JMAP
REGENTS BY STATE
STANDARD: TOPIC

NY Algebra II Regents Exam Questions from Spring 2015 to August 2019 by State Standard: Topic

www.jmap.org
<table>
<thead>
<tr>
<th>TOPIC</th>
<th>STANDARD</th>
<th>SUBTOPIC</th>
<th>QUESTION #</th>
</tr>
</thead>
<tbody>
<tr>
<td>RATE</td>
<td>F.IF.B.6</td>
<td>Rate of Change</td>
<td>1-10</td>
</tr>
<tr>
<td>QUADRATICS</td>
<td>A.REI.B.4</td>
<td>Solving Quadratics</td>
<td>11-20</td>
</tr>
<tr>
<td></td>
<td>A.REI.B.4</td>
<td>Using the Discriminant</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>A.REI.B.4</td>
<td>Complex Conjugate Root Theorem</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>G.GPE.A.2</td>
<td>Graphing Quadratic Functions</td>
<td>23-32</td>
</tr>
<tr>
<td>POWERS</td>
<td>A.SSE.B.3</td>
<td>Modeling Exponential Functions</td>
<td>33-42</td>
</tr>
<tr>
<td></td>
<td>F.BF.A.1</td>
<td>Modeling Exponential Functions</td>
<td>43-46</td>
</tr>
<tr>
<td></td>
<td>F.LE.A.2</td>
<td>Modeling Exponential Functions</td>
<td>47-50</td>
</tr>
<tr>
<td></td>
<td>F.LE.B.5</td>
<td>Modeling Exponential Functions</td>
<td>51-54</td>
</tr>
<tr>
<td></td>
<td>F.IF.B.4</td>
<td>Evaluating Logarithmic Expressions</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>F.IF.C.7</td>
<td>Graphing Exponential Functions</td>
<td>56-60</td>
</tr>
<tr>
<td></td>
<td>F.IF.C.7</td>
<td>Graphing Logarithmic Functions</td>
<td>61-67</td>
</tr>
<tr>
<td></td>
<td>A.CED.A.1</td>
<td>Exponential Growth</td>
<td>68-72</td>
</tr>
<tr>
<td></td>
<td>F.LE.A.4</td>
<td>Exponential Equations</td>
<td>73-74</td>
</tr>
<tr>
<td></td>
<td>F.LE.A.4</td>
<td>Exponential Growth</td>
<td>75-84</td>
</tr>
<tr>
<td>POLYNOMIALS</td>
<td>A.SSE.A.2</td>
<td>Factoring Polynomials</td>
<td>85-98</td>
</tr>
<tr>
<td></td>
<td>A.APR.B.3</td>
<td>Solving Polynomial Equations</td>
<td>99-102</td>
</tr>
<tr>
<td></td>
<td>A.APR.B.3</td>
<td>Graphing Polynomial Functions</td>
<td>103-107</td>
</tr>
<tr>
<td></td>
<td>F.IF.B.4</td>
<td>Graphing Polynomial Functions</td>
<td>108-113</td>
</tr>
<tr>
<td></td>
<td>F.IF.C.7</td>
<td>Graphing Polynomial Functions</td>
<td>114-123</td>
</tr>
<tr>
<td></td>
<td>A.APR.B.2</td>
<td>Remainder Theorem</td>
<td>124-135</td>
</tr>
<tr>
<td></td>
<td>A.APR.C.4</td>
<td>Polynomial Identities</td>
<td>136-145</td>
</tr>
<tr>
<td>RADICALS</td>
<td>N.RN.A.2</td>
<td>Operations with Radicals</td>
<td>146-149</td>
</tr>
<tr>
<td></td>
<td>A.REI.A.2</td>
<td>Solving Radicals</td>
<td>150-161</td>
</tr>
<tr>
<td></td>
<td>N.RN.A.1</td>
<td>Radicals and Rational Exponents</td>
<td>162-165</td>
</tr>
<tr>
<td></td>
<td>N.RN.A.2</td>
<td>Radicals and Rational Exponents</td>
<td>166-176</td>
</tr>
<tr>
<td></td>
<td>N.CN.A.2</td>
<td>Operations with Complex Numbers</td>
<td>177-188</td>
</tr>
<tr>
<td>RATIONALS</td>
<td>A.APR.D.6</td>
<td>Undefined Rationals</td>
<td>189</td>
</tr>
<tr>
<td></td>
<td>A.APR.D.6</td>
<td>Expressions with Negative Expressions</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td>A.APR.D.6</td>
<td>Rational Expressions</td>
<td>191-204</td>
</tr>
<tr>
<td></td>
<td>A.APR.D.7</td>
<td>Addition and Subtraction of Rationals</td>
<td>205</td>
</tr>
<tr>
<td></td>
<td>A.CED.A.1</td>
<td>Modeling Rationals</td>
<td>206-210</td>
</tr>
<tr>
<td></td>
<td>A.REL.A.2</td>
<td>Solving Rationals</td>
<td>211-222</td>
</tr>
<tr>
<td>SYSTEMS</td>
<td>A.REI.C.6</td>
<td>Solving Linear Systems</td>
<td>223-229</td>
</tr>
<tr>
<td></td>
<td>A.REI.C.7</td>
<td>Quadratic-Linear Systems</td>
<td>230-237</td>
</tr>
<tr>
<td></td>
<td>A.REI.D.11</td>
<td>Quadratic-Linear Systems</td>
<td>238</td>
</tr>
<tr>
<td></td>
<td>A.REI.D.11</td>
<td>Other Systems</td>
<td>239-255</td>
</tr>
<tr>
<td>FUNCTIONS</td>
<td>F.BF.A.1</td>
<td>Operations with Functions</td>
<td>256-261</td>
</tr>
<tr>
<td></td>
<td>F.LE.A.2</td>
<td>Families of Functions</td>
<td>262-264</td>
</tr>
<tr>
<td></td>
<td>F.IF.C.9</td>
<td>Comparing Functions</td>
<td>265-270</td>
</tr>
<tr>
<td></td>
<td>F.IF.B.4</td>
<td>Relating Graphs to Events</td>
<td>271</td>
</tr>
<tr>
<td></td>
<td>F.BF.B.3</td>
<td>Transformations with Functions</td>
<td>272</td>
</tr>
<tr>
<td></td>
<td>F.BF.B.3</td>
<td>Even and Odd Functions</td>
<td>273-277</td>
</tr>
<tr>
<td></td>
<td>F.BF.B.4</td>
<td>Inverse of Functions</td>
<td>278-287</td>
</tr>
<tr>
<td>SEQUENCES AND SERIES</td>
<td>F.LE.A.2</td>
<td>Sequences .......................................................... 288-294</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>---------</td>
<td>--------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.IF.A.3</td>
<td>Sequences .......................................................... 295-297</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.BF.A.2</td>
<td>Sequences .......................................................... 298-308</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.BF.B.6</td>
<td>Sigma Notation ..................................................... 309</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A.SSE.B.4</td>
<td>Series ........................................................... 310-320</td>
<td></td>
</tr>
<tr>
<td>TRIGONOMETRY</td>
<td>F.TF.A.1</td>
<td>Unit Circle ......................................................... 321</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.TF.A.2</td>
<td>Unit Circle ......................................................... 322-323</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.TF.A.2</td>
<td>Reciprocal Trigonometric Relationships ..................... 324</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.TF.A.2</td>
<td>Reference Angles .................................................. 325</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.TF.A.2</td>
<td>Determining Trigonometric Functions ......................... 326-330</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.TF.C.8</td>
<td>Determining Trigonometric Functions ......................... 331-335</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.TF.C.8</td>
<td>Simplifying Trigonometric Identities ......................... 336</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.TF.B.5</td>
<td>Modeling Trigonometric Functions ............................. 337-340</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.IF.B.4</td>
<td>Graphing Trigonometric Functions ............................. 341-349</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F.IF.C.7</td>
<td>Graphing Trigonometric Functions ............................. 350-365</td>
<td></td>
</tr>
<tr>
<td>CONICS</td>
<td>G.GPE.A.1</td>
<td>Equations of Conics ............................................. 366</td>
<td></td>
</tr>
<tr>
<td>GRAPHS AND STATISTICS</td>
<td>S.IC.A.2</td>
<td>Analysis of Data ................................................... 367-374</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.IC.B.3</td>
<td>Analysis of Data ................................................... 375-385</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.IC.B.4</td>
<td>Analysis of Data ................................................... 386-389</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.IC.B.5</td>
<td>Analysis of Data ................................................... 390-398</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.IC.B.6</td>
<td>Analysis of Data ................................................... 399-400</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.ID.B.6</td>
<td>Regression .......................................................... 401-404</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.ID.A.4</td>
<td>Normal Distributions .............................................. 405-417</td>
<td></td>
</tr>
<tr>
<td>PROBABILITY</td>
<td>S.CP.B.7</td>
<td>Theoretical Probability ......................................... 418-419</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.CP.A.2</td>
<td>Probability of Compound Events ............................... 420-422</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.CP.A.1</td>
<td>Venn Diagrams ...................................................... 423</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.CP.A.3</td>
<td>Conditional Probability .......................................... 424-429</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.CP.A.4</td>
<td>Conditional Probability .......................................... 430-435</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.CP.B.6</td>
<td>Conditional Probability .......................................... 436-438</td>
<td></td>
</tr>
</tbody>
</table>
1. Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of $B$ dollars after $m$ months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after $m$ months.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.00</td>
</tr>
<tr>
<td>10</td>
<td>1172.00</td>
</tr>
<tr>
<td>19</td>
<td>1352.00</td>
</tr>
<tr>
<td>36</td>
<td>1770.80</td>
</tr>
<tr>
<td>60</td>
<td>2591.90</td>
</tr>
<tr>
<td>69</td>
<td>2990.00</td>
</tr>
<tr>
<td>72</td>
<td>3135.80</td>
</tr>
<tr>
<td>73</td>
<td>3186.00</td>
</tr>
</tbody>
</table>

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

1. month 10 to month 60
2. month 19 to month 69
3. month 36 to month 72
4. month 60 to month 73

2. The function $N(t) = 100e^{-0.023t}$ models the number of grams in a sample of cesium-137 that remain after $t$ years. On which interval is the sample's average rate of decay the fastest?

1. $[1, 10]$
2. $[10, 20]$
3. $[15, 25]$
4. $[1, 30]$

3. The equation $t = \frac{1}{0.0105} \ln \left( \frac{A}{5000} \right)$ relates time, $t$, in years, to the amount of money, $A$, earned by a $5000 investment. Which statement accurately describes the relationship between the average rates of change of $t$ on the intervals $[6000, 8000]$ and $[9000, 12,000]$?

1. A comparison cannot be made because the intervals are different sizes.
2. The average rate of change is equal for both intervals.
3. The average rate of change is larger for the interval $[6000, 8000]$.
4. The average rate of change is larger for the interval $[9000, 12,000]$. 
4  The value of a new car depreciates over time. Greg purchased a new car in June 2011. The value, $V$, of his car after $t$ years can be modeled by the equation 
$$\log_{0.8} \left( \frac{V}{17000} \right) = t.$$ What is the average decreasing rate of change per year of the value of the car from June 2012 to June 2014, to the nearest ten dollars per year?

1  1960  
2  2180  
3  2450  
4  2770

5  The function $f(x) = 2^{-0.25x} \cdot \sin \left( \frac{\pi}{2} x \right)$ represents a damped sound wave function. What is the average rate of change for this function on the interval $[-7, 7]$, to the nearest hundredth?

1  $-3.66$  
2  $-0.30$  
3  $-0.26$  
4  3.36

6  A cardboard box manufacturing company is building boxes with length represented by $x + 1$, width by $5 - x$, and height by $x - 1$. The volume of the box is modeled by the function below.

Over which interval is the volume of the box changing at the fastest average rate?

1  $[1, 2]$  
2  $[1, 3.5]$  
3  $[1, 5]$  
4  $[0, 3.5]$

7  The distance needed to stop a car after applying the brakes varies directly with the square of the car’s speed. The table below shows stopping distances for various speeds.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>6.25</td>
<td>25</td>
<td>56.25</td>
<td>100</td>
<td>156.25</td>
<td>225</td>
<td>306.25</td>
</tr>
</tbody>
</table>

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.
8 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

<table>
<thead>
<tr>
<th>Month</th>
<th>Hours of Daylight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>9.4</td>
</tr>
<tr>
<td>Feb.</td>
<td>10.6</td>
</tr>
<tr>
<td>March</td>
<td>11.9</td>
</tr>
<tr>
<td>April</td>
<td>13.9</td>
</tr>
<tr>
<td>May</td>
<td>14.7</td>
</tr>
<tr>
<td>June</td>
<td>15.4</td>
</tr>
<tr>
<td>July</td>
<td>15.1</td>
</tr>
<tr>
<td>Aug.</td>
<td>13.9</td>
</tr>
<tr>
<td>Sept.</td>
<td>12.5</td>
</tr>
<tr>
<td>Oct.</td>
<td>11.1</td>
</tr>
<tr>
<td>Nov.</td>
<td>9.7</td>
</tr>
<tr>
<td>Dec.</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st? Interpret what this means in the context of the problem.

9 The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function \( p(t) = 2560e^{0.017185t} \), where \( t \) is time in years after 1950 and \( p(t) \) is the population in millions. Determine the average rate of change of \( p(t) \) in millions of people per year, from 4 \( \leq t \leq 8 \). Round your answer to the nearest hundredth.

10 The average monthly high temperature in Buffalo, in degrees Fahrenheit, can be modeled by the function \( B(t) = 25.29\sin(0.4895t - 1.9752) + 55.2877 \), where \( t \) is the month number (January = 1). State, to the nearest tenth, the average monthly rate of temperature change between August and November. Explain its meaning in the given context.

**QUADRATICS**

A.REI.B.4: SOLVING QUADRATICS

11 The solutions to the equation \( -\frac{1}{2}x^2 = -6x + 20 \) are

- 1 \( -6 \pm 2i \)
- 2 \( -6 \pm 2\sqrt{19} \)
- 3 \( 6 \pm 2i \)
- 4 \( 6 \pm 2\sqrt{19} \)

12 A solution of the equation \( 2x^2 + 3x + 2 = 0 \) is

- 1 \( -\frac{3}{4} + \frac{1}{4}i\sqrt{7} \)
- 2 \( -\frac{3}{4} + \frac{1}{4}i \)
- 3 \( -\frac{3}{4} + \frac{1}{4}\sqrt{7} \)
- 4 \( \frac{1}{2} \)
13. The solution to the equation $18x^2 - 24x + 87 = 0$ is
   1. $\frac{-2}{3} \pm 6i\sqrt{158}$
   2. $\frac{-2}{3} \pm \frac{1}{6}i\sqrt{158}$
   3. $\frac{2}{3} \pm 6i\sqrt{158}$
   4. $\frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$

14. The solution to the equation $4x^2 + 98 = 0$ is
   1. $\pm 7$
   2. $\pm 7i$
   3. $\pm \frac{7\sqrt{2}}{2}$
   4. $\pm \frac{7i\sqrt{2}}{2}$

15. The roots of the equation $x^2 + 2x + 5 = 0$ are
   1. $-3$ and $1$
   2. $-1$, only
   3. $-1 + 2i$ and $-1 - 2i$
   4. $-1 + 4i$ and $-1 - 4i$

16. The roots of the equation $3x^2 + 2x = -7$ are
   1. $-2, \frac{1}{3}$
   2. $\frac{7}{3}, 1$
   3. $\frac{1}{3} \pm \frac{2i\sqrt{5}}{3}$
   4. $\frac{1}{3} \pm \frac{\sqrt{11}}{3}$

17. The solutions to the equation $5x^2 - 2x + 13 = 9$ are
   1. $\frac{1}{5} \pm \frac{\sqrt{21}}{5}$
   2. $\frac{1}{5} \pm \frac{\sqrt{19}}{5}i$
   3. $\frac{1}{5} \pm \frac{\sqrt{66}}{5}i$
   4. $\frac{1}{5} \pm \frac{\sqrt{66}}{5}$

18. What is the solution when the equation $wx^2 + w = 0$ is solved for $x$, where $w$ is a positive integer?
   1. $-1$
   2. $0$
   3. $6$
   4. $\pm i$

19. Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

20. a) Algebraically determine the roots, in simplest $a + bi$ form, to the equation below.
    $$x^2 - 2x + 7 = 4x - 10$$

b) Consider the system of equations below.
    $$y = x^2 - 2x + 7$$
    $$y = 4x - 10$$
    The graph of this system confirms the solution from part a is imaginary. Explain why.
A.REI.B.4: USING THE DISCRIMINANT

21 Which representation of a quadratic has imaginary roots?

1 \[ (x + 3)^2 = 64 \]

2 \[ 2(x + 3)^2 = 64 \]

