Dear Sir

I have to acknowledge the receipt of your favor of May 14. in which you mention that you have finished the 6. first books of Euclid, plane trigonometry, surveying & algebra and ask whether I think a further pursuit of that branch of science would be useful to you. there are some propositions in the latter books of Euclid, & some of Archimedes, which are useful, & I have no doubt you have been made acquainted with them. trigonometry, so far as this, is most valuable to every man, there is scarcely a day in which he will not resort to it for some of the purposes of common life. the science of calculation also is indispensible as far as the extraction of the square & cube roots; Algebra as far as the quadratic equation & the use of logarithms are often of value in ordinary cases: but all beyond these is but a luxury; a delicious luxury indeed; but not to be indulged in by one who is to have a profession to follow for his subsistence. in this light I view the conic sections, curves of the higher orders, perhaps even spherical trigonometry, Algebraical operations beyond the 2d dimension, and fluxions.

Letter from Thomas Jefferson to William G. Munford, Monticello, June 18, 1799.
Geometry Regents at Random
Answer Section

1 ANS: 3 PTS: 2 REF: fall0816ge TOP: Planes

2 ANS: 1
(x, y) → (x + 3, y + 1)

PTS: 2 REF: fall0803ge TOP: Translations

3 ANS: 1
∠DCB and ∠ADC are supplementary adjacent angles of a parallelogram. 180 – 120 = 60. ∠2 = 60 – 45 = 15.

PTS: 2 REF: 080907ge TOP: Parallelograms

4 ANS:
6. The centroid divides each median into segments whose lengths are in the ratio 2 : 1. \( \overline{TD} = 6 \) and \( \overline{DB} = 3 \)

PTS: 2 REF: 011034ge TOP: Centroid

5 ANS: 1
\[ V = \pi r^2 h \]
1000 = \( \pi r^2 \cdot 8 \)
\[ r^2 = \frac{1000}{8\pi} \]
\[ r \approx 6.3 \]

PTS: 2 REF: 080926ge TOP: Volume

6 ANS: 3 PTS: 2 REF: 011028ge TOP: Conditional Statements

7 ANS: 4 PTS: 2 REF: fall0818ge TOP: Analytical Representations of Transformations

8 ANS: 2
The slope of \( 2x + 3y = 12 \) is \( \frac{A}{B} = \frac{2}{3} \). The slope of a perpendicular line is \( \frac{3}{2} \). Rewritten in slope intercept form, (2) becomes \( y = \frac{3}{2} x + 3 \).

PTS: 2 REF: 060926ge TOP: Parallel and Perpendicular Lines

9 ANS: 3 PTS: 2 REF: fall0804ge TOP: Constructions
10 ANS:

\[A''(8,2), B''(2,0), C''(6,-8)\]

PTS: 4 REF: 081036ge TOP: Compositions of Transformations

11 ANS: 1

\[A'(2,4)\]

PTS: 2 REF: 011023ge TOP: Compositions of Transformations

KEY: basic

12 ANS: 2

\[\frac{3}{7} = \frac{6}{x}\]

\[3x = 42\]

\[x = 14\]

PTS: 2 REF: 081027ge TOP: Side Splitter Theorem

13 ANS: 4 PTS: 2 REF: 061008ge TOP: Trapezoids

14 ANS:

4. \[l_1w_1h_1 = l_2w_2h_2\]

\[10 \times 2 \times h = 5 \times w_2 \times h\]

\[20 = 5w_2\]

\[w_2 = 4\]

PTS: 2 REF: 011030ge TOP: Volume

15 ANS: 1

\[M_x = \frac{-2+6}{2} = 2, \ M_y = \frac{3+3}{2} = 3\]. The center is \((2,3)\). \[d = \sqrt{(-2-6)^2 + (3-3)^2} = \sqrt{64+0} = 8\]. If the diameter is 8, the radius is 4 and \(r^2 = 16\).

PTS: 2 REF: fall0820ge TOP: Equations of Circles

16 ANS: 4

\[d = \sqrt{(146-(-4))^2 + (52-2)^2} = \sqrt{25,000} \approx 158.1\]

PTS: 2 REF: 061021ge TOP: Distance
17 ANS: True. The first statement is true and the second statement is false. In a disjunction, if either statement is true, the disjunction is true.

18 ANS: True. The first statement is true and the second statement is false. In a disjunction, if either statement is true, the disjunction is true.

19 ANS: 2

\[ M_x = \frac{3x + 5 + x - 1}{2} = \frac{4x + 4}{2} = 2x + 2, \quad M_y = \frac{3y + (-y)}{2} = \frac{2y}{2} = y. \]

20 ANS: 4

Corresponding angles of similar triangles are congruent.

21 ANS: 2

The slope of a line in standard form is \(-\frac{A}{B}\) so the slope of this line is \(-\frac{5}{3}\). Perpendicular lines have slope that are the opposite and reciprocal of each other.

22 ANS: 1

Parallel lines intercept congruent arcs.

23 ANS: 25.

\[ d = \sqrt{(-3 - 4)^2 + (1 - 25)^2} = \sqrt{49 + 576} = \sqrt{625} = 25. \]

24 ANS: 2

Parallel chords intercept congruent arcs. \(m\overparen{AC} = m\overparen{BD} = 30. \quad 180 - 30 - 30 = 120. \)
25 ANS: 1
\[ x + 2x + 2 + 3x + 4 = 180 \]
\[ 6x + 6 = 180 \]
\[ x = 29 \]

PTS: 2  REF: 011002ge  TOP: Interior and Exterior Angles of Triangles

26 ANS: 1  PTS: 2  REF: fall0807ge  TOP: Constructions

27 ANS: 

\[ D'(-1,1), E'(-1,5), G'(-4,5) \]

PTS: 4  REF: 080937ge  TOP: Properties of Transformations

28 ANS: 2  PTS: 2  REF: 011011ge  TOP: Locus

29 ANS: 1
\( \triangle PRT \) and \( \triangle SRQ \) share \( \angle R \) and it is given that \( \angle RPT \cong \angle RSQ \).

