{1}{4}\sqrt{7} \)  
4) \( \frac{1}{2} \)

138 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

<table>
<thead>
<tr>
<th>Rule</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>((m + p)^2 = m^2 + 2mp + p^2)</td>
</tr>
<tr>
<td>II</td>
<td>((x + y)^3 = x^3 + 3xy + y^3)</td>
</tr>
<tr>
<td>III</td>
<td>((a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2)</td>
</tr>
</tbody>
</table>

1) I, only  
2) I and II  
3) II and III  
4) I and III
Monthly mortgage payments can be found using the formula below:

\[
M = \frac{P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n}{ \left( 1 + \frac{r}{12} \right)^n - 1}
\]

- \(M\) = monthly payment
- \(P\) = amount borrowed
- \(r\) = annual interest rate
- \(n\) = number of monthly payments

The Banks family would like to borrow $120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the fewest number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than $720.

According to a pricing website, Indroid phones lose 58% of their cash value over 1.5 years. Which expression can be used to estimate the value of a $300 Indroid phone in 1.5 years?

1) \(300e^{-0.87}\)
2) \(300e^{-0.63}\)
3) \(300e^{-0.58}\)
4) \(300e^{-0.42}\)

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

The graph of \(p(x)\) is shown below.

What is the remainder when \(p(x)\) is divided by \(x + 4\)?

1) \(x - 4\)
2) \(-4\)
3) \(0\)
4) \(4\)
Algebra II Common Core State Standards Regents at Random
Answer Section

1 ANS:

![Graph](image_url)

PTS: 2  REF: 061628aii  NAT: F.IF.C.7  TOP: Graphing Trigonometric Functions
KEY: graph

2 ANS: 1
(2) is not recursive

PTS: 2  REF: 081608aii  NAT: F.LE.A.2  TOP: Sequences

3 ANS: 2

\[
\left( m^{\frac{5}{3}} \right)^{-\frac{1}{2}} = m^{\frac{5}{6}} = \frac{1}{\sqrt[6]{m^5}}
\]

PTS: 2  REF: 011707aii  NAT: N.RN.A.2  TOP: Radicals and Rational Exponents

4 ANS:

\[
a_n = x^{n-1}(x + 1) \quad x^{n-1} = 0 \quad x + 1 = 0
\]
\[
x = 0 \quad x = -1
\]

PTS: 4  REF: spr1511aii  NAT: F.LE.A.2  TOP: Sequences

5 ANS: 3

\[
\frac{1}{J} = \frac{1}{F} - \frac{1}{W}
\]
\[
\frac{1}{J} = \frac{W - F}{FW}
\]
\[
J = \frac{FW}{W - F}
\]

PTS: 2  REF: 081617aii  NAT: A.REI.A.2  TOP: Solving Rationals
KEY: rational solutions
6 ANS: 1
The zeros of the polynomial are at \(-b\), and \(c\). The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as \(x\) goes to negative infinity, \(f(x)\) goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

PTS: 2 REF: spr1501aii NAT: A.APR.B.3 TOP: Zeros of Polynomials
KEY: All

7 ANS: 4
\[ m^5 + m^3 - 6m = m(m^4 + m^2 - 6) = m(m^2 + 3)(m^2 + 2) \]

PTS: 2 REF: 011703aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: higher power

8 ANS:
Let \(x\) equal the first integer and \(x + 1\) equal the next. \((x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1\). \(2x + 1\) is an odd integer.

PTS: 2 REF: fall1511aii NAT: A.APR.C.4 TOP: Polynomial Identities

9 ANS: 4
\[ \frac{m(c)}{g(c)} = \frac{c + 1}{1 - c^2} = \frac{c + 1}{(1 + c)(1 - c)} = \frac{1}{1 - c} \]

PTS: 2 REF: 061608aii NAT: F.BF.A.1 TOP: Operations with Functions

10 ANS:
\[
7 = 20(0.5)^{\frac{t}{8.02}} \\
\log 0.35 = \log 0.5^{\frac{t}{8.02}} \\
\log 0.35 = \frac{t \log 0.5}{8.02} \\
\frac{8.02 \log 0.35}{\log 0.5} = t \\
t \approx 12
\]

PTS: 4 REF: 081634aii NAT: F.LE.A.4 TOP: Exponential Decay

11 ANS: 1 PTS: 2 REF: 011704aii NAT: F.TF.C.8 TOP: Proving Trigonometric Identities
12 ANS: 4

\[
x(x + 7) \left[ \frac{3x + 25}{x + 7} - \frac{5}{x} \right] = 3\]

\[
x(3x + 25) - 5x(x + 7) = 3(x + 7)
\]

\[
3x^2 + 25x - 5x^2 - 35x = 3x + 21
\]

\[
2x^2 + 13x + 21 = 0
\]

\[
(2x + 7)(x + 3) = 0
\]

\[
x = -\frac{7}{2}, -3
\]

PTS: 2  REF: fall1501aii  NAT: A.REI.A.2  TOP: Solving Rationals

KEY: rational solutions

13 ANS:

\[
100 = 325 + (68 - 325)e^{-2k} \quad T = 325 - 257e^{-0.066t}
\]

\[
-225 = -257e^{-2k} \quad T = 325 - 257e^{-0.066(t)} \approx 163
\]

\[
\ln\left(\frac{-225}{-257}\right) = \frac{\ln(257)}{-2}
\]

\[
k \approx 0.066
\]