3 \[ 2x^2 + 32 = 0 \]

A.REI.B.4: COMPLEX CONJUGATE ROOT THEOREM

22 Which equation has \(1 - i\) as a solution?

1 \[ x^2 + 2x - 2 = 0 \]

2 \[ x^2 + 2x + 2 = 0 \]

3 \[ x^2 - 2x - 2 = 0 \]

4 \[ x^2 - 2x + 2 = 0 \]

G.GPE.A.2: GRAPHING QUADRATIC FUNCTIONS

23 Which equation represents the set of points equidistant from line \(l\) and point \(R\) shown on the graph below?

1 \[ y = \frac{1}{8}(x + 2)^2 + 1 \]

2 \[ y = \frac{1}{8}(x + 2)^2 - 1 \]

3 \[ y = \frac{1}{8}(x - 2)^2 + 1 \]

4 \[ y = \frac{1}{8}(x - 2)^2 - 1 \]

24 Which equation represents a parabola with a focus of \((0, 4)\) and a directrix of \(y = 2\)?

1 \[ y = x^2 + 3 \]

2 \[ y = -x^2 + 1 \]

3 \[ y = \frac{x^2}{2} + 3 \]

4 \[ y = \frac{x^2}{4} + 3 \]
25 A parabola has its focus at (1,2) and its directrix is \( y = -2 \). The equation of this parabola could be
1. \( y = 8(x + 1)^2 \)
2. \( y = \frac{1}{8} (x + 1)^2 \)
3. \( y = 8(x - 1)^2 \)
4. \( y = \frac{1}{8} (x - 1)^2 \)

26 Which equation represents a parabola with the focus at (0, -1) and the directrix of \( y = 1 \)?
1. \( x^2 = -8y \)
2. \( x^2 = -4y \)
3. \( x^2 = 8y \)
4. \( x^2 = 4y \)

27 What is the equation of the directrix for the parabola \(-8(y - 3) = (x + 4)^2\)?
1. \( y = 5 \)
2. \( y = 1 \)
3. \( y = -2 \)
4. \( y = -6 \)

28 Which equation represents a parabola with a focus of \((-2, 5)\) and a directrix of \( y = 9 \)?
1. \((y - 7)^2 = 8(x + 2)\)
2. \((y - 7)^2 = -8(x + 2)\)
3. \((x + 2)^2 = 8(y - 7)\)
4. \((x + 2)^2 = -8(y - 7)\)

29 The parabola described by the equation \( y = \frac{1}{12} (x - 2)^2 + 2 \) has the directrix at \( y = -1 \). The focus of the parabola is
1. \((2, -1)\)
2. \((2, 2)\)
3. \((2, 3)\)
4. \((2, 5)\)

30 Which equation represents the equation of the parabola with focus \((-3, 3)\) and directrix \( y = 7 \)?
1. \( y = \frac{1}{8} (x + 3)^2 - 5 \)
2. \( y = \frac{1}{8} (x - 3)^2 + 5 \)
3. \( y = -\frac{1}{8} (x + 3)^2 + 5 \)
4. \( y = -\frac{1}{8} (x - 3)^2 + 5 \)

31 Determine an equation for the parabola with focus \((4, -1)\) and directrix \( y = -5 \). (Use of the grid below is optional.)
32 The directrix of the parabola \(12(y + 3) = (x - 4)^2\) has the equation \(y = -6\). Find the coordinates of the focus of the parabola.

33 A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, \(B(t)\), can be represented by the function \(B(t) = 750(1.16)^t\), where the \(t\) represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

1. \(B(t) = 750(1.012)^t\)
2. \(B(t) = 750(1.012)^{12t}\)
3. \(B(t) = 750(1.16)^{\frac{t}{12}}\)
4. \(B(t) = 750(1.16)^{\frac{t}{12}}\)

34 A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model \(P = 714(0.75)^d\), where \(P\) is the population, in thousands, \(d\) decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after \(y\) years. Suzanne's model is best represented by

1. \(P = 714(0.6500)^y\)
2. \(P = 714(0.8500)^y\)
3. \(P = 714(0.9716)^y\)
4. \(P = 714(0.9750)^y\)

35 Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, \(A\), of Iridium-192 present after \(t\) days would be \(A = 100\left(\frac{1}{2}\right)^{\frac{t}{73.83}}\). Which equation approximates the amount of Iridium-192 present after \(t\) days?

1. \(A = 100\left(\frac{73.83}{2}\right)^t\)
2. \(A = 100\left(\frac{1}{147.66}\right)^t\)
3. \(A = 100(0.990656)^t\)
4. \(A = 100(0.116381)^t\)

36 On average, college seniors graduating in 2012 could compute their growing student loan debt using the function \(D(t) = 29,400(1.068)^t\), where \(t\) is time in years. Which expression is equivalent to \(29,400(1.068)^t\) and could be used by students to identify an approximate daily interest rate on their loans?

1. \(29,400\left(\frac{1}{365}\right)^{t}\)
2. \(29,400\left(\frac{1.068}{365}\right)^{365t}\)
3. \(29,400\left(1 + \frac{0.068}{365}\right)^{365t}\)
4. \(29,400\left(1.068^{\frac{1}{365}}\right)^{365t}\)
37 The half-life of iodine-131 is 8 days. The percent of the isotope left in the body \(d\) days after being introduced is 
\[ I = 100 \left( \frac{1}{2} \right)^{\frac{d}{8}}. \] 
When this equation is written in terms of the number \(e\), the base of the natural logarithm, it is equivalent to 
\[ I = 100e^{\frac{k}{d}}. \] 
What is the approximate value of the constant, \(k\)?

- 1 \(-0.087\)
- 2 \(0.087\)
- 3 \(-11.542\)
- 4 \(11.542\)

38 Stephanie found that the number of white-winged cross bills in an area can be represented by the formula 
\[ C = 550(1.08)^t, \] where \(t\) represents the number of years since 2010. Which equation correctly represents the number of white-winged cross bills in terms of the monthly rate of population growth?

- 1 \(C = 550(1.00643)^{12t}\)
- 2 \(C = 550(1.00643)^{t}\)
- 3 \(C = 550(1.00643)^{t} + 12\)
- 4 \(C = 550(1.00643)^{t+12}\)

39 Julia deposits $2000 into a savings account that earns 4% interest per year. The exponential function that models this savings account is 
\[ y = 2000(1.04)^t, \] where \(t\) is the time in years. Which equation correctly represents the amount of money in her savings account in terms of the monthly growth rate?

- 1 \(y = 166.67(1.04)^{0.12t}\)
- 2 \(y = 2000(1.01)^t\)
- 3 \(y = 2000(1.0032737)^{12t}\)
- 4 \(y = 166.67(1.0032737)^t\)

40 Kelly-Ann has $20,000 to invest. She puts half of the money into an account that grows at an annual rate of 0.9% compounded monthly. At the same time, she puts the other half of the money into an account that grows continuously at an annual rate of 0.8%. Which function represents the value of Kelly-Ann's investments after \(t\) years?

- 1 \(f(t) = 10,000(1.9)^t + 10,000e^{0.8t}\)
- 2 \(f(t) = 10,000(1.009)^t + 10,000e^{0.008t}\)
- 3 \(f(t) = 10,000(1.075)^{12t} + 10,000e^{0.8t}\)
- 4 \(f(t) = 10,000(1.00075)^{12t} + 10,000e^{0.008t}\)

41 A study of black bears in the Adirondacks reveals that their population can be represented by the function 
\[ P(t) = 3500(1.025)^t, \] where \(t\) is the number of years since the study began. Which function is correctly rewritten to reveal the monthly growth rate of the black bear population?

- 1 \(P(t) = 3500(1.00206)^{12t}\)
- 2 \(P(t) = 3500(1.00206)^{t}\)
- 3 \(P(t) = 3500(1.34489)^{12t}\)
- 4 \(P(t) = 3500(1.34489)^{t}\)

42 For a given time, \(x\), in seconds, an electric current, \(y\), can be represented by 
\[ y = 2.5 \left( 1 - 2.7^{-10x} \right). \] 
Which equation is not equivalent?

- 1 \(y = 2.5 - 2.5 \left( 2.7^{-10x} \right)\)
- 2 \(y = 2.5 - 2.5 \left( 2.7^{2^{-0.5x}} \right)\)
- 3 \(y = 2.5 - 2.5 \left( \frac{1}{2.7^{10x}} \right)\)
- 4 \(y = 2.5 - 2.5 \left( 2.7^{-2} \right) \left( 2.7^{0.5x} \right)\)
43. Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]

1. \((1.0525)^m\)
2. \((1.0525)^{\frac{m}{12}}\)
3. \((1.00427)^m\)
4. \((1.00427)^{\frac{m}{12}}\)

44. A payday loan company makes loans between $100 and $1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a $300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

1. \(300(0.30)^{\frac{14}{365}}\)
2. \(300(1.30)^{\frac{14}{365}}\)
3. \(300(0.30)^{\frac{365}{14}}\)
4. \(300(1.30)^{\frac{365}{14}}\)

45. According to a pricing website, Indroid phones lose 58% of their cash value over 1.5 years. Which expression can be used to estimate the value of a $300 Indroid phone in 1.5 years?

1. \(300e^{-0.87}\)
2. \(300e^{-0.63}\)
3. \(300e^{-0.58}\)
4. \(300e^{-0.42}\)

46. Camryn puts $400 into a savings account that earns 6% annually. The amount in her account can be modeled by \(C(t) = 400(1.06)^t\) where \(t\) is the time in years. Which expression best approximates the amount of money in her account using a weekly growth rate?

1. \(400(1.001153846)^{52t}\)
2. \(400(1.001121184)^{52t}\)
3. \(400(1.001153846)^t\)
4. \(400(1.001121184)^t\)

47. A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If \(t\) represents the time, in weeks, and \(P(t)\) is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?

1. 56
2. 152
3. 3688
4. 81,920
48 Sodium iodide-131, used to treat certain medical conditions, has a half-life of 1.8 hours. The data table below shows the amount of sodium iodide-131, rounded to the nearest thousandth, as the dose fades over time.

<table>
<thead>
<tr>
<th>Number of Half Lives</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of Sodium Iodide-131</td>
<td>139.000</td>
<td>69.500</td>
<td>34.750</td>
<td>17.375</td>
<td>8.688</td>
</tr>
</tbody>
</table>

What approximate amount of sodium iodide-131 will remain in the body after 18 hours?

- 1 0.001
- 2 0.136

49 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the nearest tenth. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.

50 A population of 950 bacteria grows continuously at a rate of 4.75% per day. Write an exponential function, \( N(t) \), that represents the bacterial population after \( t \) days and explain the reason for your choice of base. Determine the bacterial population after 36 hours, to the nearest bacterium.

F.LE.B.5: MODELING EXPONENTIAL FUNCTIONS

51 The function \( p(t) = 110e^{0.03922t} \) models the population of a city, in millions, \( t \) years after 2010. As of today, consider the following two statements:
   I. The current population is 110 million.
   II. The population increases continuously by approximately 3.9% per year.
This model supports

- 1 I, only
- 2 II, only
- 3 both I and II
- 4 neither I nor II

52 An equation to represent the value of a car after \( t \) months of ownership is \( v = 32,000(0.81)^{\frac{t}{12}} \). Which statement is not correct?

- 1 The car lost approximately 19% of its value each month.
- 2 The car maintained approximately 98% of its value each month.
- 3 The value of the car when it was purchased was $32,000.
- 4 The value of the car 1 year after it was purchased was $25,920.
53. A certain pain reliever is taken in 220 mg dosages and has a half-life of 12 hours. The function

\[ A = 220 \left( \frac{1}{2} \right)^{\frac{t}{12}} \]

can be used to model this situation, where \( A \) is the amount of pain reliever in milligrams remaining in the body after \( t \) hours. According to this function, which statement is true?

1. Every hour, the amount of pain reliever remaining is cut in half.
2. In 12 hours, there is no pain reliever remaining in the body.
3. In 24 hours, there is no pain reliever remaining in the body.
4. In 12 hours, 110 mg of pain reliever is remaining.

54. A savings account, \( S \), has an initial value of $50. The account grows at a 2% interest rate compounded \( n \) times per year, \( t \), according to the function below.

\[ S(t) = 50 \left( 1 + \frac{0.02}{n} \right)^{nt} \]

Which statement about the account is correct?

1. As the value of \( n \) increases, the amount of interest per year decreases.
2. As the value of \( n \) increases, the value of the account approaches the function \( S(t) = 50e^{0.02t} \).
3. As the value of \( n \) decreases to one, the amount of interest per year increases.
4. As the value of \( n \) decreases to one, the value of the account approaches the function \( S(t) = 50(1 - 0.02)^t \).

F.IF.B.4: EVALUATING LOGARITHMIC EXPRESSIONS

55. The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity \( I_0 \) to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, \( I \), and the decibel rating, \( d \), of this sound is found using \( d = 10 \log \frac{I}{I_0} \). The threshold sound audible to the average person is \( 1.0 \times 10^{-12} \text{ W/m}^2 \) (watts per square meter). Consider the following sound level classifications:

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderate</td>
<td>45-69 dB</td>
</tr>
<tr>
<td>Loud</td>
<td>70-89 dB</td>
</tr>
<tr>
<td>Very loud</td>
<td>90-109 dB</td>
</tr>
<tr>
<td>Deafening</td>
<td>&gt;110 dB</td>
</tr>
</tbody>
</table>

How would a sound with intensity \( 6.3 \times 10^{-3} \text{ W/m}^2 \) be classified?

1. moderate
2. loud
3. very loud
4. deafening
F.IF.C.7: GRAPHING EXPONENTIAL FUNCTIONS

56 Which function represents exponential decay?
1. \( y = 2^{0.3t} \)
2. \( y = 1.2^{3t} \)
3. \( y = \left(\frac{1}{2}\right)^{-t} \)
4. \( y = 5^{-t} \)

57 If the function \( g(x) = ab^x \) represents exponential growth, which statement about \( g(x) \) is false?
1. \( a > 0 \) and \( b > 1 \)
2. The \( y \)-intercept is \((0, a)\).
3. The asymptote is \( y = 0 \).
4. The \( x \)-intercept is \((b, 0)\).

58 Which statement is true about the graph of \( f(x) = \left(\frac{1}{8}\right)^x \)?
1. The graph is always increasing.
2. The graph is always decreasing.
3. The graph passes through \((1, 0)\).
4. The graph has an asymptote, \( x = 0 \).

59 The function \( M(t) \) represents the mass of radium over time, \( t \), in years.

\[
M(t) = 100e^{\frac{\ln\frac{1}{2}}{1590}t}
\]

Determine if the function \( M(t) \) represents growth or decay. Explain your reasoning.

60 Graph \( y = 400(0.85)^t - 6 \) on the set of axes below.

F.IF.C.7: GRAPHING LOGARITHMIC FUNCTIONS

61 The graph of \( y = \log_3 x \) is translated to the right 1 unit and down 1 unit. The coordinates of the \( x \)-intercept of the translated graph are
1. \((0, 0)\)
2. \((1, 0)\)
3. \((2, 0)\)
4. \((3, 0)\)

62 If \( f(x) = \log_3 x \) and \( g(x) \) is the image of \( f(x) \) after a translation five units to the left, which equation represents \( g(x) \)?
1. \( g(x) = \log_3(x + 5) \)
2. \( g(x) = \log_3(x + 5) \)
3. \( g(x) = \log_3(x - 5) \)
4. \( g(x) = \log_3(x - 5) \)
63 Which statement about the graph of \( c(x) = \log_6 x \) is false?
1. The asymptote has equation \( y = 0 \).
2. The graph has no \( y \)-intercept.
3. The domain is the set of positive reals.
4. The range is the set of all real numbers.

64 Which sketch best represents the graph of \( x = 3^y \)?

65 Graph \( y = \log_2 (x + 3) - 5 \) on the set of axes below. Use an appropriate scale to include both intercepts.

Describe the behavior of the given function as \( x \) approaches \(-3\) and as \( x \) approaches positive infinity.
66 On the grid below, graph the function $y = \log_2(x - 3) + 1$

67 Graph $f(x) = \log_2(x + 6)$ on the set of axes below.

A.CED.A.1: EXPONENTIAL GROWTH

68 If $5000 is put into a savings account that pays 3.5% interest compounded monthly, how much money, to the nearest ten cents, would be in that account after 6 years, assuming no money was added or withdrawn?

1 $5177.80
2 $5941.30
3 $6146.30
4 $6166.50
69 Monthly mortgage payments can be found using the formula below:

\[
M = \frac{P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n}{\left( 1 + \frac{r}{12} \right)^n - 1}
\]

\[M = \text{monthly payment}\]
\[P = \text{amount borrowed}\]
\[r = \text{annual interest rate}\]
\[n = \text{number of monthly payments}\]

The Banks family would like to borrow $120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the fewest number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than $720.