PTS: 2  REF: fall0821ge  TOP: Similarity Proofs

30 ANS: 4  PTS: 2  REF: 060913ge  TOP: Conditional Statements

31 ANS: 4
Let \( AD = x \).  \( 36x = 12^2 \)
\[ x = 4 \]

PTS: 2  REF: 080922ge  TOP: Similarity  KEY: leg

32 ANS: 1

\[ 3x + 15 + 2x - 1 = 6x + 2 \]
\[ 5x + 14 = 6x + 2 \]
\[ x = 12 \]

PTS: 2  REF: 011021ge  TOP: Exterior Angle Theorem
33 ANS:

\[
\begin{array}{c}
\text{ID: A} \\
\end{array}
\]

PTS: 4 REF: 060937ge TOP: Compositions of Transformations
KEY: grids

34 ANS: 4 PTS: 2 REF: 080925ge
TOP: Centroid, Orthocenter, Incenter and Circumcenter

35 ANS:

\[AC. \ m\angle BCA = 63\text{ and } m\angle ABC = 80. \ AC\] is the longest side as it is opposite the largest angle.

36 ANS: 2 PTS: 2 REF: 080934ge TOP: Angle Side Relationship

37 ANS: 4

\[L = 2\pi rh = 2\pi \cdot 5 \cdot 11 \approx 345.6\]

38 ANS: 1

\[y = x^2 - 4x = (4)^2 - 4(4) = 0. \ (4,0)\] is the only intersection.

39 ANS: 1

\[V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 4^2 \cdot 12 \approx 201\]

40 ANS:

\[452. \ SA = 4\pi r^2 = 4\pi \cdot 6^2 = 144\pi \approx 452\]

41 ANS: 3 PTS: 2 REF: fall0814ge TOP: Equations of Circles
42 ANS: 4

\[ SA = 4\pi r^2 \quad V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \cdot 6^3 = 288\pi \]

\[ 144\pi = 4\pi r^2 \]

\[ 36 = r^2 \]

\[ 6 = r \]

PTS: 2 
REF: 081020ge 
TOP: Volume and Surface Area

43 ANS: 3 
PTS: 2
REF: 060905ge 
TOP: Reflections

KEY: basic

44 ANS: 3

\[ m = \frac{-A}{B} = \frac{5}{2}, \quad m = \frac{-A}{B} = \frac{10}{4} = \frac{5}{2} \]

PTS: 2 
REF: 011014ge 
TOP: Parallel and Perpendicular Lines

45 ANS: 1

PTS: 2
REF: 080911ge 
TOP: Equations of Circles

46 ANS: 2

PTS: 2
REF: 080927ge 
TOP: Planes

47 ANS: 4

Median \( BF \) bisects \( AC \) so that \( CF \cong FA \).

PTS: 2
REF: fall0810ge 
TOP: Statements

48 ANS:

20. \( 5x + 10 = 4x + 30 \)

\[ x = 20 \]

PTS: 2 
REF: 060934ge 
TOP: Similarity 
KEY: basic

49 ANS: 3

PTS: 2
REF: 081002ge 
TOP: Planes

50 ANS:

26. \( x + 3x + 5x - 54 = 180 \)

\[ 9x = 234 \]

\[ x = 26 \]

PTS: 2 
REF: 080933ge 
TOP: Interior and Exterior Angles of Triangles

51 ANS: 3

PTS: 2 
REF: 060902ge 
TOP: Triangle Congruency

52 ANS: 2

PTS: 2 
REF: 081015ge 
TOP: Properties of Transformations
53 ANS:
34. \(2x - 12 + x + 90 = 180\)
\[3x + 78 = 90\]
\[3x = 102\]
\[x = 34\]

PTS: 2 REF: 061031ge TOP: Interior and Exterior Angles of Triangles

54 ANS: 4 PTS: 2 REF: fall0802ge TOP: Negations

55 ANS: 1 PTS: 2 REF: 060903ge TOP: Identifying Transformations

56 ANS: 4 PTS: 2 REF: 081005ge TOP: Constructions

57 ANS:
y = \frac{2}{3}x - 9. The slope of \(2x - 3y = 11\) is \(\frac{A}{B} = \frac{-2}{-3} = \frac{2}{3}\):
\[-5 = \left(\frac{2}{3}\right)(6) + b\]
\[-5 = 4 + b\]
\[b = -9\]

PTS: 2 REF: 080931ge TOP: Parallel and Perpendicular Lines

58 ANS: 4 PTS: 2 REF: 011009ge TOP: Constructions

59 ANS: 4
The slope of \(y = \frac{2}{3}x - 5\) is \(-\frac{2}{3}\). Perpendicular lines have slope that are opposite reciprocals.