PTS: 4  REF: fall1513aii  NAT: F.LE.A.4  TOP: Exponential Growth

14 ANS: 1

\[
\frac{157}{25 + 47 + 157}
\]

PTS: 2  REF: 081607aii  NAT: S.CP.A.4  TOP: Conditional Probability
15 ANS:
\[ x = (y - 3)^3 + 1 \]
\[ x - 1 = (y - 3)^3 \]
\[ 3\sqrt{x - 1} = y - 3 \]
\[ 3\sqrt{x - 1} + 3 = y \]
\[ f^{-1}(x) = 3\sqrt{x - 1} + 3 \]
PTS: 2 REF: fall1509aii NAT: F.BF.B.4 TOP: Inverse of Functions
KEY: equations

16 ANS: 1

PTS: 2

17 ANS:
Jillian’s plan, because distance increases by one mile each week. 
\[ a_1 = 10 \quad a_n = n + 12 \]
\[ a_n = a_{n-1} + 1 \]

PTS: 4

18 ANS:
\[ 0 = \sqrt{t} - 2t + 6 - 2\left(\frac{9}{4}\right) - 6 < 0, \text{ so } \frac{9}{4} \text{ is extraneous.} \]
\[ 2t - 6 = \sqrt{t} \]
\[ 4t^2 - 24t + 36 = t \]
\[ 4t^2 - 25t + 36 = 0 \]
\[ (4t - 9)(t - 4) = 0 \]
\[ t = \frac{9}{4}, 4 \]
\[ (\sqrt{1} - 2(1) + 6) - (\sqrt{3} - 2(3) + 6) = 5 - \sqrt{3} \approx 3.268 \text{ mph} \]

PTS: 6

KEY: context
19 ANS:
csc \( \theta = \frac{1}{\sin \theta} \), and \( \sin \theta \) on a unit circle represents the \( y \) value of a point on the unit circle. Since \( y = \sin \theta \),
csc \( \theta = \frac{1}{y} \).

PTS: 2    REF: 011727aii    NAT: F.TF.A.2    TOP: Unit Circle

20 ANS:
Using a 95% level of confidence, \( x \pm 2 \) standard deviations sets the usual wait time as 150-302 seconds. 360 seconds is unusual.

PTS: 2    REF: 081629aii    NAT: S.IC.B.6    TOP: Analysis of Data

21 ANS: 3
\( d = 10 \log \frac{6.3 \times 10^{-3}}{1.0 \times 10^{-12}} \approx 98 \)

PTS: 2    REF: 011715aii    NAT: F.IF.B.4    TOP: Evaluating Logarithmic Expressions

22 ANS:
\[ P(P/K) = \frac{P(P\cap K)}{P(K)} = \frac{1.9}{2.3} \approx 82.6\% \] A key club member has an 82.6% probability of being enrolled in AP Physics.

PTS: 4    REF: 011735aii    NAT: S.CP.B.6    TOP: Conditional Probability

23 ANS: 3
\[-2 \left( \frac{1}{2} x^2 = -6x + 20 \right)\]
\[ x^2 - 12x = -40 \]
\[ x^2 - 12x + 36 = -40 + 36 \]
\[ (x - 6)^2 = -4 \]
\[ x - 6 = \pm 2i \]
\[ x = 6 \pm 2i \]

PTS: 2    REF: fall1504aii    NAT: A.REI.B.4    TOP: Solving Quadratics
KEY: complex solutions | completing the square
24 ANS:

\[
\begin{pmatrix}
\frac{5}{3} \\
x
\end{pmatrix} = \begin{pmatrix}
\frac{5}{6} \\
y
\end{pmatrix}
\]

\[
x^2 = y
\]

PTS: 2 REF: 011730aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

25 ANS: 2 PTS: 2 REF: 081610aii NAT: F.IF.B.4 TOP: Graphing Trigonometric Functions KEY: increasing/decreasing

26 ANS:

\[
\begin{align*}
-2x + 1 &= -2x^2 + 3x + 1 \\
2x^2 - 5x &= 0 \\
x(2x - 5) &= 0 \\
x &= 0, \frac{5}{2}
\end{align*}
\]

PTS: 2 REF: fall1507aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems KEY: AII

27 ANS: 2

PTS: 2 REF: 011712aii NAT: A.REI.D.11 TOP: Other Systems KEY: AII
28 ANS:

29 ANS: 2

\[(2 - yi)(2 - yi) = 4 - 4yi + y^2 i^2 = -y^2 - 4yi + 4\]

30 ANS: 3

31 ANS:

\[\frac{f(4) - f(-2)}{4 - (-2)} = \frac{80 - 1.25}{6} = 13.125 \quad g(x) \text{ has a greater rate of change}\]

\[\frac{g(4) - g(-2)}{4 - (-2)} = \frac{179 - 49}{6} = 38\]

32 ANS: 2

33 ANS:

\[\frac{8}{3} = x^y\]

\[\frac{x}{4^3} = x^y\]

\[\frac{4}{3} = y\]
34 ANS: 1  
A reference triangle can be sketched using the coordinates (−4,3) in the second quadrant to find the value of \( \sin \theta \).