70 Seth’s parents gave him $5000 to invest for his 16th birthday. He is considering two investment options. Option A will pay him 4.5% interest compounded annually. Option B will pay him 4.6% compounded quarterly. Write a function of option A and option B that calculates the value of each account after \(n\) years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option B will earn than option A to the nearest cent. Algebraically determine, to the nearest tenth of a year, how long it would take for option B to double Seth’s initial investment.

71 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \(C(t)\), that represents the amount of money in the account \(t\) years after the account is opened, given that no more money is deposited into or withdrawn from the account. Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

72 Tony is evaluating his retirement savings. He currently has $318,000 in his account, which earns an interest rate of 7% compounded annually. He wants to determine how much he will have in the account in the future, even if he makes no additional contributions to the account. Write a function, \(A(t)\), to represent the amount of money that will be in his account in \(t\) years. Graph \(A(t)\) where \(0 \leq t \leq 20\) on the set of axes below.

Tony's goal is to save $1,000,000. Determine algebraically, to the nearest year, how many years it will take for him to achieve his goal. Explain how your graph of \(A(t)\) confirms your answer.
F.LE.A.4: EXPONENTIAL EQUATIONS

73 What is the solution to $8(2^{x+3}) = 48$?

1. $x = \frac{\ln 6}{\ln 2} - 3$
2. $x = 0$
3. $x = \frac{\ln 48}{\ln 16} - 3$
4. $x = \ln 4 - 3$

74 The solution of $87e^{0.3x} = 5918$, to the nearest thousandth, is

1. 0.583
2. 1.945
3. 4.220
4. 14.066

F.LE.A.4: EXPONENTIAL GROWTH

75 If $ae^{bt} = c$, where $a$, $b$, and $c$ are positive, then $t$ equals

1. $\ln \left( \frac{c}{ab} \right)$
2. $\ln \left( \frac{cb}{a} \right)$
3. $\ln \left( \frac{c}{a} \right) - \frac{b}{\ln b}$
4. $\frac{\ln \left( \frac{c}{a} \right)}{\ln b}$

76 Judith puts $5000 into an investment account with interest compounded continuously. Which approximate annual rate is needed for the account to grow to $9110 after 30 years?

1. 2%
2. 2.2%
3. 0.02%
4. 0.022%

77 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the nearest percent.

78 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was $1.25 an hour and in 2015, it was $8.75. Algebraically determine the rate of growth to the nearest percent.

79 Determine, to the nearest tenth of a year, how long it would take an investment to double at a $3 \frac{3}{4}$ % interest rate, compounded continuously.

80 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the nearest day, the amount of time needed before the amount of I–131 in the patient’s body is approximately 7 milligrams.
81 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form \( A = A_0 \left( \frac{1}{2} \right)^{\frac{t}{h}} \) that models this situation, where \( h \) is the constant representing the number of hours in the half-life, \( A_0 \) is the initial mass, and \( A \) is the mass \( t \) hours after 3 p.m. Using this equation, solve for \( h \), to the nearest ten thousandth. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the nearest tenth of an hour.

82 The half-life of a radioactive substance is 15 years. Write an equation that can be used to determine the amount, \( s(t) \), of 200 grams of this substance that remains after \( t \) years. Determine algebraically, to the nearest year, how long it will take for \( \frac{1}{10} \) of this substance to remain.

83 The Fahrenheit temperature, \( F(t) \), of a heated object at time \( t \), in minutes, can be modeled by the function below. \( F_s \) is the surrounding temperature, \( F_0 \) is the initial temperature of the object, and \( k \) is a constant.

\[
F(t) = F_s + (F_0 - F_s) e^{-kt}
\]

Coffee at a temperature of 195°F is poured into a container. The room temperature is kept at a constant 68°F and \( k = 0.05 \). Coffee is safe to drink when its temperature is, at most, 120°F. To the nearest minute, how long will it take until the coffee is safe to drink?

1 7
2 10
3 11
4 18

84 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton’s Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

\[
T = T_o + (T_0 - T_o) e^{-kt}
\]

\( T_o \) = the temperature surrounding the object
\( T_0 \) = the initial temperature of the object
\( t \) = the time in hours
\( T \) = the temperature of the object after \( t \) hours
\( k \) = decay constant

The turkey reaches the temperature of approximately 100°F after 2 hours. Find the value of \( k \), to the nearest thousandth, and write an equation to determine the temperature of the turkey after \( t \) hours. Determine the Fahrenheit temperature of the turkey, to the nearest degree, at 3 p.m.

85 What is the completely factored form of \( k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48 \)?

1 \( (k - 2)(k + 3)(k + 4) \)
2 \( (k - 2)(k - 2)(k + 6)(k + 2) \)
3 \( (k + 2)(k - 2)(k + 3)(k + 4) \)
4 \( (k + 2)(k - 2)(k + 6)(k + 2) \)
86 The completely factored form of
\[2d^4 + 6d^3 - 18d^2 - 54d\] is
1 \[2d(d^2 - 9)(d + 3)\]
2 \[2d(d^2 + 9)(d + 3)\]
3 \[2d(d + 3)^2(d - 3)\]
4 \[2d(d - 3)^2(d + 3)\]

87 Factored completely, \(m^5 + m^3 - 6m\) is equivalent to
1 \((m + 3)(m - 2)\)
2 \((m^2 + 3m)(m^2 - 2)\)
3 \(m(m^4 + m^2 - 6)\)
4 \(m(m^2 + 3)(m^2 - 2)\)

88 If \(x - 1\) is a factor of \(x^3 - kx^2 + 2x\), what is the value of \(k\)?
1 \(0\)
2 \(2\)
3 \(3\)
4 \(-3\)

89 The completely factored form of
\[n^4 - 9n^2 + 4n^2 - 36n - 12n^2 + 108\] is
1 \((n^2 - 9)(n + 6)(n - 2)\)
2 \((n + 3)(n - 3)(n + 6)(n - 2)\)
3 \((n - 3)(n - 3)(n + 6)(n - 2)\)
4 \((n + 3)(n - 3)(n - 6)(n + 2)\)

90 When the expression \((x + 2)^2 + 4(x + 2) + 3\) is rewritten as the product of two binomials, the result is
1 \((x + 3)(x + 1)\)
2 \((x + 5)(x + 3)\)
3 \((x + 2)(x + 2)\)
4 \((x + 6)(x + 1)\)

91 If \((a^3 + 27) = (a + 3)(a^2 + ma + 9)\), then \(m\) equals
1 \(-9\)
2 \(-3\)
3 \(3\)
4 \(6\)

92 Which factorization is incorrect?
1 \(4k^2 - 49 = (2k + 7)(2k - 7)\)
2 \(a^3 - 8b^3 = (a - 2b)(a^2 + 2ab + 4b^2)\)
3 \(m^3 + 3m^2 - 4m + 12 = (m - 2)^2(m + 3)\)
4 \(t^3 + 5t^2 + 6t + r^2 + 5t + 6 = (t + 1)(t + 2)(t + 3)\)

93 Which expression has been rewritten correctly to form a true statement?
1 \((x + 2)^2 + 2(x + 2) - 8 = (x + 6)x\)
2 \(x^4 + 4x^2 + 9x^2y^2 - 36y^2 = (x + 3y)^2(x - 2)^2\)
3 \(x^3 + 3x^2 - 4xy^2 - 12y^2 = (x - 2y)(x + 3)^2\)
4 \((x^2 - 4)^2 - 5(x^2 - 4) - 6 = (x^2 - 7)(x^2 - 6)\)

94 Which expression is equivalent to \(x^6y^4(x^4 - 16) - 9(x^4 - 16)\)?
1 \(x^{10}y^4 - 16x^6y^4 - 9x^4 - 144\)
2 \((x^6y^4 - 9)(x + 2)^3(x - 2)\)
3 \((x^3y^2 + 3)(x^3y^2 - 3)(x + 2)^2(x - 2)^2\)
4 \((x^3y^2 + 3)(x^3y^2 - 3)(x^2 + 4)(x^2 - 4)\)

95 Rewrite the expression
\[(4x^2 + 5x)^2 - 5(4x^2 + 5x) - 6\] as a product of four linear factors.
96 Over the set of integers, factor the expression 
\[4x^3 - x^2 + 16x - 4\] completely.

97 Completely factor the following expression:
\[x^2 + 3xy + 3x^3 + y\]

98 Over the set of integers, factor the expression 
\[x^4 - 4x^2 - 12\] completely.

A.APR.B.3: SOLVING POLYNOMIAL EQUATIONS

99 The zeros for 
\[f(x) = x^4 - 4x^3 - 9x^2 + 36x\] are
1. \(\{0, \pm 3, 4\}\)
2. \(\{0, 3, 4\}\)
3. \(\{0, \pm 3, -4\}\)
4. \(\{0, 3, -4\}\)

100 What are the zeros of 
P\((m) = (m^2 - 4)(m^2 + 1)\)?
1. \(2\) and \(-2\), only
2. \(2, -2,\) and \(-4\)
3. \(\pm 4, i,\) and \(-i\)
4. \(2, -2, i,\) and \(-i\)

101 Given 
\[c(m) = m^3 - 2m^2 + 4m - 8\] , the solution of 
\[c(m) = 0\] is
1. \(\pm 2\)
2. \(2,\) only
3. \(2i, 2\)
4. \(\pm 2i, 2\)

102 When factoring to reveal the roots of the equation 
\[x^3 + 2x^2 - 9x - 18 = 0\] , which equations can be used?
1. \(x^2 (x + 2) - 9(x + 2) = 0\)
2. \(x(x^2 - 9) + 2(x^2 - 9) = 0\)
3. \((x - 2)(x^2 - 9) = 0\)
4. \(1\) and \(2\), only
5. \(1\) and \(3\), only
6. \(2\) and \(3\), only
7. \(1,\) \(2,\) and \(3\), only

A.APR.B.3: GRAPHING POLYNOMIAL EQUATIONS

103 Evan graphed a cubic function, 
\[f(x) = ax^3 + bx^2 + cx + d\] , and determined the roots of 
f\((x)\) to be \(\pm 1\) and \(2\). What is the value of \(b\), if 
a = 1?
1. \(1\)
2. \(2\)
3. \(-1\)
4. \(-2\)
104 The graph of the function \( p(x) \) is sketched below.

Which equation could represent \( p(x) \)?

1. \( p(x) = (x^2 - 9)(x - 2) \)
2. \( p(x) = x^3 - 2x^2 + 9x + 18 \)
3. \( p(x) = (x^2 + 9)(x - 2) \)
4. \( p(x) = x^3 + 2x^2 - 9x - 18 \)

105 A sketch of \( r(x) \) is shown below.

An equation for \( r(x) \) could be

1. \( r(x) = (x - a)(x + b)(x + c) \)
2. \( r(x) = (x + a)(x - b)(x - c)^2 \)
3. \( r(x) = (x + a)(x - b)(x - c) \)
4. \( r(x) = (x - a)(x + b)(x + c)^2 \)

106 Which graph represents a polynomial function that contains \( x^2 + 2x + 1 \) as a factor?
107 The graph of \( y = f(x) \) is shown below. The function has a leading coefficient of 1.

Write an equation for \( f(x) \). The function \( g \) is formed by translating function \( f \) left 2 units. Write an equation for \( g(x) \).

F.IF.B.4: GRAPHING POLYNOMIAL FUNCTIONS

108 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.

Which statement about this function is incorrect?
1. The degree of the polynomial is even.
2. There is a positive leading coefficient.
3. At two pH values, there is a relative maximum value.
4. There are two intervals where the function is decreasing.
109 A polynomial equation of degree three, \( p(x) \), is used to model the volume of a rectangular box. The graph of \( p(x) \) has \( x \) intercepts at \(-2, 10, \) and \(14\). Which statements regarding \( p(x) \) could be true?
A. The equation of \( p(x) = (x - 2)(x + 10)(x + 14) \).
B. The equation of \( p(x) = -(x + 2)(x - 10)(x - 14) \).
C. The maximum volume occurs when \( x = 10 \).
D. The maximum volume of the box is approximately 56.

1  A and C  
2  A and D  
3  B and C  
4  B and D  

110 The function below models the average price of gas in a small town since January 1st.
\[
G(t) = -0.0049t^4 + 0.0923t^3 - 0.56t^2 + 1.166t + 3.23,
\]
where \( 0 \leq t \leq 10 \).
If \( G(t) \) is the average price of gas in dollars and \( t \) represents the number of months since January 1st, the absolute maximum \( G(t) \) reaches over the given domain is about
1  $1.60  
2  $3.92  
3  $4.01  
4  $7.73  

111 An estimate of the number of milligrams of a medication in the bloodstream \( t \) hours after 400 mg has been taken can be modeled by the function below.
\[
I(t) = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t,
\]
where \( 0 \leq t \leq 6 \)
Over what time interval does the amount of medication in the bloodstream strictly increase?
1  0 to 2 hours  
2  0 to 3 hours  
3  2 to 6 hours  
4  3 to 6 hours  

112 Which description could represent the graph of \( f(x) = 4x^2(x + a) - x - a \), if \( a \) is an integer?
1  As \( x \to -\infty, f(x) \to \infty \), as \( x \to \infty, f(x) \to \infty \), and the graph has 3 \( x \)-intercepts.
2  As \( x \to -\infty, f(x) \to -\infty \), as \( x \to \infty, f(x) \to \infty \), and the graph has 3 \( x \)-intercepts.
3  As \( x \to -\infty, f(x) \to \infty \), as \( x \to \infty, f(x) \to -\infty \), and the graph has 4 \( x \)-intercepts.
4  As \( x \to -\infty, f(x) \to -\infty \), as \( x \to \infty, f(x) \to \infty \), and the graph has 4 \( x \)-intercepts.

113 Factor completely over the set of integers: 
\( 16x^4 - 81 \). Sara graphed the polynomial \( y = 16x^4 - 81 \) and stated “All the roots of \( y = 16x^4 - 81 \) are real.” Is Sara correct? Explain your reasoning.
F.IF.C.7: GRAPHING POLYNOMIAL FUNCTIONS

114 If $a$, $b$, and $c$ are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x + b)(x^2 - 2cx + c^2)$?

115 Which graph has the following characteristics?
- three real zeros
- as $x \to -\infty$, $f(x) \to -\infty$
- as $x \to \infty$, $f(x) \to \infty$
116 A 4th degree polynomial has zeros $-5, 3, i,$ and $-i$. Which graph could represent the function defined by this polynomial?

117 On the axes below, sketch a possible function $p(x) = (x - a)(x - b)(x + c)$, where $a$, $b$, and $c$ are positive, $a > b$, and $p(x)$ has a positive $y$-intercept of $d$. Label all intercepts.

118 The zeros of a quartic polynomial function are 2, $-2$, 4, and $-4$. Use the zeros to construct a possible sketch of the function, on the set of axes below.
119 Find algebraically the zeros for 
\[ p(x) = x^3 + x^2 - 4x - 4. \] On the set of axes below, graph \( y = p(x) \).

120 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

121 The zeros of a quartic polynomial function \( h \) are 
-1, \( \pm 2 \), and 3. Sketch a graph of \( y = h(x) \) on the grid below.

122 On the grid below, graph the function 
\[ f(x) = x^3 - 6x^2 + 9x + 6 \] on the domain \(-1 \leq x \leq 4\).
123 A major car company analyzes its revenue, \( R(x) \), and costs \( C(x) \), in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, \( x \), using the given functions.

\[
R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000 \\
C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000
\]

The company’s profits can be represented as the difference between its revenue and costs. Write the profit function, \( P(x) \), as a polynomial in standard form. Graph \( y = P(x) \) on the set of axes below over the domain \( 2 \leq x \leq 16 \).

Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

A.APR.B.2: REMAINDER THEOREM

124 The graph of \( p(x) \) is shown below.

What is the remainder when \( p(x) \) is divided by \( x + 4 \)?
1. \( x - 4 \)
2. \( -4 \)
3. \( 0 \)
4. \( 4 \)

125 When \( g(x) \) is divided by \( x + 4 \), the remainder is 0. Given \( g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8 \), which conclusion about \( g(x) \) is true?
1. \( g(4) = 0 \)
2. \( g(-4) = 0 \)
3. \( x - 4 \) is a factor of \( g(x) \).
4. No conclusion can be made regarding \( g(x) \).
126 If \( p(x) = 2x^3 - 3x + 5 \), what is the remainder of \( p(x) \div (x - 5) \)?