PTS: 2 REF: 080917ge TOP: Parallel and Perpendicular Lines

60 ANS: 4 PTS: 2 REF: 011019ge TOP: Similarity Proofs

61 ANS: 3 PTS: 2 REF: 060928ge TOP: Planes

62 ANS: 3
The diagonals of an isosceles trapezoid are congruent. \(5x + 3 = 11x - 5\).
\[6x = 18\]
\[x = 3\]

PTS: 2 REF: fall0801ge TOP: Trapezoids

63 ANS: 1 PTS: 2 REF: 061009ge TOP: Converse

64 ANS: 2
\[4(4x - 3) = 3(2x + 8)\]
\[16x - 12 = 6x + 24\]
\[10x = 36\]
\[x = 3.6\]

PTS: 2 REF: 080923ge TOP: Segments Intercepted by Circle

KEY: two chords
65 ANS: 2
Parallel chords intercept congruent arcs. $m\widehat{AD} = m\widehat{BC} = 60$. $m\angle CDB = \frac{1}{2} m\widehat{BC} = 30$.

PTS: 2 REF: 060906ge TOP: Chords

66 ANS: 1
The centroid divides each median into segments whose lengths are in the ratio 2:1. $\overline{GC} = 2\overline{FG}$

$$\frac{GC}{FG} = 2$$

$$\frac{2FG + FG}{FG} = 24$$

$$\frac{3FG}{FG} = 24$$

$$FG = 8$$

PTS: 2 REF: 081018ge TOP: Centroid

67 ANS: 1
If $\angle A$ is at minimum (50°) and $\angle B$ is at minimum (90°), $\angle C$ is at maximum of 40° ($180° - (50° + 90°)$). If $\angle A$ is at maximum (60°) and $\angle B$ is at maximum (100°), $\angle C$ is at minimum of 20° ($180° - (60° + 100°)$).

PTS: 2 REF: 060901ge TOP: Interior and Exterior Angles of Triangles

68 ANS: 3
$$375\pi = \pi rl = \pi (15)(25) = 375\pi$$

PTS: 2 REF: 081030ge TOP: Volume and Lateral Area

69 ANS: 3
The lateral edges of a prism are parallel.

PTS: 2 REF: fall0808ge TOP: Solids

70 ANS: 4
$$180 - (40 + 40) = 100$$

PTS: 2 REF: 080903ge TOP: Isosceles Triangle Theorem

71 ANS: 3 PTS: 2 REF: 011007ge TOP: Isosceles Triangle Theorem

72 ANS: 2
73 ANS:

PTS: 2    REF: 011032ge    TOP: Constructions

74 ANS:

PTS: 4    REF: 011037ge    TOP: Locus

75 ANS:
Yes, \( m\angle ABD = m\angle BDC = 44 \)  
\[ 180 - (93 + 43) = 44 \]  
\[ x + 19 + 2x + 6 + 3x + 5 = 180. \]
Because alternate interior \( \angle ABD \) and \( \angle CDB \) are congruent, \( \overline{AB} \) is parallel to \( \overline{DC} \).

PTS: 4    REF: 081035ge    TOP: Parallel Lines and Transversals

76 ANS:
\( \overline{JK} \cong \overline{LM} \) because opposite sides of a parallelogram are congruent. \( \overline{LM} \cong \overline{LN} \) because of the Isosceles Triangle Theorem. \( \overline{LM} \cong \overline{JM} \) because of the transitive property. \( \overline{JKL} \) is a rhombus because all sides are congruent.

PTS: 4    REF: 011036ge    TOP: Special Quadrilaterals
77 ANS: [Image]

PTS: 2 REF: 061032ge TOP: Reflections KEY: grids

78 ANS: 1
\[
d = \sqrt{(-4 - 2)^2 + (5 - (-5))^2} = \sqrt{36 + 100} = \sqrt{136} = \sqrt{4 \cdot 34} = 2\sqrt{34}.
\]

PTS: 2 REF: 080919ge TOP: Distance

79 ANS: 4
\[
y = 2x + 1 \quad y = \frac{1}{2}x - 5
\]

PTS: 2 REF: fall0822ge TOP: Parallel and Perpendicular Lines

80 ANS: 4
Longest side of a triangle is opposite the largest angle. Shortest side is opposite the smallest angle.

PTS: 2 REF: 081011ge TOP: Angle Side Relationship

81 ANS: 2
\[
x^2 = 3(x + 18)
\]
\[
x^2 - 3x - 54 = 0
\]
\[
(x - 9)(x + 6) = 0
\]
\[
x = 9
\]

PTS: 2 REF: fall0817ge TOP: Segments Intercepted by Circle KEY: tangent and secant

82 ANS: 1 PTS: 2 REF: 061013ge TOP: Tangents KEY: point of tangency

83 ANS: 2
\[
M_x = \frac{2 + (-4)}{2} = -1. \quad M_y = \frac{-3 + 6}{2} = \frac{3}{2}.
\]

PTS: 2 REF: fall0813ge TOP: Midpoint

84 ANS: 4 PTS: 2 REF: 061015ge TOP: Identifying Transformations
85 ANS: 2

Because the triangles are similar, \( \frac{m\angle A}{m\angle D} = 1 \)

PTS: 2     REF: 011022ge     TOP: Similarity     KEY: perimeter and area

86 ANS:
Contrapositive-If two angles of a triangle are not congruent, the sides opposite those angles are not congruent.

PTS: 2     REF: fall0834ge     TOP: Conditional Statements

87 ANS: 2

\[
\begin{align*}
M_x &= \frac{-2 + 6}{2} = 2, \\
M_y &= \frac{-4 + 2}{2} = -1
\end{align*}
\]

PTS: 2     REF: 080910ge     TOP: Midpoint

88 ANS: 3     PTS: 2     REF: 081026ge     TOP: Contrapositive

89 ANS: 1

\[
\angle A = \frac{(n - 2)180}{n} = \frac{(5 - 2)180}{5} = 108, \quad \angle AEB = \frac{180 - 108}{2} = 36
\]