PTS: 2  REF: spr1503aii  NAT: F.TF.A.2  TOP: Determining Trigonometric Functions

35 ANS: 4  
The scenario represents a decreasing geometric sequence with a common ratio of 0.80.

PTS: 2  REF: 061610aii  NAT: F.BF.A.2  TOP: Sequences

36 ANS: 
\[(4 - 3i)(5 + 2yi - 5 + 2yi)\]
\[(4 - 3i)(4yi)\]
\[16yi - 12yi^2\]
\[12y + 16yi\]

PTS: 2  REF: spr1506aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

37 ANS: 
\[
\frac{2x^2 + 6x + 23}{x - 5} \quad \frac{2x^3 - 4x^2 - 7x - 10}{2x^3 - 10x^2} \quad \frac{6x^2 - 7x}{6x^2 - 30x} \quad \frac{23x - 10}{23x - 115} \quad 105
\]

Since there is a remainder, \( x - 5 \) is not a factor.

PTS: 2  REF: 061627aii  NAT: A.APR.B.2  TOP: Remainder Theorem

38 ANS: 3  PTS: 2  REF: 061602aii  NAT: A.CED.A.1  
TOP: Modeling Rationals

39 ANS: 4  
The vertex is (2,−1) and \( p = 2 \). \( y = -\frac{1}{4(2)} (x - 2)^2 - 1 \)

PTS: 2  REF: 081619aii  NAT: G.GPE.A.2  TOP: Graphing Quadratic Functions
40  ANS: 1  
\[ P(28) = 5(2)^{\frac{98}{28}} \approx 56 \]  
PTS: 2  
REF: 011702aaii  
NAT: F.LE.A.2  
TOP: Modeling Exponential Functions  
KEY: AII

41  ANS: 3  
PTS: 2  
REF: 011706aaii  
NAT: S.IC.B.3  
TOP: Analysis of Data  
KEY: type

42  ANS: 1  
\[
\frac{3x^2 + 4x - 1}{2x + 3} = \frac{16x^3 + 17x^2 + 10x + 2}{6x^3 + 9x^2} - \frac{8x^2 + 10x}{8x^2 + 12x} - \frac{2x + 2}{2x - 3} = \frac{5}{5} 
\]  
PTS: 2  
REF: fall1503aaii  
NAT: A.APR.D.6  
TOP: Rational Expressions  
KEY: remainder

43  ANS: 1  
PTS: 2  
REF: 061618aaii  
NAT: F.IF.C.7  
TOP: Graphing Exponential and Logarithmic Functions

44  ANS: 1  
The car lost approximately 19% of its value each year.  
PTS: 2  
REF: 081613aaii  
NAT: F.LE.B.5  
TOP: Modeling Exponential Functions

45  ANS: 4  
PTS: 2  
REF: 081622aaii  
NAT: F.BF.A.1  
TOP: Modeling Exponential Functions  
KEY: AII

46  ANS: 2  
PTS: 2  
REF: 081603aaii  
NAT: A.REI.D.11  
TOP: Other Systems  
KEY: AII
47 ANS:
Some of the students who did not drink energy drinks read faster than those who did drink energy drinks.
17.7 – 19.1 = −1.4 Differences of -1.4 and less occur \( \frac{25}{232} \) or about 10% of the time, so the difference is not unusual.

PTS: 4 REF: 081636aii NAT: S.IC.B.5 TOP: Analysis of Data

48 ANS: 2

PTS: 2 REF: 011716aii NAT: A.REI.D.11 TOP: Other Systems

KEY: AII

49 ANS:

\[
20000 = PMT \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right) \quad 21000 - x = 300 \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right)
\]

\[
PMT \approx 400.76 \quad x \approx 6028
\]

PTS: 4 REF: 011736aii NAT: A.SSE.B.4 TOP: Series

50 ANS: 4

\[496 \pm 2(115)\]

PTS: 2 REF: 011718aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: interval

51 ANS:

\[A(t) = 100(0.5)^{\frac{t}{63}},\] where \( t \) is time in years, and \( A(t) \) is the amount of titanium-44 left after \( t \) years.

\[
\frac{A(40) - A(0)}{10 - 0} = \frac{89.58132 - 100}{10} = -1.041868 \quad \text{The estimated mass at } t = 40 \text{ is } 100 - 40(-1.041868) \approx 58.3. \quad \text{The actual mass is } A(40) = 100(0.5)^{\frac{40}{63}} \approx 64.3976. \quad \text{The estimated mass is less than the actual mass.}
\]

PTS: 6 REF: fall1517aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions

KEY: AII

52 ANS:

\[2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k \quad h = -2 \]

\[-2x^2 + 8x + 5 = hx^2 - 4hx + k \quad k = 5
\]

PTS: 4 REF: 011733aii NAT: A.APR.C.4 TOP: Polynomial Identities
II. Ninth graders drive to school less often; III. Students know little about adults; IV. Calculus students love math!

\[ \sqrt{2x - 7} = (5 - x)^2 \]
\[ 2(4) - 7 + 4 = 5 \]
\[ 2(8) - 7 + 8 = 5 \]
\[ 2x - 7 = 25 - 10x + x^2 \]
\[ \sqrt{1} = 1 \]
\[ \sqrt{9} \neq -3 \]
\[ 0 = x^2 - 12x + 32 \]
\[ 0 = (x - 8)(x - 4) \]
\[ x = 4, 8 \]