1. -230
2. 0
3. 40
4. 240

127 Which binomial is a factor of \( x^4 - 4x^2 - 4x + 8 \)?

1. \( x - 2 \)
2. \( x + 2 \)
3. \( x - 4 \)
4. \( x + 4 \)

128 Which binomial is not a factor of the expression \( x^3 - 11x^2 + 16x + 84 \)?

1. \( x + 2 \)
2. \( x + 4 \)
3. \( x - 6 \)
4. \( x - 7 \)

129 Given \( P(x) = x^3 - 3x^2 - 2x + 4 \), which statement is true?

1. \( (x - 1) \) is a factor because \( P(-1) = 2 \).
2. \( (x + 1) \) is a factor because \( P(1) = 2 \).
3. \( (x + 1) \) is a factor because \( P(1) = 0 \).
4. \( (x - 1) \) is a factor because \( P(1) = 0 \).

130 Use an appropriate procedure to show that \( x - 4 \) is a factor of the function \( f(x) = 2x^3 - 5x^2 - 11x - 4 \). Explain your answer.

131 Given \( z(x) = 6x^3 + bx^2 - 52x + 15 \), \( z(2) = 35 \), and \( z(-5) = 0 \), algebraically determine all the zeros of \( z(x) \).

132 Evaluate \( j(-1) \) given \( j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48 \). Explain what your answer tells you about \( x + 1 \) as a factor. Algebraically find the remaining zeros of \( j(x) \).

133 Determine if \( x - 5 \) is a factor of \( 2x^3 - 4x^2 - 7x - 10 \). Explain your answer.

134 Given \( r(x) = x^3 - 4x^2 + 4x - 6 \), find the value of \( r(2) \). What does your answer tell you about \( x - 2 \) as a factor of \( r(x) \)? Explain.

135 Determine for which polynomial(s) \( (x + 2) \) is a factor. Explain your answer.

\[ P(x) = x^4 - 3x^3 - 16x - 12 \]
\[ Q(x) = x^3 - 3x^2 - 16x - 12 \]

A.APR.C.4: POLYNOMIAL IDENTITIES

136 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

1. \((m+p)^2 = m^2 + 2mp + p^2\)
2. \((x+y)^3 = x^3 + 3xy + y^3\)
3. \((a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2\)

1. I, only
2. I and II
3. II and III
4. I and III
137 Which statement(s) are true for all real numbers?

I \((x-y)^2 = x^2 + y^2\)
II \((x+y)^3 = x^3 + 3xy + y^3\)

1 I, only
2 II, only
3 I and II
4 neither I nor II

138 The expression \((x + a)(x + b)\) can not be written as

1 \(a(x + b) + x(x + b)\)
2 \(x^2 + abx + ab\)
3 \(x^2 + (a + b)x + ab\)
4 \(x(x + a) + b(x + a)\)

139 Which expression can be rewritten as \((x + 7)(x - 1)\)?

1 \((x + 3)^2 - 16\)
2 \((x + 3)^2 - 10(x + 3) - 2(x + 3) + 20\)
3 \(\frac{(x - 1)(x^2 - 6x - 7)}{x + 1}\)
4 \(\frac{(x + 7)(x^2 + 4x + 3)}{x + 3}\)

140 Given the following polynomials

\(x = (a + b + c)^2\)
\(y = a^2 + b^2 + c^2\)
\(z = ab + bc + ac\)

Which identity is true?

1 \(x = y - z\)
2 \(x = y + z\)
3 \(x = y - 2z\)
4 \(x = y + 2z\)

141 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

142 Algebraically prove that \(\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}\)
where \(x \neq -2\).

143 Algebraically determine the values of \(h\) and \(k\) to correctly complete the identity stated below.

\[2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k\]

144 Verify the following Pythagorean identity for all values of \(x\) and \(y\):

\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\]

145 Erin and Christa were working on cubing binomials for math homework. Erin believed they could save time with a shortcut. She wrote down the rule below for Christa to follow.

\[(a + b)^3 = a^3 + b^3\]

Does Erin's shortcut always work? Justify your result algebraically.
146 For positive values of $x$, which expression is equivalent to $\sqrt[3]{16x^2 \cdot x^\frac{2}{3} + 8x^3}$?

1. $6\sqrt[3]{x^3}$
2. $6\sqrt[3]{x^5}$
3. $4\sqrt[3]{x^2 + 2\sqrt[3]{x^5}}$
4. $4\sqrt[3]{x^3 + 2\sqrt[3]{x^3}}$

147 For $x > 0$, which expression is equivalent to $\frac{3\sqrt[3]{x^2} \cdot \sqrt[5]{x^5}}{\sqrt[6]{x}}$?

1. $x$
2. $x^\frac{3}{2}$
3. $x^3$
4. $x^{10}$

148 Given $y > 0$, the expression $\sqrt[3]{3x^2y} \cdot \sqrt[5]{27x^3y^2}$ is equivalent to

1. $81x^5y^3$
2. $3^{\frac{5}{2}}x^2y^\frac{5}{3}$
3. $\frac{1}{2}x^2y^\frac{5}{2}$
4. $3^2x^3y^6$

149 Write $\sqrt[3]{x} \cdot \sqrt{x}$ as a single term with a rational exponent.

A.REI.A.2: SOLVING RADICALS

150 The solution set for the equation $\sqrt{56 - x} = x$ is

1. $\{-8, 7\}$
2. $\{-7, 8\}$
3. $\{7\}$
4. $\{\}$

151 The solution set for the equation $\sqrt{x + 14} - \sqrt{2x + 5} = 1$ is

1. $\{-6\}$
2. $\{2\}$
3. $\{18\}$
4. $\{2, 22\}$

152 What is the solution set for $x$ in the equation below?

$\sqrt{x + 1} - 1 = x$

1. $\{1\}$
2. $\{0\}$
3. $\{-1, 0\}$
4. $\{0, 1\}$

153 The value(s) of $x$ that satisfy $\sqrt{x^2 - 4x - 5} = 2x - 10$ are

1. $\{5\}$
2. $\{7\}$
3. $\{5, 7\}$
4. $\{3, 5, 7\}$

154 The solution set for the equation $b = \sqrt{2b^2 - 64}$ is

1. $\{-8\}$
2. $\{8\}$
3. $\{\pm 8\}$
4. $\{\}$
155 The Beaufort Wind Scale was devised by British Rear Admiral Sir Francis Beaufort, in 1805 based upon observations of the effects of the wind. Beaufort numbers, $B$, are determined by the equation

$$B = 1.69s + 4.45 - 3.49,$$

where $s$ is the speed of the wind in mph, and $B$ is rounded to the nearest integer from 0 to 12.

<table>
<thead>
<tr>
<th>Beaufort Wind Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beaufort Number</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

Using the table above, classify the force of wind at a speed of 30 mph. Justify your answer. In 1946, the scale was extended to accommodate strong hurricanes. A strong hurricane received a $B$ value of exactly 15. Algebraically determine the value of $s$, to the nearest mph. Any $B$ values that round to 10 receive a Beaufort number of 10. Using technology, find an approximate range of wind speeds, to the nearest mph, associated with a Beaufort number of 10.

156 Solve algebraically for all values of $x$:

$$\sqrt{x - 5} + x = 7$$

157 Solve the equation $\sqrt{2x - 7} + x = 5$ algebraically, and justify the solution set.

158 Solve algebraically for all values of $x$:

$$\sqrt{x + 4} + x = 6$$

159 Solve algebraically for all values of $x$:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

160 Solve the given equation algebraically for all values of $x$.

$$3\sqrt{x - 2} = -5$$
161 The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation
\[ s = \sqrt{t - 2t + 6} \]
where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \). How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

162 Explain how \( \left( \frac{1}{3} \right)^{\frac{2}{3}} \) can be written as the equivalent radical expression \( \sqrt[3]{9} \).

163 Explain how \( \left( -8 \right)^{\frac{4}{3}} \) can be evaluated using properties of rational exponents to result in an integer answer.

164 Explain why \( 81^{\frac{3}{4}} \) equals 27.

165 Explain what a rational exponent, such as \( \frac{5}{2} \), means. Use this explanation to evaluate \( 9^{\frac{5}{2}} \).

166 For \( x \neq 0 \), which expressions are equivalent to one divided by the sixth root of \( x \)?

\[
\begin{align*}
I. & \quad \frac{1}{\sqrt[6]{x}} \\
II. & \quad \frac{1}{x^{\frac{1}{6}}} \\
III. & \quad x^{-\frac{1}{6}}
\end{align*}
\]

1. I and II, only
2. I and III, only
3. II and III, only
4. I, II, and III

167 When \( b > 0 \) and \( d \) is a positive integer, the expression \( (3b)^{\frac{2}{d}} \) is equivalent to

\[
\begin{align*}
1. & \quad \frac{1}{\left( \sqrt[d]{3b} \right)^2} \\
2. & \quad \left( \sqrt[d]{3b} \right)^d \\
3. & \quad \frac{1}{\sqrt[d]{3b}^d} \\
4. & \quad \left( \frac{2}{\sqrt[d]{3b}} \right)^2
\end{align*}
\]

168 The expression \( \left( \frac{m^2}{3} \right)^{-\frac{1}{2}} \) is equivalent to

\[
\begin{align*}
1. & \quad -\sqrt[6]{m^5} \\
2. & \quad \frac{1}{\sqrt[6]{m^5}} \\
3. & \quad -m^\frac{5}{6} \sqrt{m} \\
4. & \quad \frac{1}{m^\frac{5}{6} \sqrt{m}}
\end{align*}
\]
169 What does \( \left( -\frac{54y^9}{y^4} \right)^{\frac{2}{3}} \) equal?

1 \( \frac{9x^{6.3} \sqrt[4]{4}}{y^{3.3} \sqrt[2]{y^2}} \)

2 \( \frac{9x^{6.3} \sqrt[4]{4}}{y^{2.3} \sqrt[2]{y^2}} \)

3 \( \frac{9x^{6.3} \sqrt[4]{4}}{y^{1.3} \sqrt{y}} \)

4 \( \frac{9x^{6.3} \sqrt[4]{4}}{y^{2.3} \sqrt[2]{y^2}} \)

170 If \( n = \sqrt[5]{a^5} \) and \( m = a \), where \( a > 0 \), an expression for \( \frac{n}{m} \) could be

1 \( \frac{5}{2} \)

2 \( a^4 \)

3 \( \sqrt[4]{a^5} \)

4 \( \sqrt[3]{a^3} \)

171 For \( x \geq 0 \), which equation is false?

1 \( \left( x^{\frac{3}{2}} \right)^2 = 4 \sqrt[3]{x^3} \)

2 \( \left( x^3 \right)^{\frac{1}{4}} = 4 \sqrt[3]{x^3} \)

3 \( \left( x^{\frac{3}{2}} \right)^{\frac{1}{2}} = 4 \sqrt[3]{x^3} \)

4 \( (x^3)^2 = 3 \sqrt[3]{x^4} \)

172 Use the properties of rational exponents to determine the value of \( y \) for the equation:

\[
\frac{\sqrt[3]{x^8}}{(x^4)^{\frac{1}{3}}} = x^y, \quad x > 1
\]

173 Given the equal terms \( \sqrt[3]{x^5} \) and \( y^\frac{5}{6} \), determine and state \( y \), in terms of \( x \).

174 Express the fraction \( \frac{2x^{\frac{3}{2}}}{\sqrt[4]{16x^4}} \) in simplest radical form.

175 Justify why \( \frac{\sqrt[3]{x^2} \cdot y^5}{\sqrt[4]{x^3} \cdot y^4} \) is equivalent to \( x^{\frac{-1}{12}} \cdot y^{\frac{2}{3}} \) using properties of rational exponents, where \( x \neq 0 \) and \( y \neq 0 \).

176 Kenzie believes that for \( x \geq 0 \), the expression \( \left( \sqrt[3]{x^7} \right) \left[ \sqrt[5]{x^3} \right] \) is equivalent to \( \sqrt[35]{x^6} \). Is she correct? Justify your response algebraically.
N.CN.A.2: OPERATIONS WITH COMPLEX NUMBERS

177 Given $i$ is the imaginary unit, $(2 - yi)^2$ in simplest form is
1 $y^2 - 4yi + 4$
2 $-y^2 - 4yi + 4$
3 $-y^2 + 4$
4 $y^2 + 4$

178 The expression $6xi^3(-4xi + 5)$ is equivalent to
1 $2x - 5i$
2 $-24x^2 - 30xi$
3 $-24x^2 + 30x - i$
4 $26x - 24x^2 i - 5i$

179 Which expression is equivalent to $(3k - 2i)^2$, where $i$ is the imaginary unit?
1 $9k^2 - 4$
2 $9k^2 + 4$
3 $9k^2 - 12ki - 4$
4 $9k^2 - 12ki + 4$

180 Where $i$ is the imaginary unit, the expression $(x + 3i)^2 - (2x - 3i)^2$ is equivalent to
1 $-3x^2$
2 $-3x^2 - 18$
3 $-3x^2 + 18xi$
4 $-3x^2 - 6xi - 18$

181 If $A = -3 + 5i$, $B = 4 - 2i$, and $C = 1 + 6i$, where $i$ is the imaginary unit, then $A - BC$ equals
1 $5 - 17i$
2 $5 + 27i$
3 $-19 - 17i$
4 $-19 + 27i$

182 Which expression is equivalent to $(2x - i)^2 - (2x - i)(2x + 3i)$ where $i$ is the imaginary unit and $x$ is a real number?
1 $-4 - 8xi$
2 $-4 - 4xi$
3 $2$
4 $8x - 4i$

183 The expression $6 - (3x - 2i)^2$ is equivalent to
1 $-9x^2 + 12xi + 10$
2 $9x^2 - 12xi + 2$
3 $-9x^2 + 10$
4 $-9x^2 + 12xi - 4i + 6$

184 Write $(5 + 2yi)(4 - 3i) - (5 - 2yi)(4 - 3i)$ in $a + bi$ form, where $y$ is a real number.

185 Simplify $xi(i - 7i)^2$, where $i$ is the imaginary unit.

186 Express $(1 - i)^3$ in $a + bi$ form.

187 Write $\frac{1}{2} i^3 \left( \sqrt{-9} - 4 \right) - 3i^2$ in simplest $a + bi$ form.
188 Elizabeth tried to find the product of \((2 + 4i)\) and \((3 - i)\), and her work is shown below.

\[
(2 + 4i)(3 - i) = 6 - 2i + 12i - 4i^2
\]
\[
= 6 + 10i - 4i^2
\]
\[
= 6 + 10i - 4(1)
\]
\[
= 6 + 10i - 4
\]
\[
= 2 + 10i
\]

Identify the error in the process shown and determine the correct product of \((2 + 4i)\) and \((3 - i)\).

**RATIONALS**

**A.APR.D.6: UNDEFINED RATIONALS**

189 The function

\[
f(x) = \frac{x - 3}{x^2 + 2x - 8}
\]

is undefined when

\(x\) equals

1 2 or -4
2 4 or -2
3 3, only
4 2, only

**A.APR.D.6: EXPRESSIONS WITH NEGATIVE EXPONENTS**

190 The expression

\[
-\frac{3x^2 - 5x + 2}{x^3 + 2x^2}
\]

can be rewritten as

1 \(-\frac{3x - 3}{x^2 + 2x}\)
2 \(-\frac{3x - 1}{x^2}\)
3 \(-3x^{-1} + 1\)
4 \(-3x^{-1} + x^{-2}\)

**A.APR.D.6: RATIONAL EXPRESSIONS**

191 The expression \(\frac{6x^3 + 17x^2 + 10x + 2}{2x + 3}\) equals

1 \(3x^2 + 4x - 1 + \frac{5}{2x + 3}\)
2 \(6x^2 + 8x - 2 + \frac{5}{2x + 3}\)
3 \(6x^2 - x + 13 - \frac{37}{2x + 3}\)
4 \(3x^2 + 13x + \frac{49}{2} + \frac{151}{2x + 3}\)

192 The expression \(\frac{4x^3 + 5x + 10}{2x + 3}\) is equivalent to

1 \(2x^2 + 3x - 7 + \frac{31}{2x + 3}\)
2 \(2x^2 - 3x - 7 - \frac{11}{2x + 3}\)
3 \(2x^2 + 2.5x + 5 + \frac{15}{2x + 3}\)
4 \(2x^2 - 2.5x - 5 - \frac{20}{2x + 3}\)

193 The expression \(\frac{x^3 + 2x^2 + x + 6}{x + 2}\) is equivalent to

1 \(x^2 + 3\)
2 \(x^2 + 1 + \frac{4}{x + 2}\)
3 \(2x^2 + x + 6\)
4 \(2x^2 + 1 + \frac{4}{x + 2}\)
194 Which expression is equivalent to \( \frac{4x^3 + 9x - 5}{2x - 1} \), where \( x \neq \frac{1}{2} \)?