PTS: 2     REF: 081022ge     TOP: Interior and Exterior Angles of Polygons

90 ANS: 

PTS: 2     REF: 081033ge     TOP: Locus

91 ANS: 3

PTS: 2     REF: fall0805ge     TOP: Quadratic-Linear Systems
92 ANS: 2
\[ x^2 + (x + 7)^2 = 13^2 \]
\[ x^2 + x^2 + 7x + 49 = 169 \]
\[ 2x^2 + 14x - 120 = 0 \]
\[ x^2 + 7x - 60 = 0 \]
\[ (x + 12)(x - 5) = 0 \]
\[ x = 5 \]
\[ 2x = 10 \]

PTS: 2 REF: 061024ge TOP: Pythagorean Theorem

93 ANS: 2
Longest side of a triangle is opposite the largest angle. Shortest side is opposite the smallest angle.

PTS: 2 REF: 060911ge TOP: Angle Side Relationship

94 ANS: 1
After the translation, the coordinates are \(A'(-1, 5)\) and \(B'(3, 4)\). After the dilation, the coordinates are \(A''(-2, 10)\) and \(B''(6, 8)\).

PTS: 2 REF: fall0823ge TOP: Compositions of Transformations

95 ANS: 4
The marked 60º angle and the angle above it are on the same straight line and supplementary. This unmarked supplementary angle is 120º. Because the unmarked 120º angle and the marked 120º angle are alternate exterior angles and congruent, \(d \parallel e\).

PTS: 2 REF: 080901ge TOP: Parallel Lines and Transversals

96 ANS:
\(\overline{AC} \cong \overline{EC}\) and \(\overline{DC} \cong \overline{BC}\) because of the definition of midpoint. \(\angle ACB \cong \angle ECD\) because of vertical angles. \(\triangle ABC \cong \triangle EDC\) because of SAS. \(\angle CDE \cong \angle CBA\) because of CPCTC. \(BD\) is a transversal intersecting \(\overline{AB}\) and \(\overline{ED}\). Therefore \(\overline{AB} \parallel \overline{DE}\) because \(\angle CDE\) and \(\angle CBA\) are congruent alternate interior angles.

PTS: 6 REF: 060938ge TOP: Triangle Proofs

97 ANS: 1 PTS: 2 REF: 060920ge TOP: Graphing Circles
98 ANS: 4

\[ \Delta ABC \sim \Delta DBE. \quad \frac{AB}{DB} = \frac{AC}{DE} \]
\[ \frac{9}{2} = \frac{x}{3} \]
\[ x = 13.5 \]

PTS: 2 REF: 060927ge TOP: Side Splitter Theorem

99 ANS: 2

The slope of a line in standard form is \( \frac{A}{B} \), so the slope of this line is \( \frac{-2}{-1} = 2 \). A parallel line would also have a slope of 2. Since the answers are in slope intercept form, find the y-intercept:

\[ y = mx + b \]
\[ -11 = 2(-3) + b \]
\[ -5 = b \]

PTS: 2 REF: fall0812ge TOP: Parallel and Perpendicular Lines

100 ANS: 1

Translations and reflections do not affect distance.

PTS: 2 REF: 080908ge TOP: Properties of Transformations

101 ANS: 1 PTS: 2 REF: 061012ge TOP: Constructions

102 ANS: 1 PTS: 2 REF: 081028ge TOP: Centroid, Orthocenter, Incenter and Circumcenter

103 ANS: 4 PTS: 2 REF: 080914ge TOP: Planes

104 ANS:

PTS: 2 REF: fall0830ge TOP: Properties of Transformations
105 ANS: 4

PTS: 2    REF: 081001ge    TOP: Triangle Congruency

106 ANS: 3

\[
\frac{36 + 20}{2} = 28
\]

PTS: 2    REF: 061019ge    TOP: Arcs Determined by Angles

107 ANS:

PTS: 2    REF: 081032ge    TOP: Constructions

108 ANS: 3    PTS: 2    REF: 011010ge    TOP: Equations of Circles

109 ANS:

PTS: 2    REF: 061033ge    TOP: Locus

110 ANS: 4

The slope of a line in standard form is \( \frac{A}{B} \), so the slope of this line is \( \frac{-4}{2} = -2 \). A parallel line would also have a slope of \(-2\). Since the answers are in slope intercept form, find the \( y \)-intercept:

\[
y = mx + b
\]

\[
3 = -2(7) + b
\]

\[
17 = b
\]

PTS: 2    REF: 081010ge    TOP: Parallel and Perpendicular Lines
111 ANS:

![Diagram](image1)

PTS: 2  REF: 080932ge  TOP: Constructions

112 ANS: 1

\[3x^2 + 18x + 24\]

\[3(x^2 + 6x + 8)\]

\[3(x + 4)(x + 2)\]

PTS: 2  REF: fall0815ge  TOP: Volume

113 ANS:

\[2\sqrt{3}. \ x^2 = 3 \cdot 4\]

\[x = \sqrt{12} = 2\sqrt{3}\]

PTS: 2  REF: fall0829ge  TOP: Similarity  KEY: altitude

114 ANS: 1

\[-2\left(-\frac{1}{2}x = 6x + 10\right)\]

\[y = -12x - 20\]

PTS: 2  REF: 061027ge  TOP: Parallel and Perpendicular Lines

115 ANS: 4

The slope of \(y = -3x + 2\) is \(-3\). The perpendicular slope is \(\frac{1}{3}\). \(-1 = \frac{1}{3}(3) + b\)

\[-1 = 1 + b\]

\[b = -2\]