Yes. The margin of error from this simulation indicates that 95% of the observations fall within ±0.12 of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to be 0.06 in the dotplot, by 2, or applying the estimated standard error formula, \( \sqrt{\frac{p(1-p)}{n}} \) or \( \sqrt{\frac{(0.25)(0.75)}{50}} \) and multiplying by 2. The interval 0.25 ± 0.12 includes plausible values for the true proportion of people who prefer Stephen’s new product. The company has evidence that the population proportion could be at least 25%. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 (9 out of 50) or less several times, even when the population proportion is 0.25, due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

\[ (1 - i)(1 - i)(1 - i) = (1 - 2i + i^2)(1 - i) = -2i(1 - i) = -2i + 2i^2 = -2 - 2i \]

\[ \frac{1}{60} = \frac{2\pi}{B} \]
\[ B = 120\pi \]
The pattern suggests an exponential pattern, not linear or sinusoidal. A 4% growth rate is accurate, while a 43% growth rate is not.

\[
A(t) = 800e^{-0.347t} \\
B(t) = 400e^{-0.231t}
\]

\[
\begin{align*}
  800e^{-0.347t} &= 400e^{-0.231t} \\
  0.15 &= e^{-0.347t} \\
  \ln 2e^{-0.347t} &= \ln e^{-0.231t} \\
  \ln 0.15 &= \ln e^{-0.347t} \\
  \ln 2 + \ln e^{-0.347t} &= \ln e^{-0.231t} \\
  \ln 0.15 &= -0.347t \cdot \ln e \\
  \ln 2 - 0.347t &= -0.231t \\
  \ln 2 &= 0.116t \\
  6 &\approx t
\end{align*}
\]

\[
P(S \cap M) = P(S) + P(M) - P(S \cup M) = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376}
\]

sample: pails of oranges; population: truckload of oranges. It is likely that about 5% of all the oranges are unsatisfactory.

\[
P(S) = \frac{5}{1376} \\
150 \times \frac{5}{1376} \\
0.00729
\]
\[
\sqrt{56-x} = x
\]

\(-8\) is extraneous.

\[
56-x = x^2
\]

\[
0 = x^2 + x - 56
\]

\[
0 = (x + 8)(x - 7)
\]

\[
x = 7
\]

PTS: 2  REF: 061605aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: extraneous solutions

64 ANS: 3

\[-33t^2 + 360t = 700 + 5t\]

\[-33t^2 + 355t - 700 = 0\]

\[
t = \frac{-355 \pm \sqrt{355^2 - 4(-33)(-700)}}{2(-33)} \approx 3, 8
\]

KEY: AII

65 ANS: 2

\[
B(t) = 750 \left( 1.16^{\frac{1}{12}} \right)^{12t} \approx 750(1.012)^{12t}
\]

\[
B(t) = 750 \left( 1 + \frac{0.16}{12} \right)^{12t}
\]

is wrong, because the growth is an annual rate that is not compounded monthly.

PTS: 2  REF: spr1504aii  NAT: A.SSE.B.3  TOP: Modeling Exponential Functions
KEY: AII

66 ANS: 1

\[d = 18; \ r = \pm \frac{5}{4}\]

PTS: 2  REF: 011714aii  NAT: F.IF.A.3  TOP: Sequences
KEY: term
The vertex of the parabola is \((4, -3)\). The \(x\)-coordinate of the focus and the vertex is the same. Since the distance from the vertex to the directrix is 3, the distance from the vertex to the focus is 3, so the \(y\)-coordinate of the focus is 0. The coordinates of the focus are \((4, 0)\).

\[
A = 5000(1.045)^n
\]

\[
B = 5000 \left(1 + \frac{0.046}{4}\right)^{4n}
\]

\[
5000\left(1 + \frac{0.046}{4}\right)^{4n} - 5000(1.045)^n \approx 6578.87 - 6511.30 \approx 67.57
\]

\[
10000 = 5000\left(1 + \frac{0.046}{4}\right)^{4n}
\]

\[
2 = 1.0115^{4n}
\]

\[
\log 2 = 4n \cdot \log 1.0115
\]

\[
n = \frac{\log 2}{4 \log 1.0115}
\]

\[
n \approx 15.2
\]

\[
x^4 - 4x^3 - 9x^2 + 36x = 0
\]

\[
x^3(x - 4) - 9x(x - 4) = 0
\]

\[
(x^3 - 9x)(x - 4) = 0
\]

\[
x(x^2 - 9)(x - 4) = 0
\]

\[
x(x + 3)(x - 3)(x - 4) = 0
\]

\[
x = 0, \pm 3, 4
\]
6x - 3y + 2z = -10  
-x + 3y + 5z = 45  
4x + 10z = 62  
4x + 4(7) = 20  

-2x + 3y + 8z = 72  
6x - 3y + 2z = -10  
4x + 4z = 20  
4x = -8  

4x + 10z = 62  
7x + 7z = 35  
6z = 42  
x = -2  

4x + 4z = 20  
z = 7  

6(-2) - 3y + 2(7) = -10  
-3y = -12  
y = 4

PTS: 4  
REF: spr1510aii  
NAT: A.REI.C.6  
TOP: Solving Linear Systems  
KEY: three variables