1. \( 2x^2 + x + 5 \)
2. \( 2x^2 + \frac{11}{2} + \frac{1}{2(2x - 1)} \)
3. \( 2x^2 - x + 5 \)
4. \( 2x^2 - x + 4 + \frac{1}{2x - 1} \)

195 What is the quotient when \( 10x^3 - 3x^2 - 7x + 3 \) is divided by \( 2x - 1 \)?

1. \( 5x^2 + x + 3 \)
2. \( 5x^2 - x + 3 \)
3. \( 5x^2 - x - 3 \)
4. \( 5x^2 + x - 3 \)

196 Written in simplest form, \( \frac{c^3 - d^3}{d^2 + cd - 2c^2} \) where \( c \neq d \), is equivalent to

1. \( \frac{c + d}{d + 2c} \)
2. \( \frac{c - d}{d + 2c} \)
3. \( \frac{-c - d}{d + 2c} \)
4. \( \frac{-c + d}{d + 2c} \)

197 For all values of \( x \) for which the expression is defined, \( \frac{x^3 + 2x^2 - 9x - 18}{x^3 - x^2 - 6x} \), in simplest form, is equivalent to

1. \( \frac{3}{2} \)
2. \( \frac{17}{2} \)
3. \( \frac{x + 3}{x} \)
4. \( \frac{x^2 - 9}{x(x - 3)} \)

198 Which expression is equivalent to \( \frac{2x^4 + 8x^3 - 25x^2 - 6x + 14}{x + 6} \)?

1. \( 2x^3 + 4x^2 + x - 12 + \frac{86}{x + 6} \)
2. \( 2x^3 - 4x^2 - x + 14 \)
3. \( 2x^3 - 4x^2 - x + \frac{14}{x + 6} \)
4. \( 2x^3 - 4x^2 - x \)

199 Which expression(s) are equivalent to \( \frac{x^2 - 4x}{2x} \), where \( x \neq 0 \)?

I. \( \frac{x}{2} - 2 \)
II. \( \frac{x - 4}{2} \)
III. \( \frac{x - 1}{2} - \frac{3}{2} \)

1. II, only
2. I and II
3. II and III
4. I, II, and III
200 The expression $\frac{9x^2 - 2}{3x + 1}$ is equivalent to

1. $3x - 1 - \frac{1}{3x + 1}$
2. $3x - 1 + \frac{1}{3x + 1}$
3. $3x + 1 - \frac{1}{3x + 1}$
4. $3x + 1 + \frac{1}{3x + 1}$

201 Given $f(x) = 3x^2 + 7x - 20$ and $g(x) = x - 2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$.

202 Determine the quotient and remainder when $(6a^3 + 11a^2 - 4a - 9)$ is divided by $(3a - 2)$. Express your answer in the form $q(a) + \frac{r(a)}{d(a)}$.

203 Given $a(x) = x^4 + 2x^3 + 4x - 10$ and $b(x) = x + 2$, determine $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$. Is $b(x)$ a factor of $a(x)$? Explain.

204 When the function $p(x)$ is divided by $x - 1$ the quotient is $x^2 + 7 + \frac{5}{x - 1}$. State $p(x)$ in standard form.

205 The expression $2 - \frac{x - 1}{x + 2}$ is equivalent to

1. $1 - \frac{3}{x + 2}$
2. $1 + \frac{3}{x + 2}$
3. $1 - \frac{1}{x + 2}$
4. $1 + \frac{1}{x + 2}$

206 A number, minus twenty times its reciprocal, equals eight. The number is

1. 10 or -2
2. 10 or 2
3. -10 or -2
4. -10 or 2

207 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, $T$, are left in the semester?

1. $\frac{255 + 93T}{3T} = 90$
2. $\frac{255 + 90T}{3T} = 93$
3. $\frac{255 + 93T}{T + 3} = 90$
4. $\frac{255 + 90T}{T + 3} = 93$
208 Mallory wants to buy a new window air conditioning unit. The cost for the unit is $329.99. If she plans to run the unit three months out of the year for an annual operating cost of $108.78, which function models the cost per year over the lifetime of the unit, \( C(n) \), in terms of the number of years, \( n \), that she owns the air conditioner.

1. \( C(n) = 329.99 + 108.78n \)
2. \( C(n) = 329.99 + 326.34n \)
3. \( C(n) = \frac{329.99 + 108.78n}{n} \)
4. \( C(n) = \frac{329.99 + 326.34n}{n} \)

209 A manufacturing plant produces two different-sized containers of peanuts. One container weighs \( x \) ounces and the other weighs \( y \) pounds. If a gift set can hold one of each size container, which expression represents the number of gift sets needed to hold 124 ounces?

1. \( \frac{124}{16x + y} \)
2. \( \frac{x + 16y}{124} \)
3. \( \frac{124}{x + 16y} \)
4. \( \frac{16x + y}{124} \)

210 A formula for work problems involving two people is shown below.

\[
\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}
\]

\( t_1 \) = the time taken by the first person to complete the job
\( t_2 \) = the time taken by the second person to complete the job
\( t_b \) = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk. Determine, to the nearest tenth of an hour, how long it would take Fred and Barney working together to build a desk.

211 What is the solution set of the equation

\[
\frac{3x + 25}{x + 7} - 5 = \frac{3}{x}
\]

1. \( \left\{ \frac{3}{2}, 7 \right\} \)
2. \( \left\{ \frac{7}{2}, -3 \right\} \)
3. \( \left\{ \frac{3}{2}, 7 \right\} \)
4. \( \left\{ \frac{7}{2}, -3 \right\} \)
212 The focal length, \( F \), of a camera’s lens is related to the distance of the object from the lens, \( J \), and the distance to the image area in the camera, \( W \), by the formula below.

\[
\frac{1}{J} + \frac{1}{W} = \frac{1}{F}
\]

When this equation is solved for \( J \) in terms of \( F \) and \( W \), \( J \) equals
1. \( F - W \)
2. \( \frac{FW}{F - W} \)
3. \( \frac{FW}{W - F} \)
4. \( \frac{1}{F} - \frac{1}{W} \)

213 What is the solution, if any, of the equation

\[
\frac{2}{x + 3} - \frac{3}{4 - x} = \frac{2x - 2}{x^2 - x - 12}
\]

1. \(-1\)
2. \(-5\)
3. all real numbers
4. no real solution

214 The solutions to \( x + 3 - \frac{4}{x - 1} = 5 \) are
1. \( \frac{3}{2} \pm \frac{\sqrt{17}}{2} \)
2. \( \frac{3}{2} \pm \frac{\sqrt{17}}{2} i \)
3. \( \frac{3}{2} \pm \frac{\sqrt{33}}{2} \)
4. \( \frac{3}{2} \pm \frac{\sqrt{33}}{2} i \)

215 What is the solution set of the equation

\[
\frac{2}{x} - \frac{3x}{x + 3} = \frac{x}{x + 3}
\]

1. \{3\}
2. \( \left\{ \frac{3}{2} \right\} \)
3. \{-2,3\}
4. \( \left\{ -1, \frac{3}{2} \right\} \)

216 What is the solution set of the equation

\[
\frac{2}{3x + 1} = \frac{1}{x} - \frac{6x}{3x + 1}
\]

1. \( \left\{ \frac{1}{3}, \frac{1}{2} \right\} \)
2. \( \left\{ \frac{1}{3} \right\} \)
3. \( \left\{ \frac{1}{2} \right\} \)
4. \( \left\{ \frac{1}{3}, -2 \right\} \)

217 What is the solution set of the equation

\[
\frac{10}{x^2 - 2x} + \frac{4}{x} = \frac{5}{x - 2}
\]

1. \{0,2\}
2. \{0\}
3. \{2\}
4. \{\}
218 To solve \( \frac{2x}{x-2} - \frac{11}{x} = \frac{8}{x^2 - 2x} \), Ren multiplied both sides by the least common denominator. Which statement is true?
1. 2 is an extraneous solution.
2. \( \frac{7}{2} \) is an extraneous solution.
3. 0 and 2 are extraneous solutions.
4. This equation does not contain any extraneous solutions.

219 Solve for \( x \):
\[
\frac{1}{x} - \frac{1}{3} = \frac{1}{3x}
\]

220 Solve for all values of \( p \):
\[
\frac{3p}{p - 5} - \frac{2}{p + 3} = \frac{p}{p + 3}
\]

221 Algebraically solve for \( x \):
\[
\frac{-3}{x + 3} + \frac{1}{2} = \frac{x}{6} - \frac{1}{2}
\]

222 Algebraically solve for \( x \):
\[
\frac{7}{2x} - \frac{2}{x + 1} = \frac{1}{4}
\]

223 For the system shown below, what is the value of \( z \)?
\[
\begin{align*}
y &= -2x + 14 \\
3x - 4z &= 2 \\
3x - y &= 16
\end{align*}
\]

224 Which value is not contained in the solution of the system shown below?
\[
\begin{align*}
a + 5b - c &= -20 \\
4a - 5b + 4c &= 19 \\
-a - 5b - 5c &= 2
\end{align*}
\]

225 Consider the system of equations below:
\[
\begin{align*}
x + y - z &= 6 \\
2x - 3y + 2z &= -19 \\
-x + 4y - z &= 17
\end{align*}
\]
Which number is not the value of any variable in the solution of the system?
1. -1
2. 2
3. 3
4. -4
226 Solve the following system of equations algebraically for all values of \(x, y,\) and \(z:\)
\[
\begin{align*}
x + 3y + 5z &= 45 \\
6x - 3y + 2z &= -10 \\
-2x + 3y + 8z &= 72
\end{align*}
\]

227 Solve the following system of equations algebraically for all values of \(x, y,\) and \(z:\)
\[
\begin{align*}
x + y + z &= 1 \\
2x + 4y + 6z &= 2 \\
-x + 3y - 5z &= 11
\end{align*}
\]

228 Solve the following system of equations algebraically for all values of \(x, y,\) and \(z:\)
\[
\begin{align*}
2x + 3y - 4z &= -1 \\
x - 2y + 5z &= 3 \\
-4x + y + z &= 16
\end{align*}
\]

229 Solve the following system of equations algebraically for all values of \(a, b,\) and \(c:\)
\[
\begin{align*}
a + 4b + 6c &= 23 \\
a + 2b + c &= 2 \\
6b + 2c &= a + 14
\end{align*}
\]

230 What is the solution to the system of equations \(y = 3x - 2\) and \(y = g(x)\) where \(g(x)\) is defined by the function below?

\[
\begin{align*}
g(x) &= \begin{cases}
0, & x = -2 \\
1, & x = 1
\end{cases}
\end{align*}
\]

231 What is the solution set of the following system of equations?
\[
\begin{align*}
y &= 3x + 6 \\
y &= (x + 4)^2 - 10
\end{align*}
\]

\[
\begin{align*}
1 & \quad \{(0, -2)\} \\
2 & \quad \{(0, -2), (1, 6)\} \\
3 & \quad \{(1, 6)\} \\
4 & \quad \{(1, 1), (6, 16)\}
\end{align*}
\]
232 What are the solution(s) to the system of equations shown below?

\[ x^2 + y^2 = 5 \]
\[ y = 2x \]

1. \( x = 1 \) and \( x = -1 \)
2. \( x = 1 \)
3. (1,2) and (-1,-2)
4. (1,2), only

233 Consider the system shown below.

\[ 2x - y = 4 \]
\[ (x + 3)^2 + y^2 = 8 \]

The two solutions of the system can be described as

1. both imaginary
2. both irrational
3. both rational
4. one rational and one irrational

234 The graphs of the equations \( y = x^2 + 4x - 1 \) and \( y + 3 = x \) are drawn on the same set of axes. One solution of this system is

1. \((-5,-2)\)
2. \((-1,-4)\)
3. \((1,4)\)
4. \((-2,-1)\)

235 Algebraically determine the values of \( x \) that satisfy the system of equations below.

\[ y = -2x + 1 \]
\[ y = -2x^2 + 3x + 1 \]

236 Solve the system of equations shown below algebraically.

\[ \begin{align*}
(x - 3)^2 + (y + 2)^2 &= 16 \\
2x + 2y &= 10
\end{align*} \]

237 Solve the following system of equations algebraically. \( x^2 + y^2 = 400 \)

\[ y = x - 28 \]

A.REI.D.11: QUADRATIC-LINEAR SYSTEMS

238 Sally’s high school is planning their spring musical. The revenue, \( R \), generated can be determined by the function \( R(t) = -33t^2 + 360t \), where \( t \) represents the price of a ticket. The production cost, \( C \), of the musical is represented by the function \( C(t) = 700 + 5t \). What is the highest ticket price, to the nearest dollar, they can charge in order not to lose money on the event?

1. \( t = 3 \)
2. \( t = 5 \)
3. \( t = 8 \)
4. \( t = 11 \)

A.REI.D.11: OTHER SYSTEMS

239 To the nearest tenth, the value of \( x \) that satisfies \( 2^x = -2x + 11 \) is

1. 2.5
2. 2.6
3. 5.8
4. 5.9
Algebra II Regents Exam Questions by State Standard: Topic

240 When \( g(x) = \frac{2}{x+2} \) and \( h(x) = \log(x+1) + 3 \) are graphed on the same set of axes, which coordinates best approximate their point of intersection?

1. \((-0.9, 1.8)\)
2. \((-0.9, 1.9)\)
3. \((1.4, 3.3)\)
4. \((1.4, 3.4)\)

241 For which values of \( x \), rounded to the nearest hundredth, will \( |x^2 - 9| - 3 = \log_3 x \)?

1. 2.29 and 3.63
2. 2.37 and 3.54
3. 2.84 and 3.17
4. 2.92 and 3.06

242 If \( p(x) = 2 \ln(x) - 1 \) and \( m(x) = \ln(x + 6) \), then what is the solution for \( p(x) = m(x) \)?

1. 1.65
2. 3.14
3. 5.62
4. no solution

243 How many solutions exist for \( \frac{1}{1-x^2} = -|3x-2| + 5 \)?

1. 1
2. 2
3. 3
4. 4

244 After examining the functions \( f(x) = \ln(x + 2) \) and \( g(x) = e^{-x} \) over the interval \((-2, 3]\), Lexi determined that the correct number of solutions to the equation \( f(x) = g(x) \) is

1. 1
2. 2
3. 3
4. 0

245 Which value, to the nearest tenth, is not a solution of \( p(x) = q(x) \) if \( p(x) = x^3 + 3x^2 - 3x - 1 \) and \( q(x) = 3x + 8 \)?

1. \(-3.9\)
2. \(-1.1\)
3. \(2.1\)
4. \(4.7\)

246 If \( f(x) = 3|x| - 1 \) and \( g(x) = 0.03x^3 - x + 1 \), an approximate solution for the equation \( f(x) = g(x) \) is

1. 1.96
2. 11.29
3. \((-0.99, 1.96)\)
4. \((11.29, 32.87)\)

247 Pedro and Bobby each own an ant farm. Pedro starts with 100 ants and says his farm is growing exponentially at a rate of 15% per month. Bobby starts with 350 ants and says his farm is steadily decreasing by 5 ants per month. Assuming both boys are accurate in describing the population of their ant farms, after how many months will they both have approximately the same number of ants?

1. 7
2. 8
3. 13
4. 36
The populations of two small towns at the beginning of 2018 and their annual population growth rate are shown in the table below.

<table>
<thead>
<tr>
<th>Town</th>
<th>Population</th>
<th>Annual Population Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jonesville</td>
<td>1240</td>
<td>6% increase</td>
</tr>
<tr>
<td>Williamstown</td>
<td>890</td>
<td>11% increase</td>
</tr>
</tbody>
</table>

Assuming the trend continues, approximately how many years after the beginning of 2018 will it take for the populations to be equal?

1 7
2 20
3 68
4 125

Selected values for the functions $f$ and $g$ are shown in the tables below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3.12</td>
<td>−4.88</td>
<td>−2.01</td>
<td>−1.01</td>
</tr>
<tr>
<td>0</td>
<td>−6</td>
<td>0</td>
<td>0.58</td>
</tr>
<tr>
<td>1.23</td>
<td>−4.77</td>
<td>8.52</td>
<td>2.53</td>
</tr>
<tr>
<td>8.52</td>
<td>2.53</td>
<td>13.11</td>
<td>3.01</td>
</tr>
<tr>
<td>9.01</td>
<td>3.01</td>
<td>16.52</td>
<td>3.29</td>
</tr>
</tbody>
</table>

A solution to the equation $f(x) = g(x)$ is

1 0
3 3.01
2 2.53
4 8.52

Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5% per month. The fox population had an initial value of 30 and grew continuously at the rate of 3% per month. Find, to the nearest tenth of a month, how long it takes for these populations to be equal.