PTS: 2  REF: 011018ge  TOP: Parallel and Perpendicular Lines

116 ANS: 1  PTS: 2  REF: 060918ge  TOP: Planes

117 ANS:

\[y = -2x + 14.\] The slope of \(2x + y = 3\) is \(\frac{-A}{B} = \frac{-2}{1} = -2\). \(y = mx + b\).

\[4 = (-2)(5) + b\]

\[b = 14\]

PTS: 2  REF: 060931ge  TOP: Parallel and Perpendicular Lines
118 ANS: 3
\[(x + 3)^2 - 4 = 2x + 5\]
\[x^2 + 6x + 9 - 4 = 2x + 5\]
\[x^2 + 4x = 0\]
\[x(x + 4) = 0\]
\[x = 0, -4\]

PTS: 2 REF: 081004ge TOP: Quadratic-Linear Systems

119 ANS: 3
\[4(x + 4) = 8^2\]
\[4x + 16 = 64\]
\[x = 12\]

PTS: 2 REF: 060916ge TOP: Segments Intercepted by Circle
KEY: tangent and secant

120 ANS:
70. \[3x + 5 + 3x + 5 + 2x + 2x = 180\]
\[10x + 10 = 360\]
\[10x = 350\]
\[x = 35\]
\[2x = 70\]

PTS: 2 REF: 081029ge TOP: Trapezoids

121 ANS: 4
\[M_x = \frac{-6 + 1}{2} = -\frac{5}{2} \quad M_y = \frac{1 + 8}{2} = \frac{9}{2} \]

PTS: 2 REF: 060919ge TOP: Midpoint

122 ANS: 

PTS: 2 REF: fall0832ge TOP: Constructions
110. \[ 6x + 20 = x + 40 + 4x - 5 \]
\[ 6x + 20 = 5x + 35 \]
\[ x = 15 \]
\[ 6((15) + 20) = 110 \]

ANS: 110.

PTS: 2
REF: 081031ge
TOP: Isosceles Triangle Theorem

124 ANS:
\[ BD \cong DB \text{ (Reflexive Property); } \triangle ABD \cong \triangle CDB \text{ (SSS); } \angle BDC \cong \angle ABD \text{ (CPCTC).} \]

PTS: 4
REF: 061035ge
TOP: Quadrilateral Proofs

125 ANS: 1
Since \( AC \cong BC \), \( m\angle A = m\angle B \) under the Isosceles Triangle Theorem.

PTS: 2
REF: fall0809ge
TOP: Triangles in the Coordinate Plane

126 ANS: 3
\[ m = \frac{-A}{B} = \frac{-3}{4} \]

PTS: 2
REF: 011025ge
TOP: Parallel and Perpendicular Lines

127 ANS: 4
\( BG \) is also an angle bisector since it intersects the concurrence of \( CD \) and \( AE \)

PTS: 2
REF: 061025ge
KEY: Centroid, Orthocenter, Incenter and Circumcenter
128 ANS:

![Diagram of a circle with a vertical line through it.]

129 ANS:

37. Since $DE$ is a midsegment, $AC = 14$. $10 + 13 + 14 = 37$

130 ANS:

18. $V = \frac{1}{3}Bh = \frac{1}{3} lwh$

$288 = \frac{1}{3} \cdot 8 \cdot 6 \cdot h$

$288 = 16h$

$18 = h$

131 ANS:

20. The sides of the triangle formed by connecting the midpoints are half the sides of the original triangle.

$5 + 7 + 8 = 20.$

132 ANS: 4  

PTS: 2  REF: 061003ge  TOP: Solids
The closer a chord is to the center of a circle, the longer the chord.

The sum of the interior angles of a pentagon is \((5 - 2)180 = 540\).

Opposite sides of a parallelogram are congruent. \(4x - 3 = x + 3\). \(SV = (2) + 3 = 5\).

\[
3x = 6
\]
\[
x = 2
\]
139 ANS:

\[ y = \frac{2}{3}x + 1 \]

\[ 2y + 3x = 6 \]

\[ y = \frac{3}{2}x + 3 \]

\[ 5 = \frac{2}{3}(6) + b \]

\[ 5 = 4 + b \]

\[ m = \frac{3}{2} \]

\[ m_\perp = \frac{2}{3} \]

PTS: 4  REF: 080936ge  TOP: Locus

140 ANS: 1  PTS: 2  REF: 061005ge  TOP: Properties of Transformations

141 ANS: 3

PTS: 2  REF: 080920ge  TOP: Midsegments

142 ANS:

\[ y = \frac{2}{3}x + 1 \]

\[ 2y + 3x = 6 \]

\[ y = \frac{3}{2}x + 3 \]

\[ 5 = \frac{2}{3}(6) + b \]

\[ 5 = 4 + b \]

\[ m = \frac{3}{2} \]

\[ y = \frac{2}{3}x + 1 \]

PTS: 4  REF: 061036ge  TOP: Parallel and Perpendicular Lines

143 ANS: 2  PTS: 2  REF: 061022ge  TOP: Parallel and Perpendicular Lines
ANS:

8x - 5 = 3x + 30.  4z - 8 = 3z.  9y + 8 + 5y - 2 = 90.

5x = 35  z = 8  14y + 6 = 90
x = 7  14y = 84
       y = 6

PTS: 6  REF: 061038ge  TOP: Special Parallelograms

ANS: 1

4x = 6 \cdot 10
x = 15

PTS: 2  REF: 081017ge  TOP: Segments Intercepted by Circle

KEY: two chords

ANS: 4  PTS: 2  REF: 060922ge  TOP: Equations of Circles

ANS: 2  PTS: 2  REF: fall0806ge  TOP: Planes

The slope of \(y = x + 2\) is 1. The slope of \(y - x = -1\) is \(\frac{-A}{B} = \frac{-(-1)}{1} = 1\).