71. ANS: 1  
(1) \( \frac{9 - 0}{2 - 1} = 9 \)  
(2) \( \frac{17 - 0}{3.5 - 1} = 6.8 \)  
(3) \( \frac{0 - 0}{5 - 1} = 0 \)  
(4) \( \frac{17 - -5}{3.5 - 1} = 6.3 \)

PTS: 2  
REF: 011724aii  
NAT: F.IF.B.6  
TOP: Rate of Change  
KEY: AII

72. ANS: 3  
PTS: 2  
REF: 011710aii  
NAT: F.BF.A.1  
TOP: Operations with Functions

73. ANS:  
The expression is of the form \( y^2 - 5y - 6 \) or \( (y - 6)(y + 1) \). Let \( y = 4x^2 + 5x \):  
\( (4x^2 + 5x - 6)(4x^2 + 5x + 1) \)  

\( (4x - 3)(x + 2)(4x + 1)(x + 1) \)

PTS: 2  
REF: fall1512aii  
NAT: A.SSE.A.2  
TOP: Factoring Polynomials  
KEY: a>1

74. ANS:  
The mean difference between the students’ final grades in group 1 and group 2 is -3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription. One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandemize the students’ final grades many (500) times. If the observed difference -3.64 is the result of the assignment of students to groups alone, then a difference of -3.64 or less should be observed fairly regularly in the simulation output. However, a difference of -3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups’ mean final grades.

PTS: 4  
REF: fall1514aii  
NAT: S.IC.B.5  
TOP: Analysis of Data
This scenario can be modeled with a Venn Diagram:

Since $P(S \cup I) = 0.2$, $P(S \cup I) = 0.8$. Then, $P(S \cap I) = P(S) + P(I) - P(S \cup I)$. If $S$ and $I$ are independent, then the

\[ P(S \cap I) = 0.5 + 0.7 - 0.8 = 0.4 \]

Product Rule must be satisfied. However, $(0.5)(0.7) \neq 0.4$. Therefore, salary and insurance have not been treated independently.

**ANS:** 4  
**REF:** spr1513aii  
**NAT:** S.CP.A.2  
**TOP:** Theoretical Probability

\[
\frac{2x^2 - 3x + 7}{2x + 3} = \frac{4x^3 + 6x^2}{4x^3 + 0x^2 + 5x + 10} - \frac{6x^2 + 5x}{6x^2 - 9x} + \frac{14x + 10}{14x + 21} - \frac{11}{4x + 10}
\]

**ANS:** 2  
**REF:** 061614aii  
**NAT:** A.APR.D.6  
**TOP:** Rational Expressions  
**KEY:** remainder

The probability of rain equals the probability of rain, given that Sean pitches.

**ANS:** 1  
**REF:** 061611aii  
**NAT:** S.CP.A.3  
**TOP:** Conditional Probability

\[
S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04} \quad S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04} \approx 660778.39
\]

**ANS:** 4  
**REF:** 061634aii  
**NAT:** A.SSE.B.4  
**TOP:** Series

\[
ME = \left( z \sqrt{\frac{p(1-p)}{n}} \right) = \left( 1.96 \sqrt{\frac{(0.55)(0.45)}{900}} \right) \approx 0.03
\]

**ANS:** 2  
**REF:** 081612aii  
**NAT:** S.IC.B.4  
**TOP:** Analysis of Data
80 ANS: 2

$h(x)$ does not have a $y$-intercept.

PTS: 2 REF: 011719a1i NAT: F.IF.B.4 TOP: Properties of Graphs of Functions

81 ANS: 4

If $1 - i$ is one solution, the other is $1 + i$.

\[(x - (1 - i))(x - (1 + i)) = 0\]

\[x^2 - x - ix - x + ix + (1 - i^2) = 0\]

\[x^2 - 2x + 2 = 0\]

PTS: 2 REF: 081601a1ii NAT: A.REI.B.4 TOP: Complex Conjugate Root Theorem

82 ANS: 4

\[x = \frac{8 \pm \sqrt{(-8)^2 - 4(6)(29)}}{2(6)} = \frac{8 \pm \sqrt{-632}}{12} = \frac{8 \pm i\sqrt{4 \cdot \sqrt{158}}}{12} = \frac{2}{3} \pm \frac{1}{6} i\sqrt{158}\]

PTS: 2 REF: 011711a1ii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | quadratic formula

83 ANS: 4

As the range is $[4,10]$, the midline is $y = \frac{4 + 10}{2} = 7$.