Given: $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$

$k(x) = −|0.7x| + 5$

State the solutions to the equation $h(x) = k(x)$, rounded to the nearest hundredth.
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website's popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth? Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2} x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?
253 Sketch the graphs of \( r(x) = \frac{1}{x} \) and \( a(x) = |x| - 3 \) on the set of axes below. Determine, to the nearest tenth, the positive solution of \( r(x) = a(x) \).

254 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function \( N(t) = N_0 (e)^{-rt} \), where \( N(t) \) is the amount left in the body, \( N_0 \) is the initial dosage, \( r \) is the decay rate, and \( t \) is time in hours.

Patient \( A \), \( A(t) \), is given 800 milligrams of a drug with a decay rate of 0.347. Patient \( B \), \( B(t) \), is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, \( A(t) \) and \( B(t) \), to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

To the nearest hour, \( t \), when does the amount of the given drug remaining in patient \( B \) begin to exceed the amount of the given drug remaining in patient \( A \)? The doctor will allow patient \( A \) to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the nearest tenth of an hour, how long patient \( A \) will have to wait to take another 800 milligram dose of the drug.
255 The value of a certain small passenger car based on its use in years is modeled by
\[ V(t) = 28482.698(0.684)^t \]
where \( V(t) \) is the value in dollars and \( t \) is the time in years. Zach had to take out a loan to purchase the small passenger car. The function \( Z(t) = 22151.327(0.778)^t \), where \( Z(t) \) is measured in dollars, and \( t \) is the time in years, models the unpaid amount of Zach's loan over time. Graph \( V(t) \) and \( Z(t) \) over the interval \( 0 \leq t \leq 5 \), on the set of axes below.

State when \( V(t) = Z(t) \), to the nearest hundredth, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a $3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the nearest year, how long will it take Zach to cancel this policy? Justify your answer.
259 If \( g(c) = 1 - c^2 \) and \( m(c) = c + 1 \), then which statement is **not** true?

1. \( g(c) \cdot m(c) = 1 + c - c^2 - c^3 \)
2. \( g(c) + m(c) = 2 + c - c^2 \)
3. \( m(c) - g(c) = c + c^2 \)
4. \( \frac{m(c)}{g(c)} = \frac{-1}{1 - c} \)

260 If \( f(x) = x^2 + 9 \) and \( g(x) = x + 3 \), which operation would not result in a polynomial expression?

1. \( f(x) + g(x) \)
2. \( f(x) - g(x) \)
3. \( f(x) \cdot g(x) \)
4. \( f(x) \div g(x) \)

261 Given: \( f(x) = 2x^2 + x - 3 \) and \( g(x) = x - 1 \)
Express \( f(x) \cdot g(x) - [f(x) + g(x)] \) as a polynomial in standard form.

F.L.E.A.2: FAMILIES OF FUNCTIONS

262 Which table best represents an exponential relationship?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
263 The graph of \( y = f(x) \) is shown below.

Which expression defines \( f(x) \)?

1. \( 2x \)
2. \( 5(2^x) \)
3. \( \frac{x}{5} \)
4. \( 5(2^{2x}) \)

264 Perry invested in property that cost him $1500. Five years later it was worth $3000, and 10 years from his original purchase, it was worth $6000. Assuming the growth rate remains the same, which type of function could he create to find the value of his investment 30 years from his original purchase?

1. exponential function
2. linear function
3. quadratic function
4. trigonometric function

F.IF.C.9: COMPARING FUNCTIONS

265 Which function shown below has a greater average rate of change on the interval \([-2,4]\)? Justify your answer.

\[
g(x) = 4x^3 - 5x^2 + 3
\]
266 The $x$-value of which function’s $x$-intercept is larger, $f$ or $h$? Justify your answer.

$$f(x) = \log(x - 4)$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$h(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$6$</td>
</tr>
<tr>
<td>$0$</td>
<td>$4$</td>
</tr>
<tr>
<td>$1$</td>
<td>$2$</td>
</tr>
<tr>
<td>$2$</td>
<td>$0$</td>
</tr>
<tr>
<td>$3$</td>
<td>$-2$</td>
</tr>
</tbody>
</table>

267 Consider the function $h(x) = 2\sin(3x) + 1$ and the function $q$ represented in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$-8$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$1$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$2$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Determine which function has the smaller minimum value for the domain $[-2, 2]$. Justify your answer.
268 Consider \( f(x) = 4x^2 + 6x - 3 \), and \( p(x) \) defined by the graph below.

The difference between the values of the maximum of \( p \) and minimum of \( f \) is

1. 0.25
2. 1.25
3. 3.25
4. 10.25

269 Consider the function \( p(x) = 3x^3 + x^2 - 5x \) and the graph of \( y = m(x) \) below.

Which statement is true?

1. \( p(x) \) has three real roots and \( m(x) \) has two real roots.
2. \( p(x) \) has one real root and \( m(x) \) has two real roots.
3. \( p(x) \) has two real roots and \( m(x) \) has three real roots.
4. \( p(x) \) has three real roots and \( m(x) \) has four real roots.

270 Which statement regarding the graphs of the functions below is untrue?

\[
\begin{align*}
  f(x) &= 3 \sin 2x, \text{ from } -\pi < x < \pi \\
  g(x) &= (x - 0.5)(x + 4)(x - 2) \\
  h(x) &= \log_2 x \\
  j(x) &= -|4x - 2| + 3
\end{align*}
\]

1. \( f(x) \) and \( j(x) \) have a maximum \( y \)-value of 3.
2. \( f(x) \), \( h(x) \), and \( j(x) \) have one \( y \)-intercept.
3. \( g(x) \) and \( j(x) \) have the same end behavior as \( x \to -\infty \).
4. \( g(x) \), \( h(x) \), and \( j(x) \) have rational zeros.
F.IF.B.4: RELATING GRAPHS TO EVENTS

271 Irma initially ran one mile in over ten minutes. She then began a training program to reduce her one-mile time. She recorded her one-mile time once a week for twelve consecutive weeks, as modeled in the graph below.

Which statement regarding Irma’s one-mile training program is correct?
1. Her one-mile speed increased as the number of weeks increased.
2. Her one-mile speed decreased as the number of weeks increased.
3. If the trend continues, she will run under a six-minute mile by week thirteen.
4. She reduced her one-mile time the most between weeks ten and twelve.

F.BF.B.3: TRANSFORMATIONS WITH FUNCTIONS

272 The graph below represents national and New York State average gas prices.

If New York State’s gas prices are modeled by \( G(x) \) and \( C > 0 \), which expression best approximates the national average \( x \) months from August 2014?
1. \( G(x + C) \)
2. \( G(x) + C \)
3. \( G(x - C) \)
4. \( G(x) - C \)
F.BF.B.3: EVEN AND ODD FUNCTIONS

273 Functions \( f, g, \) and \( h \) are given below.

\[
f(x) = \sin(2x)
\]
\[
g(x) = f(x) + 1
\]

Which statement is true about functions \( f, g, \) and \( h \)?
1. \( f(x) \) and \( g(x) \) are odd, \( h(x) \) is even.
2. \( f(x) \) and \( g(x) \) are even, \( h(x) \) is odd.
3. \( f(x) \) is odd, \( g(x) \) is neither, \( h(x) \) is even.
4. \( f(x) \) is even, \( g(x) \) is neither, \( h(x) \) is odd.

274 Which equation represents an odd function?
1. \( y = \sin x \)
2. \( y = \cos x \)
3. \( y = (x + 1)^3 \)
4. \( y = e^{3x} \)

275 Which function is even?
1. \( f(x) = \sin x \)
2. \( f(x) = x^2 - 4 \)
3. \( f(x) = |x - 2| + 5 \)
4. \( f(x) = x^4 + 3x^3 + 4 \)

276 If \( f(x) \) is an even function, which function must also be even?
1. \( f(x - 2) \)
2. \( f(x) + 3 \)
3. \( f(x + 1) \)
4. \( f(x + 1) + 3 \)

277 Algebraically determine whether the function \( f(x) = x^4 - 3x^2 - 4 \) is odd, even, or neither.

F.BF.B.4: INVERSE OF FUNCTIONS

278 Given \( f^{-1}(x) = \frac{3}{4}x + 2 \), which equation represents \( f(x) \)?
1. \( f(x) = \frac{4}{3}x - \frac{8}{3} \)
2. \( f(x) = -\frac{4}{3}x + \frac{8}{3} \)
3. \( f(x) = \frac{3}{4}x - 2 \)
4. \( f(x) = -\frac{3}{4}x + 2 \)

279 What is the inverse of the function \( y = \log_3 x \)?
1. \( y = x^3 \)
2. \( y = \log_3 3 \)
3. \( y = 3^x \)
4. \( x = 3^y \)
280 The inverse of the function \( f(x) = \frac{x + 1}{x - 2} \) is

1. \( f^{-1}(x) = \frac{x + 1}{x + 2} \)
2. \( f^{-1}(x) = \frac{2x + 1}{x - 1} \)
3. \( f^{-1}(x) = \frac{x + 1}{x - 2} \)
4. \( f^{-1}(x) = \frac{x - 1}{x + 1} \)

281 What is the inverse of \( f(x) = -6(x - 2) \)?

1. \( f^{-1}(x) = -2 - \frac{x}{6} \)
2. \( f^{-1}(x) = 2 - \frac{x}{6} \)
3. \( f^{-1}(x) = \frac{1}{-6(x - 2)} \)
4. \( f^{-1}(x) = 6(x + 2) \)

282 What is the inverse of \( f(x) = x^3 - 2 \)?

1. \( f^{-1}(x) = \sqrt[3]{x + 2} \)
2. \( f^{-1}(x) = \pm \sqrt[3]{x + 2} \)
3. \( f^{-1}(x) = \frac{x}{x - 2} \)
4. \( f^{-1}(x) = \frac{x}{x - 2} \)

283 Given \( f(x) = \frac{1}{2}x + 8 \), which equation represents the inverse, \( g(x) \)?

1. \( g(x) = 2x - 8 \)
2. \( g(x) = 2x - 16 \)
3. \( g(x) = -\frac{1}{2}x + 8 \)
4. \( g(x) = -\frac{1}{2}x - 16 \)

284 If \( f(x) = a^x \) where \( a > 1 \), then the inverse of the function is

1. \( f^{-1}(x) = \log_a x \)
2. \( f^{-1}(x) = a \log x \)
3. \( f^{-1}(x) = \log_a x \)
4. \( f^{-1}(x) = x \log a \)

285 What is the inverse of the function \( y = 4x + 5 \)?

1. \( x = \frac{1}{4}y - \frac{5}{4} \)
2. \( y = \frac{1}{4}x - \frac{5}{4} \)
3. \( y = 4x - 5 \)
4. \( y = \frac{1}{4x + 5} \)

286 What is the inverse of \( f(x) = \frac{x}{x + 2} \), where \( x \neq -2 \)?

1. \( f^{-1}(x) = \frac{2x}{x - 1} \)
2. \( f^{-1}(x) = \frac{-2x}{x - 1} \)
3. \( f^{-1}(x) = \frac{x}{x - 2} \)
4. \( f^{-1}(x) = \frac{-x}{x - 2} \)

287 For the function \( f(x) = (x - 3)^3 + 1 \), find \( f^{-1}(x) \).
F.L.E.A.2: SEQUENCES

288 After Roger’s surgery, his doctor administered pain medication in the following amounts in milligrams over four days.

<table>
<thead>
<tr>
<th>Day (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dosage (m)</td>
<td>2000</td>
<td>1680</td>
<td>1411.2</td>
<td>1185.4</td>
</tr>
</tbody>
</table>

How can this sequence best be modeled recursively?

1. \( m_1 = 2000 \)
2. \( m_n = m_{n-1} - 320 \)
3. \( m_1 = 2000 \)
4. \( m_n = (0.84)m_{n-1} \)
5. \( m_n = 2000(0.84)^{n-1} \)

289 A recursive formula for the sequence 18, 9, 4.5, . . . is

1. \( f_1 = 18 \)
2. \( f_n = \frac{1}{2} f_{n-1} \)
3. \( g_1 = 18 \)
4. \( g_n = 2g_{n-1} \)
5. \( g_1 = 18 \)
6. \( g_n = 18 \left( \frac{1}{2} \right)^{n-1} \)

290 The sequence \( a_1 = 6, a_n = 3a_{n-1} \) can also be written as

1. \( a_n = 6 \cdot 3^n \)
2. \( a_n = 6 \cdot 3^{n+1} \)
3. \( a_n = 2 \cdot 3^n \)
4. \( a_n = 2 \cdot 3^{n+1} \)

291 Given \( f(9) = -2 \), which function can be used to generate the sequence \(-8, -7.25, -6.5, -5.75, \ldots \)?

1. \( f(n) = -8 + 0.75n \)
2. \( f(n) = -8 - 0.75(n - 1) \)
3. \( f(n) = -8.75 + 0.75n \)
4. \( f(n) = -0.75 + 8(n - 1) \)

292 Write a recursive formula, \( a_n \), to describe the sequence graphed below.

293 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79, .... Write a recursive formula for Candy's sequence.
Determine the eighth term in Candy's sequence.
294 Write an explicit formula for \( a_n \), the \( n \)th term of the recursively defined sequence below.

\[
\begin{align*}
a_1 &= x + 1 \\
a_n &= x(a_{n-1})
\end{align*}
\]

For what values of \( x \) would \( a_n = 0 \) when \( n > 1 \)?

F.IF.A.3: SEQUENCES

295 When a ball bounces, the heights of consecutive bounces form a geometric sequence. The height of the first bounce is 121 centimeters and the height of the third bounce is 64 centimeters. To the nearest centimeter, what is the height of the fifth bounce?

\[
\begin{array}{c|c}
1 & 25 \\
2 & 34 \\
3 & 36 \\
4 & 42 \\
\end{array}
\]

F.BF.A.2: SEQUENCES

298 The population of Jamesburg for the years 2010-2013, respectively, was reported as follows: 250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

\[
\begin{array}{c|c}
1 & \text{not possible} \\
2 & \text{not possible} \\
3 & \text{not possible} \\
4 & \text{not possible} \\
\end{array}
\]

299 In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State \( t \) years after 2010?

\[
\begin{array}{c|c}
1 & \text{not possible} \\
2 & \text{not possible} \\
3 & \text{not possible} \\
4 & \text{not possible} \\
\end{array}
\]

300 At her job, Pat earns $25,000 the first year and receives a raise of $1000 each year. The explicit formula for the \( n \)th term of this sequence is \( a_n = 25,000 + (n - 1)1000 \). Which rule best represents the equivalent recursive formula?

\[
\begin{array}{c|c}
1 & a_n = 24,000 + 1000n \\
2 & a_n = 25,000 + 1000n \\
3 & a_1 = 25,000, a_n = a_{n-1} + 1000 \\
4 & a_1 = 25,000, a_n = a_{n+1} + 1000 \\
\end{array}
\]
301 The Rickerts decided to set up an account for their daughter to pay for her college education. The day their daughter was born, they deposited $1000 in an account that pays 1.8% compounded annually. Beginning with her first birthday, they deposit an additional $750 into the account on each of her birthdays. Which expression correctly represents the amount of money in the account $n$ years after their daughter was born?

1. $a_n = 1000(1.018)^n + 750$
2. $a_n = 1000(1.018)^n + 750n$
3. $a_0 = 1000$
4. $a_0 = 1000$

\[a_n = a_{n-1}(1.018) + 750\]

302 The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of $75,000, which model is a recursive formula representing the value of the boat $n$ years after it was purchased?

1. $a_n = 75,000(0.08)^n$
2. $a_0 = 75,000$
3. $a_n = 75,000(1.08)^n$
4. $a_0 = 75,000$

\[a_n = 0.92(a_{n-1})\]

303 The formula below can be used to model which scenario?

\[a_1 = 3000\]
\[a_n = 0.80a_{n-1}\]

1. The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2. The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3. A bank account starts with a deposit of $3000, and each year it grows by 80%.
4. The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

304 Savannah just got contact lenses. Her doctor said she can wear them 2 hours the first day, and can then increase the length of time by 30 minutes each day. If this pattern continues, which formula would not be appropriate to determine the length of time, in either minutes or hours, she could wear her contact lenses on the $n$th day?

1. $a_1 = 120$
2. $a_n = a_{n-1} + 30$
3. $a_1 = 2$
4. $a_n = a_{n-1} + 0.5$

\[a_n = 2.5 + 0.5n\]
305 Which situation could be modeled using a geometric sequence?

1. A cell phone company charges $30.00 per month for 2 gigabytes of data and $12.50 for each additional gigabyte of data.

2. The temperature in your car is 79°. You lower the temperature of your air conditioning by 2° every 3 minutes in order to find a comfortable temperature.

3. David’s parents have set a limit of 50 minutes per week that he may play online games during the school year. However, they will increase his time by 5% per week for the next ten weeks.