PTS: 2  REF: 080909ge  TOP: Parallel and Perpendicular Lines

ANS:

22.4.  \[ V = \pi r^2 h \]

12566.4 = \pi r^2 \cdot 8

\[ r^2 = \frac{12566.4}{8\pi} \]

\[ r \approx 22.4 \]

PTS: 2  REF: fall0833ge  TOP: Volume

ANS: 3  PTS: 2  REF: 061004ge  TOP: Isosceles Triangle Theorem
151 ANS:

\[ C_x = \frac{Q_x + R_x}{2}, \quad C_y = \frac{Q_y + R_y}{2}. \]

\[ 3.5 = \frac{1 + R_x}{2}, \quad 2 = \frac{8 + R_y}{2} \]

\[ 7 = 1 + R_x, \quad 4 = 8 + R_y \]

\[ 6 = R_x, \quad -4 = R_y \]

PTS: 2 REF: 011031ge TOP: Midpoint

152 ANS: 2

The slope of \( y = \frac{1}{2} x + 5 \) is \( \frac{1}{2} \). The slope of a perpendicular line is \( -2 \).

\[ y = mx + b \]

\[ 5 = (-2)(-2) + b \]

\[ b = 1 \]

PTS: 2 REF: 060907ge TOP: Parallel and Perpendicular Lines

153 ANS: 3

\[ V = \pi r^2 h = \pi \cdot 6^2 \cdot 27 = 972\pi \]

PTS: 2 REF: 011027ge TOP: Volume

154 ANS:

\[ 5. \quad \frac{3}{x} = \frac{6 + 3}{15} \]

\[ 9x = 45 \]

\[ x = 5 \]

PTS: 2 REF: 011033ge TOP: Side Splitter Theorem

155 ANS: 3 PTS: 2 REF: 060925ge TOP: Constructions

156 ANS:

3. The non-parallel sides of an isosceles trapezoid are congruent. \( 2x + 5 = 3x + 2 \)

\[ x = 3 \]

PTS: 2 REF: 080929ge TOP: Trapezoids

157 ANS:

\( \angle D, \angle G \) and \( 24^\circ \) or \( \angle E, \angle F \) and \( 84^\circ \). \[ \frac{2}{15} \times 360 = 48. \] Since the chords forming \( \angle D \) and \( \angle G \) are intercepted by \( FE \), their measure is \( 24^\circ \). \[ \frac{7}{15} \times 360 = 168. \] Since the chords forming \( \angle E \) and \( \angle F \) are intercepted by \( GD \), their measure is \( 84^\circ \).

PTS: 4 REF: fall0836ge TOP: Arcs Determined by Angles
\[ a^2 + (5\sqrt{2})^2 = (2\sqrt{15})^2 \]
\[ a^2 + 25 \times 2 = 4 \times 15 \]
\[ a^2 + 50 = 60 \]
\[ a^2 = 10 \]
\[ a = \sqrt{10} \]

PTS: 2          REF: 011016ge        TOP: Pythagorean Theorem

159 ANS: 1          PTS: 2          REF: 080918ge        TOP: Special Quadrilaterals

160 ANS: 4          PTS: 2          REF: 080905ge        TOP: Triangle Congruency

161 ANS: 1          PTS: 2          REF: 011024ge        TOP: Planes

162 ANS: 3
\[ 2y = -6x + 8 \] Perpendicular lines have slope the opposite and reciprocal of each other.
\[ y = -3x + 4 \]
\[ m = -3 \]
\[ m_{\perp} = \frac{1}{3} \]

PTS: 2          REF: 081024ge        TOP: Parallel and Perpendicular Lines

163 ANS: 2          PTS: 2          REF: 061020ge        TOP: Constructions

164 ANS: \[ 15 + 5\sqrt{5} \]

PTS: 4          REF: 060936ge        TOP: Triangles in the Coordinate Plane

165 ANS: 2          PTS: 2          REF: 011003ge        TOP: Properties of Transformations

166 ANS: 4
\[ (n - 2)180 = (8 - 2)180 = 1080 \]
\[ \frac{1080}{8} = 135 \]

PTS: 2          REF: fall0827ge        TOP: Interior and Exterior Angles of Polygons
The length of the midsegment of a trapezoid is the average of the lengths of its bases. \[ \frac{x + 30}{2} = 44. \]
\[ x + 30 = 88 \]
\[ x = 58 \]

PTS: 2  REF: 011001ge  TOP: Trapezoids

Adjacent sides of a rectangle are perpendicular and have opposite and reciprocal slopes.

PTS: 2  REF: 061028ge  TOP: Quadrilaterals in the Coordinate Plane

\[(d + 4)4 = 12(6)\]
\[4d + 16 = 72\]
\[d = 14\]
\[r = 7\]

PTS: 2  REF: 061023ge  TOP: Segments Intercepted by Circle

KEY: two secants

Because \( OC \) is a radius, its length is 5. Since \( CE = 2 \) \( OE = 3 \). \( \triangle EDO \) is a 3-4-5 triangle. If \( ED = 4 \), \( BD = 8 \).

PTS: 2  REF: fall0811ge  TOP: Chords

\[ \frac{36 - 20}{2} = 8. \sqrt{17^2 - 8^2} = 15 \]

PTS: 2  REF: 061016ge  TOP: Trapezoids

A dilation affects distance, not angle measure.

PTS: 2  REF: 080906ge  TOP: Identifying Transformations

ANS: 2  PTS: 2  REF: 061007ge  TOP: Parallel Lines and Transversals

Because \( OC \) is a radius, its length is 5. Since \( CE = 2 \) \( OE = 3 \). \( \triangle EDO \) is a 3-4-5 triangle. If \( ED = 4 \), \( BD = 8 \).