PTS: 2 REF: fall1506a1i NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions

KEY: mixed

84 ANS: 4

\[y = g(x) = (x - 2)^2\]

\[(x - 2)^2 = 3x - 2\hspace{1em} y = 3(6) - 2 = 16\]

\[x^2 - 4x + 4 = 3x - 2\hspace{1em} y = 3(1) - 2 = 1\]

\[x^2 - 7x + 6 = 0\]

\[(x - 6)(x - 1) = 0\]

\[x = 6,1\]

PTS: 2 REF: 011705a1ii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

KEY: AII

85 ANS: 4

\[\frac{B(60) - B(10)}{60 - 10} \approx 28\% \hspace{1em} \frac{B(69) - B(19)}{69 - 19} \approx 33\% \hspace{1em} \frac{B(72) - B(36)}{72 - 36} \approx 38\% \hspace{1em} \frac{B(73) - B(60)}{73 - 60} \approx 46\%\]

PTS: 2 REF: 011721a1ii NAT: F.I.F.B.6 TOP: Rate of Change

KEY: AII
86 ANS: 4 PTS: 2 REF: 061601aii NAT: N.RN.A.2
TOP: Radicals and Rational Exponents
87 ANS: 3 PTS: 2 REF: 061607aii NAT: S.IC.A.2
TOP: Analysis of Data
88 ANS:
\[\sin^2 \theta + (-0.7)^2 = 1\]
Since \(\theta\) is in Quadrant II, \(\sin \theta = \sqrt{.51}\) and \(\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{.51}}{-0.7} \approx -1.02\)
\[
\sin^2 \theta = .51
\]
\[
\sin \theta = \pm \sqrt{.51}
\]
PTS: 2 REF: 081628aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions
89 ANS: 2
\[
x + 2\sigma \text{ represents approximately } 48\% \text{ of the data.}
\]
PTS: 2 REF: 061609aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: percent
90 ANS: 2 PTS: 2 REF: 011709aii NAT: S.IC.B.5
TOP: Analysis of Data
91 ANS: 4 PTS: 2 REF: 081624aii NAT: F.LE.A.2
TOP: Sequences
92 ANS: 2
\[x = -\frac{3}{4}y + 2\]
\[-4x = 3y - 8\]
\[-4x + 8 = 3y\]
\[-\frac{4}{3}x + \frac{8}{3} = y\]
PTS: 2 REF: 061616aii NAT: F.BF.B.4 TOP: Inverse of Functions
KEY: equations
93 ANS: 4
\[4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324\]
\[4(x - 3)^2 + 4(y + 9)^2 = 436\]
94 ANS: 1 PTS: 2 REF: 081609aii NAT: A.SSE.B.4
TOP: Series KEY: sigma notation
95 ANS: 4
\[ k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48 \]
\[ k^2(k^2 - 4) + 8k(k^2 - 4) + 12(k^2 - 4) \]
\[ (k^2 - 4)(k^2 + 8k + 12) \]
\[ (k + 2)(k - 2)(k + 6)(k + 2) \]
PTS: 2 REF: fall1505a TOP: Factoring Polynomials
KEY: factoring by grouping

96 ANS:
\[ 0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 \]
\[ z(x) = 6x^3 + 19x^2 - 52x + 15 \]
\[ 0 = -750 + 25b + 260 + 15 \]
\[ 475 = 25b \]
\[ 19 = b \]
\[
\begin{array}{c|cccc}
& 6 & 19 & -52 & 15 \\
\hline
-5 & & & & \\
\hline
6 & -30 & 55 & 15 \\
\end{array}
\]
\[ 6x^2 - 11x + 3 = 0 \]
\[ (2x - 3)(3x - 1) = 0 \]
\[ x = \frac{3}{2}, \frac{1}{3}, -5 \]
PTS: 4 REF: fall1515a TOP: Remainder Theorem

97 ANS: 3
\[ (m - 2)^2(m + 3) = (m^2 - 4m + 4)(m + 3) = m^3 + 3m^2 - 4m^2 - 12m + 4m + 12 = m^3 - m^2 - 8m + 12 \]
PTS: 2 REF: 081605a TOP: Factoring Polynomials
KEY: factoring by grouping

98 ANS: 3
\[ f(x) = -f(x), \text{ so } f(x) \text{ is odd. } g(-x) \neq g(x), \text{ so } g(x) \text{ is not even. } g(-x) \neq -g(x), \text{ so } g(x) \text{ is not odd. } h(-x) = h(x), \text{ so } h(x) \text{ is even.} \]
PTS: 2 REF: fall1502a TOP: Even and Odd Functions
The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of $-12$ and a maximum of 12. The value of $A$ is $-12$ since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter $B$. By experimentation with technology or using the relation $P = \frac{2\pi}{B}$ (where $P$ is the period), it is determined that $B = \frac{2\pi}{13}$.

$$f(t) = -12 \cos \left( \frac{2\pi}{13} t \right)$$

In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from $t = 13$ to $t = 19.5$ (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.

**PTS: 6**  
**REF: spr1514a**  
**NAT: F.IF.C.7**  
**TOP: Graphing Trigonometric Functions**  
**KEY: graph**

100. **ANS:**

```
\text{normcdf}(510, 540, 480, 24) = 0.0994 \quad z = \frac{510 - 480}{24} = 1.25 \quad 1.25 = \frac{x - 510}{20} \quad 2.5 = \frac{x - 510}{20} \quad 535-560
```

```
z = \frac{540 - 480}{24} = 2.5 \quad x = 535 \quad x = 560
```

**PTS: 4**  
**REF: fall1516a**  
**NAT: S.ID.A.4**  
**TOP: Normal Distributions**  
**KEY: probability**

101. **ANS:** 3

(3) repeats 3 times over $2\pi$.