4. Sarah has $100.00 in her piggy bank and saves an additional $15.00 each week.

306 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

307 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed $2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed $6.25 to replace his library card and pay the fine for the overdue book. Suppose the total amount Simon owes when the book is \( n \) days late can be determined by an arithmetic sequence. Determine a formula for \( a_n \), the \( n \)th term of this sequence. Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.
Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer. Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose. Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.
F.BF.B.6: SIGMA NOTATION

309 Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1 \[ \sum_{n=1}^{6} 8(1.10)^{n-1} \]

2 \[ \sum_{n=1}^{6} 8(1.10)^{n} \]

3 \[ \frac{8 - 8(1.10)^{6}}{0.90} \]

4 \[ \frac{8 - 8(0.10)^{n}}{1.10} \]

A.SSE.B.4: SERIES

310 Jake wants to buy a car and hopes to save at least $5000 for a down payment. The table below summarizes the amount of money he plans to save each week.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Saved, in Dollars</td>
<td>2</td>
<td>5</td>
<td>12.5</td>
<td>31.25</td>
<td>...</td>
</tr>
</tbody>
</table>

Based on this plan, which expression should he use to determine how much he has saved in \( n \) weeks?

1 \[ \frac{2 - 2(2.5^{n})}{1 - 2.5} \]

2 \[ \frac{2 - 2(2.5^{n-1})}{1 - 2.5} \]

3 \[ \frac{1 - 2.5^{n}}{1 - 2.5} \]

4 \[ \frac{1 - 2.5^{n-1}}{1 - 2.5} \]
311. A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

1. 29
2. 58
3. 120
4. 149

312. A 7-year lease for office space states that the annual rent is $85,000 for the first year and will increase by 6% each additional year of the lease. What will the total rent expense be for the entire 7-year lease?

1. $42,809.63
2. $90,425.53
3. $595,000.00
4. $713,476.20

313. Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

1. $11,622,614.67
2. $17,433,922.00
3. $116,226,146.80
4. $1,743,392,200.00

314. The first term of a geometric sequence is 8 and the fourth term is 216. What is the sum of the first 12 terms of the corresponding series?

1. 236,192
2. 708,584
3. 2,125,760
4. 6,377,288

315. Jasmine decides to put $100 in a savings account each month. The account pays 3% annual interest, compounded monthly. How much money, S, will Jasmine have after one year?

1. \( S = 100(1.03)^{12} \)
2. \( S = \frac{100 - 100(1.0025)^{12}}{1 - 1.0025} \)
3. \( S = 100(1.0025)^{12} \)
4. \( S = \frac{100 - 100(1.03)^{12}}{1 - 1.03} \)

316. Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, \( S_n \), for Alexa's total earnings over \( n \) years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the nearest cent.

317. Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[
P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right)
\]

\( P_n \) = present amount borrowed
\( n \) = number of monthly pay periods
\( PMT \) = monthly payment
\( i \) = interest rate per month

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.
318 Jim is looking to buy a vacation home for $172,600 near his favorite southern beach. The formula to compute a mortgage payment, $M$, is

$$M = P \cdot \frac{r(1+r)^N}{(1+r)^N - 1}$$

where $P$ is the principal amount of the loan, $r$ is the monthly interest rate, and $N$ is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With no down payment, determine Jim's mortgage payment, rounded to the nearest dollar. Algebraically determine and state the down payment, rounded to the nearest dollar, that Jim needs to make in order for his mortgage payment to be $1100.

319 The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is $152,500 and they will make a $15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the nearest dollar.

$$M = \frac{P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n}{\left( 1 + \frac{r}{12} \right)^n - 1}$$

$M$ = monthly payment
$P$ = amount borrowed
$r$ = annual interest rate
$n$ = total number of monthly payments

320 Rowan is training to run in a race. He runs 15 miles in the first week, and each week following, he runs 3% more than the week before. Using a geometric series formula, find the total number of miles Rowan runs over the first ten weeks of training, rounded to the nearest thousandth.

TRIGONOMETRY
F.TF.A.1: UNIT CIRCLE

321 Which diagram shows an angle rotation of 1 radian on the unit circle?
322 The terminal side of θ, an angle in standard position, intersects the unit circle at \( P\left( -\frac{1}{3}, -\frac{\sqrt{8}}{3} \right) \).

What is the value of sec θ?
1  \(-3\)
2  \(\frac{3\sqrt{8}}{8}\)
3  \(\frac{1}{3}\)
4  \(-\frac{\sqrt{8}}{3}\)

323 Point \( M\left( t, \frac{4}{7} \right) \) is located in the second quadrant on the unit circle. Determine the exact value of \( t \).

324 Using the unit circle below, explain why csc θ = \( \frac{1}{y} \).

325 Which diagram represents an angle, \( \alpha \), measuring \( \frac{13\pi}{20} \) radians drawn in standard position, and its reference angle, \( \theta \)?
326 If the terminal side of angle $\theta$, in standard position, passes through point $(-4,3)$, what is the numerical value of $\sin \theta$?

1. $\frac{3}{5}$
2. $\frac{4}{5}$
3. $\frac{3}{5}$
4. $\frac{4}{5}$

327 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, $\theta$, intercepts the circle in Quadrant II at point $C$. The $y$-coordinate of point $C$ is 8. What is the value of $\cos \theta$?

1. $\frac{3}{5}$
2. $\frac{3}{4}$
3. $\frac{3}{5}$
4. $\frac{4}{5}$

328 The hours of daylight, $y$, in Utica in days, $x$, from January 1, 2013 can be modeled by the equation $y = 3.06 \sin(0.017x - 1.40) + 12.23$. How many hours of daylight, to the nearest tenth, does this model predict for February 14, 2013?

1. 9.4
2. 10.4
3. 12.1
4. 12.2

329 The temperature, in degrees Fahrenheit, in Times Square during a day in August can be predicted by the function $T(x) = 8 \sin(0.3x - 3) + 74$, where $x$ is the number of hours after midnight. According to this model, the predicted temperature, to the nearest degree Fahrenheit, at 7 P.M. is

1. 68
2. 74
3. 77
4. 81

330 An angle, $\theta$, is in standard position and its terminal side passes through the point $(2,-1)$. Find the exact value of $\sin \theta$.

331 Given that $\sin^2 \theta + \cos^2 \theta = 1$ and $\sin \theta = -\frac{\sqrt{2}}{5}$, what is a possible value of $\cos \theta$?

1. $\frac{5 + \sqrt{2}}{5}$
2. $\frac{\sqrt{23}}{5}$
3. $\frac{3\sqrt{2}}{5}$
4. $\frac{\sqrt{35}}{5}$
332 Given \( \cos \theta = \frac{7}{25} \), where \( \theta \) is an angle in standard position terminating in quadrant IV, and \( \sin^2 \theta + \cos^2 \theta = 1 \), what is the value of \( \tan \theta \)?

1. \( \frac{24}{25} \)
2. \( \frac{24}{7} \)
3. \( \frac{24}{25} \)
4. \( \frac{24}{7} \)

333 If \( \cos \theta = -\frac{3}{4} \) and \( \theta \) is in Quadrant III, then \( \sin \theta \) is equivalent to

1. \( -\frac{\sqrt{7}}{4} \)
2. \( \frac{\sqrt{7}}{4} \)
3. \( -\frac{5}{4} \)
4. \( \frac{5}{4} \)

334 Using the identity \( \sin^2 \theta + \cos^2 \theta = 1 \), find the value of \( \tan \theta \), to the nearest hundredth, if \( \cos \theta = -0.7 \) and \( \theta \) is in Quadrant II.

335 Given \( \tan \theta = \frac{7}{24} \), and \( \theta \) terminates in Quadrant III, determine the value of \( \cos \theta \).

F.TF.C.8: SIMPLIFYING TRIGONOMETRIC IDENTITIES

336 If \( \sin^2 (32^\circ) + \cos^2 (M) = 1 \), then \( M \) equals

1. \( 32^\circ \)
2. \( 58^\circ \)
3. \( 68^\circ \)
4. \( 72^\circ \)

F.TF.B.5: MODELING TRIGONOMETRIC FUNCTIONS

337 Which equation is represented by the graph shown below?

1. \( y = \frac{1}{2} \cos 2x \)
2. \( y = \cos x \)
3. \( y = \frac{1}{2} \cos x \)
4. \( y = 2 \cos \frac{1}{2} x \)
338 The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below. If the depth, \( d \), is measured in feet and time, \( t \), is measured in hours since midnight, what is an equation for the depth of the water at the marker?

1 \[ d = 5 \cos \left( \frac{\pi}{6} t \right) + 9 \]

2 \[ d = 9 \cos \left( \frac{\pi}{6} t \right) + 5 \]

3 \[ d = 9 \sin \left( \frac{\pi}{6} t \right) + 5 \]

4 \[ d = 5 \sin \left( \frac{\pi}{6} t \right) + 9 \]

339 The function \( f(x) = a \cos bx + c \) is plotted on the graph shown below.

What are the values of \( a \), \( b \), and \( c \)?

1 \( a = 2, b = 6, c = 3 \)

2 \( a = 2, b = 3, c = 1 \)

3 \( a = 4, b = 6, c = 5 \)

4 \( a = 4, b = \frac{\pi}{3}, c = 3 \)

340 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Which equation best represents the value of the voltage as it flows through the electric wires, where \( t \) is time in seconds?

1 \( V = 120 \sin (t) \)

2 \( V = 120 \sin (60t) \)

3 \( V = 120 \sin (60\pi t) \)

4 \( V = 120 \sin (120\pi t) \)
341 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, $H$, in feet, above the ground of one of the six-person cars can be modeled by

$$H(t) = 70\sin\left(\frac{2\pi}{7}(t - 1.75)\right) + 80,$$

where $t$ is time, in minutes. Using $H(t)$ for one full rotation, this car's minimum height, in feet, is

1) 150
2) 70
3) 10
4) 0

342 Relative to the graph of $y = 3\sin x$, what is the shift of the graph of $y = 3\sin\left(x + \frac{\pi}{3}\right)$?

1) $\frac{\pi}{3}$ right
2) $\frac{\pi}{3}$ left
3) $\frac{\pi}{3}$ up
4) $\frac{\pi}{3}$ down

343 Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if $a$ and $b$ are positive constants?

1) right $a$ units, up $b$ units
2) right $a$ units, down $b$ units
3) left $a$ units, up $b$ units
4) left $a$ units, down $b$ units

344 A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave decreasing, only?

1) (0, 200)
2) (100, 300)
3) (200, 400)
4) (300, 400)

345 Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914\sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238\sin(0.525x - 2.148) + 86.729$. Which statement can not be concluded based on the average monthly temperature models $x$ months after starting data collection?

1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.
2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.
3) The maximum average monthly temperature for Bar Harbor is $79^\circ$F, to the nearest degree.
4) The minimum average monthly temperature for Phoenix is $20^\circ$F, to the nearest degree.

346 As $x$ increases from 0 to $\frac{\pi}{2}$, the graph of the equation $y = 2\tan x$ will

1) increase from 0 to 2
2) decrease from 0 to $-2$
3) increase without limit
4) decrease without limit
347 Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

1 $y = -4 \cos \left( \frac{\pi}{4} x \right) - 3$

2 $y = -4 \cos \left( \frac{\pi}{4} x \right) + 5$

3 $y = -4 \cos(8x) - 3$

4 $y = -4 \cos(8x) + 5$

348 The height, $h(t)$ in cm, of a piston, is given by the equation $h(t) = 12 \cos \left( \frac{\pi}{3} t \right) + 8$, where $t$ represents the number of seconds since the measurements began. Determine the average rate of change, in cm/sec, of the piston's height on the interval $1 \leq t \leq 2$. At what value(s) of $t$, to the nearest tenth of a second, does $h(t) = 0$ in the interval $1 \leq t \leq 5$? Justify your answer.

349 A person’s lung capacity can be modeled by the function $C(t) = 250 \sin \left( \frac{2\pi}{5} t \right) + 2450$, where $C(t)$ represents the volume in mL present in the lungs after $t$ seconds. State the maximum value of this function over one full cycle, and explain what this value represents.
351 Which sinusoid has the greatest amplitude?

1  
2  \( y = 3 \sin(\theta - 3) + 5 \)

3  
4  \( y = -5 \sin(\theta - 1) - 3 \)

352 The height above ground for a person riding a Ferris wheel after \( t \) seconds is modeled by

\[ h(t) = 150 \sin\left(\frac{\pi}{45} t + 67.5\right) + 160 \text{ feet}. \] How many seconds does it take to go from the bottom of the wheel to the top of the wheel?

1 10  
2 45  
3 90  
4 150

353 Which statement is incorrect for the graph of the function \( y = -3 \cos\left(\frac{\pi}{3} (x - 4)\right) + 7? \)

1 The period is 6.  
2 The amplitude is 3.  
3 The range is \([4,10]\).  
4 The midline is \( y = -4 \).

354 Tides are a periodic rise and fall of ocean water. On a typical day at a seaport, to predict the time of the next high tide, the most important value to have would be the

1 time between consecutive low tides  
2 time when the tide height is 20 feet  
3 average depth of water over a 24-hour period  
4 difference between the water heights at low and high tide

355 The average monthly temperature of a city can be modeled by a cosine graph. Melissa has been living in Phoenix, Arizona, where the average annual temperature is 75°F. She would like to move, and live in a location where the average annual temperature is 62°F. When examining the graphs of the average monthly temperatures for various locations, Melissa should focus on the

1 amplitude  
2 horizontal shift  
3 period  
4 midline

356 The volume of air in a person’s lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

357 On July 21, 2016, the water level in Puget Sound, WA reached a high of 10.1 ft at 6 a.m. and a low of −2 ft at 12:30 p.m. Across the country in Long Island, NY, Shinnecock Bay’s water level reached a high of 2.5 ft at 10:42 p.m. and a low of −0.1 ft at 5:31 a.m. The water levels of both locations are affected by the tides and can be modeled by sinusoidal functions. Determine the difference in amplitudes, in feet, for these two locations.
358 The graph below represents the height above the ground, \( h \), in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, \( t \), in seconds.

Identify the period of the graph and describe what the period represents in this context.

359 Graph \( t(x) = 3\sin(2x) + 2 \) over the domain \([0, 2\pi]\) on the set of axes below.

360 On the axes below, graph one cycle of a cosine function with amplitude 3, period \( \frac{\pi}{2} \), midline \( y = -1 \), and passing through the point \((0,2)\).
361 Write an equation for a sine function with an amplitude of 2 and a period of $\frac{\pi}{2}$. On the grid below, sketch the graph of the equation in the interval 0 to $2\pi$.

362 a) On the axes below, sketch at least one cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of $2\pi$.

b) Explain any differences between a sketch of $y = 2 \sin \left( x - \frac{\pi}{3} \right) - \frac{3}{2}$ and the sketch from part a.
363 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form \( f(t) = A \cos(Bt) \), where \( A \) and \( B \) are real numbers, that models the water level, \( f(t) \), in inches above or below the average Carter Beach sea level, as a function of the time measured in \( t \) hours since 8:30 a.m. On the grid below, graph one cycle of this function.

People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

364 The resting blood pressure of an adult patient can be modeled by the function \( P \) below, where \( P(t) \) is the pressure in millimeters of mercury after time \( t \) in seconds.

\[ P(t) = 24\cos(3\pi t) + 120 \]

On the set of axes below, graph \( y = P(t) \) over the domain \( 0 \leq t \leq 2 \).

Determine the period of \( P \). Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.
Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function \( f(t) = -13 \cos(0.8\pi t) + 13 \), where \( t \) represents the time (in seconds) since the nail first became caught in the tire. Determine the period of \( f(t) \). Interpret what the period represents in this context. On the grid below, graph at least one cycle of \( f(t) \) that includes the \( y \)-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

**CONICS**

**G.GPE.A.1: EQUATIONS OF CIRCLES**

The equation \( 4x^2 - 24x + 4y^2 + 72y = 76 \) is equivalent to:

1. \( 4(x - 3)^2 + 4(y + 9)^2 = 76 \)
2. \( 4(x - 3)^2 + 4(y + 9)^2 = 121 \)
3. \( 4(x - 3)^2 + 4(y + 9)^2 = 166 \)
4. \( 4(x - 3)^2 + 4(y + 9)^2 = 436 \)

### GRAPHS AND STATISTICS

**S.I.C.A.2: ANALYSIS OF DATA**

Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Given the results of her coin flips and of her computer simulation, which statement is most accurate?

1. 73 of the computer's next 100 coin flips will be heads.
2. 50 of her next 100 coin flips will be heads.
3. Her coin is not fair.
4. Her coin is fair.

A game spinner is divided into 6 equally sized regions, as shown in the diagram below.

For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times, assuming the probability of winning each spin is \( \frac{1}{6} \).