PTS: 2  REF: fall0811ge  TOP: Chords

ANS: 3

\[ \frac{180 - 46}{2} = 67 \]

PTS: 2  REF: 011029ge  TOP: Isosceles Triangle Theorem
176 ANS: 4  PTS: 2  REF: 060912ge  TOP: Locus
177 ANS: 3  PTS: 2  REF: 080913ge  TOP: Triangle Congruency
178 ANS: 3  PTS: 2  REF: fall0825ge  TOP: Centroid, Orthocenter, Incenter and Circumcenter
179 ANS: 4
The radius is 4.  \( r^2 = 16 \).

PTS: 2  REF: 061014ge  TOP: Equations of Circles
180 ANS: 4
\[ x^2 = (4 + 5) \times 4 \]
\[ x^2 = 36 \]
\[ x = 6 \]

PTS: 2  REF: 011008ge  TOP: Segments Intercepted by Circle
KEY: tangent and secant
181 ANS: 1

PTS: 2  REF: 081003ge  TOP: Midsegments

182 ANS: 2
\[ y + \frac{1}{2}x = 4 \]
\[ 3x + 6y = 12 \]
\[ y = -\frac{1}{2}x + 4 \]
\[ 6y = -3x + 12 \]
\[ y = -\frac{3}{6}x + 2 \]
\[ m = \frac{1}{2} \]
\[ y = -\frac{1}{2}x + 2 \]

PTS: 2  REF: 081014ge  TOP: Parallel and Perpendicular Lines
183 ANS: 4
\[ 180 - (50 + 30) = 100 \]

PTS: 2  REF: 081006ge  TOP: Similarity  KEY: basic
184 ANS: 1  PTS: 2  REF: 081012ge  TOP: Tangents
KEY: two tangents
185 ANS: 1
\[ AB = 10 \text{ since } \triangle ABC \text{ is a 6-8-10 triangle. } \]
\[ 6^2 = 10x \]
\[ 3.6 = x \]

PTS: 2 REF: 060915ge TOP: Similarity KEY: leg

186 ANS: 2

PTS: 2 REF: 081007ge TOP: Triangle Congruency

187 ANS:
2.4. \[ 5a = 4^2 \quad 5b = 3^2 \quad h^2 = ab \]
\[ a = 3.2 \quad b = 1.8 \quad h^2 = 3.2 \cdot 1.8 \]
\[ h = \sqrt{5.76} = 2.4 \]

PTS: 4 REF: 081037ge TOP: Similarity KEY: altitude

188 ANS:
Because \( AB \parallel DC \), \( \widehat{AD} \cong \widehat{BC} \) since parallel chords intersect congruent arcs. \( \angle BDC \cong \angle ACD \) because inscribed angles that intercept congruent arcs are congruent. \( AD \cong BC \) since congruent chords intersect congruent arcs. \( DC \cong CD \) because of the reflexive property. Therefore, \( \triangle ACD \cong \triangle BDC \) because of SAS.

PTS: 6 REF: fall0838ge TOP: Circle Proofs

189 ANS:

PTS: 2 REF: 060930ge TOP: Constructions
18. If the ratio of $TA$ to $AC$ is 1:3, the ratio of $TE$ to $ES$ is also 1:3. $x + 3x = 24$. $3(6)$ = 18.

$x = 6$

PTS: 4 REF: 060935ge TOP: Tangents KEY: common tangency

ANS: 4 PTS: 2 REF: 081023ge TOP: Similarity

KEY: perimeter and area

ANS: 4 PTS: 2 REF: 060904ge TOP: Solids

KEY: common tangency

ANS: 3 PTS: 2 REF: 080928ge TOP: Tangents

ANS: 3 PTS: 2 REF: 061017ge TOP: Planes

2016. $V = \frac{1}{3} Bh = \frac{1}{3} s^2 h = \frac{1}{3} 12^2 \cdot 42 = 2016$

PTS: 2 REF: 080930ge TOP: Volume

ANS: 2 PTS: 2 REF: 011004ge TOP: Constructions

ANS: 4 PTS: 2 REF: 061018ge TOP: Identifying Transformations

ANS: 4 PTS: 2 REF: fall0824ge TOP: Tangents

ANS: 4 PTS: 2 REF: 080915ge TOP: Identifying Transformations

ANS: 2 PTS: 2 REF: 060910ge TOP: Equations of Circles

ANS:

\[ FE \cong FE \text{ (Reflexive Property)}; \quad AE - FE \cong FC - EF \text{ (Line Segment Subtraction Theorem)}; \quad AF \cong CE \text{ (Substitution)}; \quad \angle BFA \cong \angle DEC \text{ (All right angles are congruent)}; \quad \triangle BFA \cong \triangle DEC \text{ (AAS)}; \quad \overline{AB} \cong \overline{CD} \text{ and } \overline{BF} \cong \overline{DE} \text{ (CPCTC)}; \quad \angle BFC \cong \angle DEA \text{ (All right angles are congruent)}; \quad \triangle BFC \cong \triangle DEA \text{ (SAS)}; \quad \overline{AD} \cong \overline{CB} \text{ (CPCTC)}; \quad ABCD \text{ is a parallelogram (opposite sides of quadrilateral } ABCD \text{ are congruent)}

PTS: 6 REF: 080938ge TOP: Special Quadrilaterals

ANS: 3 PTS: 2 REF: 081021ge TOP: Properties of Transformations

ANS:

36, because a dilation does not affect angle measure. 10, because a dilation does affect distance.