**PTS: 2**  
**REF: 011722a**  
**NAT: F.IF.C.7**  
**TOP: Graphing Trigonometric Functions**  
**KEY: recognize**
\[
\sqrt{x - 5} = -x + 7 \quad \sqrt{x - 5} = -9 + 7 = -2 \text{ is extraneous.}
\]
\[
x - 5 = x^2 - 14x + 49
\]
\[
0 = x^2 - 15x + 54
\]
\[
0 = (x - 6)(x - 9)
\]
\[
x = 6, 9
\]
106 ANS: 3
\[ 2d(d^3 + 3d^2 - 9d - 27) \]
\[ 2d(d^2(d + 3) - 9(d + 3)) \]
\[ 2d(d^2 - 9)(d + 3) \]
\[ 2d(d + 3)(d - 3) \]
\[ 2d(d + 3)^2(d - 3) \]

PTS: 2 REF: 081615aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: factoring by grouping

107 ANS: 1

PTS: 2 REF: 081616aii NAT: F.TF.A.1 TOP: Unit Circle

108 ANS:
\[ \frac{x^3 + 9}{x^3 + 8} \]
\[ = \frac{x^3 + 8 + 1}{x^3 + 8} \]
\[ = \frac{x^3 + 9}{x^3 + 8} \]

PTS: 2 REF: 061631aii NAT: A.APR.C.4 TOP: Polynomial Identities

109 ANS:
\[ f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 128 - 80 - 44 - 4 = 0 \]
Any method that demonstrates 4 is a zero of \( f(x) \) confirms that \( x - 4 \) is a factor, as suggested by the Remainder Theorem.

PTS: 2 REF: spr1507aii NAT: A.APR.B.2 TOP: Remainder Theorem

110 ANS: 3

\[ H(t) \text{ is at a minimum at } 70(-1) + 80 = 10 \]

PTS: 2 REF: 061613aii NAT: F.IF.B.4 TOP: Graphing Trigonometric Functions
KEY: maximum/minimum
111 ANS: \[\frac{1}{12} \approx 1.00427\]

PTS: 2 REF: 061621aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions

KEY: AII

112 ANS:

\[
\begin{align*}
x - 2 \left( 3x^2 + 7x - 20 \right) & = 3x + 13 + \frac{6}{x - 2} \\
3x^2 - 6x & = 13x - 20 \\
3x^2 - 6x & = 13x - 26 \\
6 & =
\end{align*}
\]

PTS: 2 REF: 011732aii NAT: A.APR.D.6 TOP: Rational Expressions

113 ANS: 2 PTS: 2 REF: 061620aii NAT: F.IF.B.4 TOP: Graphing Polynomial Functions

114 ANS: 2 PTS: 2 REF: 011701aii NAT: F.IF.B.4 TOP: Graphing Trigonometric Functions

115 ANS:

![Graph](image)

PTS: 2 REF: 011729aii NAT: A.APR.B.3 TOP: Zeros of Polynomials

116 ANS:

\[A = Pe^{rt}\]

\[135000 = 100000e^{5r}\]

\[1.35 = e^{5r}\]

\[\ln 1.35 = \ln e^{5r}\]

\[\ln 1.35 = 5r\]

\[.06 \approx r \text{ or } 6\%\]

PTS: 2 REF: 061632aii NAT: F.LE.A.4 TOP: Exponential Growth
ANS:
Randomly assign participants to two groups. One group uses the toothpaste with ingredient X and the other group uses the toothpaste without ingredient X.

PTS: 2  REF: 061626a1i  NAT: S.IC.B.3  TOP: Analysis of Data  KEY: type

ANS: 4

\[
\frac{1}{x} - \frac{1}{3} = \frac{-1}{3x} \\
\frac{3-x}{3x} = \frac{-1}{3x} \\
3-x = -1 \\
x = 4
\]

PTS: 2  REF: 061625a1i  NAT: A.REI.A.2  TOP: Solving Rationals  KEY: rational solutions
\[ 0 = x^2(x + 1) - 4(x + 1) \]
\[ 0 = (x^2 - 4)(x + 1) \]
\[ 0 = (x + 2)(x - 2)(x + 1) \]
\[ x = -2, -1, 2 \]

**PTS: 4**  
**REF: 081633aii**  
**NAT: F.IF.C.7**  
**TOP: Graphing Polynomial Functions**

\[ x^2 + 0x + 1 \]
\[ x + 2 \]
\[ \overline{x^3 + 2x^2 + x + 6} \]
\[ x^3 + 2x^2 \]
\[ 0x^2 + x \]
\[ 0x^2 + 0x \]
\[ x + 6 \]
\[ x + 2 \]
\[ 4 \]

**PTS: 2**  
**REF: 081611aii**  
**NAT: A.APR.D.6**  
**TOP: Rational Expressions**  
**KEY: remainder**

\[ xi(-6i)^2 = xi(36i^2) = 36xi^3 = -36xi \]

**PTS: 2**  
**REF: 081627aii**  
**NAT: N.CN.A.2**  
**TOP: Operations with Complex Numbers**

**ANS:**

No, because \( P(M / R) \neq P(M) \)
\[ \frac{70}{180} \neq \frac{230}{490} \]
\[ 0.38 \neq 0.47 \]

**PTS: 2**  
**REF: 011731aii**  
**NAT: S.CP.A.4**  
**TOP: Conditional Probability**
ANS:

Applying the commutative property, \( \left( \frac{1}{5} \right)^2 \) can be rewritten as \( \left( \frac{1}{5} \right)^2 \) or \( 9^\frac{1}{5} \). A fractional exponent can be rewritten as a radical with the denominator as the index, or \( 9^\frac{1}{5} = \sqrt[5]{9} \).

PTS: 2  REF: 081626aii  NAT: N.RN.A.1  TOP: Radicals and Rational Exponents

ANS:

\[ y = -x + 5 \quad y = -7 + 5 = -2 \]
\[ (x - 3)^2 + (-x + 5 + 2)^2 = 16 \quad y = -3 + 5 = 2 \]
\[ x^2 - 6x + 9 + x^2 - 14x + 49 = 16 \]
\[ 2x^2 - 20x + 42 = 0 \]
\[ x^2 - 10x + 21 = 0 \]
\[ (x - 7)(x - 3) = 0 \]
\[ x = 7, 3 \]

PTS: 4  REF: 061633aii  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems

KEY: AII

ANS: 1

The graph of \( y = \sin x \) is unchanged when rotated 180º about the origin.

PTS: 2  REF: 081614aii  NAT: F.BF.B.3  TOP: Even and Odd Functions

ANS:

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ A \text{ and } B \text{ are independent since } P(A \cap B) = P(A) \cdot P(B) \]
\[ 0.8 = 0.6 + 0.5 - P(A \cap B) \]
\[ 0.3 = 0.6 \cdot 0.5 \]
\[ P(A \cap B) = 0.3 \]
\[ 0.3 = 0.3 \]

PTS: 2  REF: 081632aii  NAT: S.CP.A.2  TOP: Theoretical Probability
128 ANS: 
\[ 0 = \log_{10}(x - 4) \] The x-intercept of \( h \) is (2,0). \( f \) has the larger value.

\[ 10^0 = x - 4 \]
\[ 1 = x - 4 \]
\[ x = 5 \]

PTS: 2 REF: 081630aii NAT: F.IF.C.9 TOP: Comparing Functions KEY: AII

129 ANS: 1

\[ \frac{2(x - 4)}{(x + 3)(x - 4)} + \frac{3(x + 3)}{(x - 4)(x + 3)} = \frac{2x - 2}{x^2 - x - 12} \]
\[ 2x - 8 + 3x + 9 = 2x - 2 \]
\[ 3x = -3 \]
\[ x = -1 \]


130 ANS: 
\[ 0.602 \pm 2 \cdot 0.066 = 0.47 - 0.73. \] Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

PTS: 4 REF: 061635aii NAT: S.IC.B.5 TOP: Analysis of Data
A parabola with a focus of $(0,4)$ and a directrix of $y = 2$ is sketched as follows:

By inspection, it is determined that the vertex of the parabola is $(0,3)$. It is also evident that the distance, $p$, between the vertex and the focus is 1. It is possible to use the formula $(x - h)^2 = 4p(y - k)$ to derive the equation of the parabola as follows:

$$(x - 0)^2 = 4(1)(y - 3)$$

$$x^2 = 4y - 12$$

$$x^2 + 12 = 4y$$

$$\frac{x^2}{4} + 3 = y$$

or A point $(x, y)$ on the parabola must be the same distance from the focus as it is from the directrix. For any such point $(x, y)$, the distance to the focus is $\sqrt{(x - 0)^2 + (y - 4)^2}$ and the distance to the directrix is $y - 2$. Setting this equal leads to:

$$x^2 + y^2 - 8y + 16 = y^2 - 4y + 4$$

$$x^2 + 16 = 4y + 4$$

$$\frac{x^2}{4} + 3 = y$$

The graph shows three real zeros, and has end behavior matching the given end behavior.
ANS: Amplitude, because the height of the graph shows the volume of the air.

PTS: 2  
REF: 081625aii  
NAT: F.IF.C.7  
TOP: Graphing Trigonometric Functions

ANS: 4

\[
y = 5^{-t} = \left(\frac{1}{5}\right)^t
\]

PTS: 2  
REF: 061615aii  
NAT: F.IF.C.8  
TOP: Modeling Exponential Functions

ANS: 1

\[
x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)} = \frac{-3 \pm \sqrt{-7}}{4} = \frac{-3 \pm i\sqrt{7}}{4}
\]

PTS: 2  
REF: 061612aii  
NAT: A.REI.B.4  
TOP: Solving Quadratics

KEY: complex solutions | quadratic formula

ANS: 4

\[(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3\]

PTS: 2  
REF: 081620aii  
NAT: A.APR.C.4  
TOP: Polynomial Identities
139 ANS:

\[
720 = \frac{120000 \left( \frac{0.048}{12} \right) \left( 1 + \frac{0.048}{12} \right)^n}{1 + \frac{0.048}{12} - 1} \\
\quad \frac{275.2}{12} \approx 23 \text{ years}
\]

\[
720(1.004)^n - 720 = 480(1.004)^n
\]

\[
240(1.004)^n = 720
\]

\[
1.004^n = 3
\]

\[
n \log 1.004 = \log 3
\]

\[
n \approx 275.2 \text{ months}
\]