PTS: 4 REF: 011035ge TOP: Properties of Transformations

ANS: 2

\[ 6 + 17 > 22 \]

PTS: 2 REF: 080916ge TOP: Triangle Inequality Theorem

ANS: 3 PTS: 2 REF: 080902ge TOP: Constructions
\[ AB \parallel CD \text{ and } AD \parallel CB \text{ because their slopes are equal. } ABCD \text{ is a parallelogram because opposite sides are parallel. } AB \neq BC. \ ABCD \text{ is not a rhombus because all sides are not equal. } AB \perp BC \text{ because their slopes are not opposite reciprocals. } ABCD \text{ is not a rectangle because } \angle ABC \text{ is not a right angle.} \]

PTS: 4  
REF: 081038ge  
TOP: Quadrilaterals in the Coordinate Plane

\[ \text{ANS: 2} \]

The centroid divides each median into segments whose lengths are in the ratio 2 : 1.

PTS: 2  
REF: 060914ge  
TOP: Centroid

\[ \text{ANS: 1} \]

In an equilateral triangle, each interior angle is 60° and each exterior angle is 120° (180° - 120°). The sum of the three interior angles is 180° and the sum of the three exterior angles is 360°.

PTS: 2  
REF: 060909ge  
TOP: Interior and Exterior Angles of Triangles

\[ \text{ANS: 2} \]

\[ \frac{87 + 35}{2} = \frac{122}{2} = 61 \]

PTS: 2  
REF: 011015ge  
TOP: Arcs Determined by Angles  
KEY: inside circle

\[ \text{ANS: 2} \]

\[ \frac{140 - RS}{2} = 40 \]

\[ 140 - RS = 80 \]

\[ RS = 60 \]

PTS: 2  
REF: 081025ge  
TOP: Arcs Determined by Angles  
KEY: outside circle

\[ \text{ANS: 4} \]

\[ d = \sqrt{(-3 - 1)^2 + (2 - 0)^2} = \sqrt{16 + 4} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5} \]

PTS: 2  
REF: 011017ge  
TOP: Distance

\[ \text{ANS: 4} \]

(4) is not true if \( \angle PQR \) is obtuse.

PTS: 2  
REF: 060924ge  
TOP: Exterior Angle Theorem
213 ANS: 2
$7 + 18 > 6 + 12$

PTS: 2  REF: fall0819ge  TOP: Triangle Inequality Theorem

214 ANS: 3  PTS: 2  REF: 080924ge  TOP: Negations

215 ANS: 1  PTS: 2  REF: 081008ge  TOP: Planes

216 ANS: 1  PTS: 2  REF: 081009ge  TOP: Equations of Circles

217 ANS: 3  PTS: 2  REF: 060908ge  TOP: Identifying Transformations

218 ANS: 2

$\angle ACB$ and $\angle ECD$ are congruent vertical angles and $\angle CAB \cong \angle CED$.  

PTS: 2  REF: 060917ge  TOP: Similarity Proofs

219 ANS: 1  PTS: 2  REF: 061010ge  TOP: Angle Side Relationship

220 ANS:
Midpoint: $\left(\frac{-4+4}{2}, \frac{2+(-4)}{2}\right) = (0,-1)$. Distance: $d = \sqrt{(-4-4)^2 + (2-(-4))^2} = \sqrt{100} = 10$

$r = 5$

$r^2 = 25$

$x^2 + (y+1)^2 = 25$

PTS: 2  REF: 061037ge  TOP: Equations of Circles

221 ANS:
$y = \frac{4}{3}x - 6$. $M_x = \frac{-1+7}{2} = 3$  

The perpendicular bisector goes through $(3,-2)$ and has a slope of $\frac{4}{3}$.

$M_y = \frac{1+(-5)}{2} = -2$

$m = \frac{1-(-5)}{-1-7} = -\frac{3}{4}$

$y - y_M = m(x - x_M)$.

$y - 1 = \frac{4}{3}(x - 2)$

PTS: 4  REF: 080935ge  TOP: Perpendicular Bisector
222 ANS:  
\[(x + 1)^2 + (y - 2)^2 = 36\]

PTS: 2  REF: 081034ge  TOP: Equations of Circles

223 ANS: 

PTS: 6  REF: 011038ge  TOP: Quadratic-Linear Systems

224 ANS: 4  
\[d = \sqrt{(-6 - 2)^2 + (4 - (-5))^2} = \sqrt{64 + 81} = \sqrt{145}\]

PTS: 2  REF: 081013ge  TOP: Distance

225 ANS: 4  
Sum of interior \( \angle s \) = sum of exterior \( \angle s \)

\[(n - 2)180 = n \left(180 - \frac{(n - 2)180}{n}\right)\]

\[180n - 360 = 180n - 180n + 360\]

\[180n = 720\]

\[n = 4\]

PTS: 2  REF: 081016ge  TOP: Interior and Exterior Angles of Polygons

226 ANS: 4  PTS: 2  REF: 011012ge  TOP: Planes

227 ANS: 2  PTS: 2  REF: 080921ge  TOP: Equations of Circles
\begin{align*}
y + x &= 4 \quad x^2 - 6x + 10 = -x + 4 \quad y + x = 4 \quad y + 2 = 4 \\
y &= -x + 4 \quad x^2 - 5x + 6 = 0 \quad y + 3 = 4 \quad y = 2 \\
(x - 3)(x - 2) &= 0 \quad y = 1 \\
x &= 3 \text{ or } 2
\end{align*}

PTS: 2 \quad REF: 080912ge \quad TOP: Quadratic-Linear Systems