The output of the simulation is shown in the diagram below.
Which explanation is appropriate for Miles and his dad to make?
1. The spinner was likely unfair, since the number 6 failed to occur in about 20% of the simulations.
2. The spinner was likely unfair, since the spinner should have landed on the number 6 by the sixth spin.
3. The spinner was likely not unfair, since the number 6 failed to occur in about 20% of the simulations.
4. The spinner was likely not unfair, since in the output the player wins once or twice in the majority of the simulations.

Based on the results of the simulation, which statement is false?
1. Five heads occurred most often, which is consistent with the theoretical probability of obtaining a heads.
2. Eight heads is unusual, as it falls outside the middle 95% of the data.
3. Obtaining three heads or fewer occurred 28% of the time.
4. Seven heads is not unusual, as it falls within the middle 95% of the data.
370 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

![Histogram of Proportion of Heads](image1)

Do the results of the simulation provide strong evidence that Robin’s coin is unfair? Explain your answer.

371 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.

![Number of Married Men Distribution](image2)

a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the nearest integer.

b) A study claims "50 percent of men 21 and older in the United States are married." Do your results from part a contradict this claim? Explain.
372 A radio station claims to its advertisers that the mean number of minutes commuters listen to the station is 30. The station conducted a survey of 500 of their listeners who commute. The sample statistics are shown below.

<table>
<thead>
<tr>
<th>( \bar{x} )</th>
<th>29.11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_x )</td>
<td>20.718</td>
</tr>
</tbody>
</table>

A simulation was run 1000 times based upon the results of the survey. The results of the simulation appear below.

Based on the simulation results, is the claim that commuters listen to the station on average 30 minutes plausible? Explain your response including an interval containing the middle 95\% of the data, rounded to the nearest hundredth.

373 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario. State one conclusion that the quality control team could make about the population if 5\% of the sample was found to be unsatisfactory.

374 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said "What are the odds I got all of that kind?" Mrs. Jones replied, "simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual." Explain how this simulation could be used to solve the problem.
S.IC.B.3: ANALYSIS OF DATA

375 Which statement(s) about statistical studies is true?
I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math.
1 I, only
2 II, only
3 I and III
4 III and IV

376 Which statement about statistical analysis is false?
1 Experiments can suggest patterns and relationships in data.
2 Experiments can determine cause and effect relationships.
3 Observational studies can determine cause and effect relationships.
4 Observational studies can suggest patterns and relationships in data.

377 Cheap and Fast gas station is conducting a consumer satisfaction survey. Which method of collecting data would most likely lead to a biased sample?
1 interviewing every 5th customer to come into the station
2 interviewing customers chosen at random by a computer at the checkout
3 interviewing customers who call an 800 number posted on the customers' receipts
4 interviewing every customer who comes into the station on a day of the week chosen at random out of a hat

378 Which scenario is best described as an observational study?
1 For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
2 A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.
3 A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
4 Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.
379 The operator of the local mall wants to find out how many of the mall's employees make purchases in the food court when they are working. She hopes to use these data to increase the rent and attract new food vendors. In total, there are 1023 employees who work at the mall. The best method to obtain a random sample of the employees would be to survey:
1. all 170 employees at each of the larger stores
2. 50% of the 90 employees of the food court
3. every employee
4. every 30th employee entering each mall entrance for one week

380 A researcher randomly divides 50 bean plants into two groups. He puts one group by a window to receive natural light and the second group under artificial light. He records the growth of the plants weekly. Which data collection method is described in this situation?
1. observational study
2. controlled experiment
3. survey
4. systematic sample

381 A random sample of 100 people that would best estimate the proportion of all registered voters in a district who support improvements to the high school football field should be drawn from registered voters in the district at a:
1. football game
2. supermarket
3. school fund-raiser
4. high school band concert

382 A sociologist reviews randomly selected surveillance videos from a public park over a period of several years and records the amount of time people spent on a smartphone. The statistical procedure the sociologist used is called:
1. a census
2. an experiment
3. an observational study
4. a sample survey

383 A veterinary pharmaceutical company plans to test a new drug to treat a common intestinal infection among puppies. The puppies are randomly assigned to two equal groups. Half of the puppies will receive the drug, and the other half will receive a placebo. The veterinarians monitor the puppies. This is an example of which study method?
1. census
2. observational study
3. survey
4. controlled experiment

384 Chuck's Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck:

Explain one statistical bias in this data collection method.

The driver who receives the highest number of positive comments will win the recognition.
Describe how a controlled experiment can be created to examine the effect of ingredient $X$ in a toothpaste.

A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the diagram below.

At a 95% confidence level, the proportion of New York City residents with hypertension and the margin of error are closest to
1. proportion $\approx .16$; margin of error $\approx .01$
2. proportion $\approx .16$; margin of error $\approx .02$
3. proportion $\approx .01$; margin of error $\approx .16$
4. proportion $\approx .02$; margin of error $\approx .16$

Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to
1. 0.01
2. 0.03
3. 0.06
4. 0.12
Stephen’s Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products \(A\), \(B\), and the new product. Nine out of fifty participants preferred Stephen’s new cola to products \(A\) and \(B\). The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen’s new product, each of sample size 50, simulated 100 times.

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.
Some smart-phone applications contain "in-app" purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.

Considering the middle 95% of the data, determine the margin of error, to the nearest hundredth, for the simulated results. In the given context, explain what this value represents.
S.IC.B.5: ANALYSIS OF DATA

Gabriel performed an experiment to see if planting 13 tomato plants in black plastic mulch leads to larger tomatoes than if 13 plants are planted without mulch. He observed that the average weight of the tomatoes from tomato plants grown in black plastic mulch was 5 ounces greater than those from the plants planted without mulch. To determine if the observed difference is statistically significant, he rerandomized the tomato groups 100 times to study these random differences in the mean weights. The output of his simulation is summarized in the dotplot below.

Given these results, what is an appropriate inference that can be drawn?

1. There was no effect observed between the two groups.
2. There was an effect observed that could be due to the random assignment of plants to the groups.
3. There is strong evidence to support the hypothesis that tomatoes from plants planted in black plastic mulch are larger than those planted without mulch.
4. There is strong evidence to support the hypothesis that tomatoes from plants planted without mulch are larger than those planted in black plastic mulch.
A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that 24% of candies in bags are blue. A simulation was run 100 times with a sample size of 50, based on the premise that 24% of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.

The simulation results in a mean of 0.254 and a standard deviation of 0.060. Based on this simulation, what is a plausible interval containing the middle 95% of the data?

1. (0.194, 0.314)
2. (0.134, 0.374)
3. (−0.448, 0.568)
4. (0.254, 0.374)

Mary bought a pack of candy. The manufacturer claims that 30% of the candies manufactured are red. In her pack, 14 of the 60 candies are red. She ran a simulation of 300 samples, assuming the manufacturer is correct. The results are shown below.

Based on the simulation, determine the middle 95% of plausible values that the proportion of red candies in a pack is within. Based on the simulation, is it unusual that Mary’s pack had 14 red candies out of a total of 60? Explain.
Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups’ final grades is shown below:

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>80.16</td>
<td>83.8</td>
</tr>
<tr>
<td>( S_x )</td>
<td>6.9</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students’ final grades were rerandomized 500 times. The results are shown below.

Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.
Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the nearest hundredth. Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.
Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

<table>
<thead>
<tr>
<th>Group 1 (seconds)</th>
<th>Group 2 (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.4</td>
<td>23.3</td>
</tr>
<tr>
<td>18.1</td>
<td>18.8</td>
</tr>
<tr>
<td>18.2</td>
<td>22.1</td>
</tr>
<tr>
<td>19.6</td>
<td>12.7</td>
</tr>
<tr>
<td>18.6</td>
<td>16.9</td>
</tr>
<tr>
<td>16.2</td>
<td>24.4</td>
</tr>
<tr>
<td>16.1</td>
<td>21.2</td>
</tr>
<tr>
<td>15.3</td>
<td>21.2</td>
</tr>
<tr>
<td>17.8</td>
<td>16.3</td>
</tr>
<tr>
<td>19.7</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Mean = 17.7  Mean = 19.1

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva’s hypothesis may be incorrect. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.

Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.
Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if 50% or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that 50% of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40, simulated 100 times.

Assume the set of data is approximately normal and the dealership wants to be 95% confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the nearest hundredth. Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is 32.5%. The dealership decides not to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

<table>
<thead>
<tr>
<th></th>
<th>Scented Paper</th>
<th>Unscented Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>23</td>
<td>18</td>
</tr>
<tr>
<td>$s_\mu$</td>
<td>2.898</td>
<td>2.408</td>
</tr>
</tbody>
</table>

Calculate the difference in means in the experimental test grades (scented - unscented). A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth. Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.
To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

Classical: 74, 83, 77, 77, 84, 82, 90, 89
Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer. To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.

Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.
S.I.C.B.6: ANALYSIS OF DATA

A public opinion poll was conducted on behalf of Mayor Ortega's reelection campaign shortly before the election. 264 out of 550 likely voters said they would vote for Mayor Ortega; the rest said they would vote for his opponent. Which statement is least appropriate to make, according to the results of the poll?

1. There is a 48% chance that Mayor Ortega will win the election.
2. The point estimate ($\hat{p}$) of voters who will vote for Mayor Ortega is 48%.
3. It is most likely that between 44% and 52% of voters will vote for Mayor Ortega.
4. Due to the margin of error, an inference cannot be made regarding whether Mayor Ortega or his opponent is most likely to win the election.

S.ID.B.6: REGRESSION

The price of a postage stamp in the years since the end of World War I is shown in the scatterplot below.

The equation that best models the price, in cents, of a postage stamp based on these data is

1. $y = 0.59x - 14.82$
2. $y = 1.04(1.43)^x$
3. $y = 1.43(1.04)^x$
4. $y = 24 \sin(14x) + 25$

Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth's wait time unusual? Justify your answer.
402 A scatterplot showing the weight, $w$, in grams, of each crystal after growing $t$ hours is shown below.

![Scatterplot](image)

The relationship between weight, $w$, and time, $t$, is best modeled by

1. $w = 4^t + 5$
2. $w = (1.4)^t + 2$
3. $w = 5(2.1)^t$
4. $w = 8(.75)^t$

403 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

<table>
<thead>
<tr>
<th>Hours (x)</th>
<th>Average Number of Spores (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>260</td>
</tr>
<tr>
<td>4</td>
<td>1130</td>
</tr>
<tr>
<td>6</td>
<td>16,380</td>
</tr>
</tbody>
</table>

Using these data, write an exponential regression equation, rounding all values to the nearest thousandth. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the nearest quarter hour, that the meat can be kept at room temperature safely.
404 A runner is using a nine-week training app to prepare for a "fun run." The table below represents the amount of the program completed, $A$, and the distance covered in a session, $D$, in miles.

<table>
<thead>
<tr>
<th>$A$</th>
<th>4/9</th>
<th>5/9</th>
<th>6/9</th>
<th>8/9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>2</td>
<td>2.25</td>
<td>3</td>
<td>3.25</td>
<td></td>
</tr>
</tbody>
</table>

Based on these data, write an exponential regression equation, rounded to the nearest thousandth, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

405 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the nearest whole percent, is

1. 6
2. 48
3. 68
4. 95

406 The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?

1. 0.3803
2. 0.4612
3. 0.8415
4. 0.9612

407 In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects 95% of the Critical Reading scores?

1. $430 \pm 115$
2. $430 \pm 230$
3. $496 \pm 115$
4. $496 \pm 230$

408 The distribution of the diameters of ball bearings made under a given manufacturing process is normally distributed with a mean of 4 cm and a standard deviation of 0.2 cm. What proportion of the ball bearings will have a diameter less than 3.7 cm?

1. 0.0668
2. 0.4332
3. 0.8664
4. 0.9500
409 There are 440 students at Thomas Paine High School enrolled in U.S. History. On the April report card, the students' grades are approximately normally distributed with a mean of 79 and a standard deviation of 7. Students who earn a grade less than or equal to 64.9 must attend summer school. The number of students who must attend summer school for U.S. History is closest to

1 3
2 5
3 10
4 22

410 The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

1 0.2257
2 0.2743
3 0.7257
4 0.7757

411 The scores on a mathematics college-entry exam are normally distributed with a mean of 68 and standard deviation 7.2. Students scoring higher than one standard deviation above the mean will not be enrolled in the mathematics tutoring program. How many of the 750 incoming students can be expected to be enrolled in the tutoring program?

1 631
2 512
3 238
4 119

412 There are 400 students in the senior class at Oak Creek High School. All of these students took the SAT. The distribution of their SAT scores is approximately normal. The number of students who scored within 2 standard deviations of the mean is approximately

1 75
2 95
3 300
4 380

413 The mean intelligence quotient (IQ) score is 100, with a standard deviation of 15, and the scores are normally distributed. Given this information, the approximate percentage of the population with an IQ greater than 130 is closest to

1 2%
2 31%
3 48%
4 95%

414 Suppose two sets of test scores have the same mean, but different standard deviations, }$\sigma_1$ and $\sigma_2$, with $\sigma_2 > \sigma_1$. Which statement best describes the variability of these data sets?

1 Data set one has the greater variability.
2 Data set two has the greater variability.
3 The variability will be the same for each data set.
4 No conclusion can be made regarding the variability of either set.

415 The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the nearest integer, weighed less than 8.25 pounds.
416 The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

417 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the nearest ten thousandth, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

PROBABILITY

S.CP.B.7: THEORETICAL PROBABILITY

418 The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

I: dependent
II: independent
III: mutually exclusive

1 I, only
2 II, only
3 I and III
4 II and III

419 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is \( \frac{974}{1376} \), what is the probability that a student participates in both sports and music?

S.CP.A.2: PROBABILITY OF COMPOUND EVENTS

420 On a given school day, the probability that Nick oversleeps is 48\% and the probability he has a pop quiz is 25\%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

1 73\%
2 36\%
3 23\%
4 12\%

421 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50\% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70\% chance that there will be an agreement on the insurance benefits. There is a 20\% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on both issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

422 Given events \( A \) and \( B \), such that \( P(A) = 0.6 \), \( P(B) = 0.5 \), and \( P(A \cup B) = 0.8 \), determine whether \( A \) and \( B \) are independent or dependent.
423 Data for the students enrolled in a local high school are shown in the Venn diagram below.

If a student from the high school is selected at random, what is the probability that the student is a sophomore given that the student is enrolled in Algebra II?

1. \( \frac{85}{210} \)
2. \( \frac{85}{295} \)
3. \( \frac{85}{405} \)
4. \( \frac{85}{1600} \)

424 A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events \( D \), that a customer uses the drive-thru, and \( F \), that a customer orders French fries, are independent. The following data are given in a report:

\[
P(F) = 0.8 \\
P(F \cap D) = 0.456
\]

Given this information, \( P(F|D) \) is

1. 0.344
2. 0.3648
3. 0.57
4. 0.8

425 Consider the probability statements regarding events \( A \) and \( B \) below.

\[
P(A \text{ or } B) = 0.3; \\
P(A \text{ and } B) = 0.2; \text{ and} \\
P(A|B) = 0.8
\]

What is \( P(B) \)?

1. 0.1
2. 0.25
3. 0.375
4. 0.667

426 Sean’s team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

1. independent
2. dependent
3. mutually exclusive
4. complements
427 Suppose events $A$ and $B$ are independent and $P(A \text{ and } B)$ is 0.2. Which statement could be true?

1. $P(A) = 0.4, P(B) = 0.3, P(A \text{ or } B) = 0.5$
2. $P(A) = 0.8, P(B) = 0.25$
3. $P(A|B) = 0.2, P(B) = 0.2$
4. $P(A) = 0.15, P(B) = 0.05$

428 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math? Are the events "the student is a junior" and "the student's favorite subject is Math" independent of each other? Explain your answer.

429 The probability that a resident of a housing community opposes spending money for community improvement on plumbing issues is 0.8. The probability that a resident favors spending money on improving walkways given that the resident opposes spending money on plumbing issues is 0.85. Determine the probability that a randomly selected resident opposes spending money on plumbing issues and favors spending money on walkways.

S.CP.A.4: CONDITIONAL PROBABILITY

430 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Text Messages per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0–10</td>
</tr>
<tr>
<td>15–18</td>
<td>4</td>
</tr>
<tr>
<td>19–22</td>
<td>6</td>
</tr>
<tr>
<td>23–60</td>
<td>25</td>
</tr>
</tbody>
</table>

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

1. $\frac{157}{229}$
2. $\frac{157}{312}$
3. $\frac{157}{384}$
4. $\frac{157}{456}$
431 The results of a poll of 200 students are shown in the table below:

<table>
<thead>
<tr>
<th>Preferred Music Style</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Techno</td>
<td>Rap</td>
<td>Country</td>
</tr>
<tr>
<td>Female</td>
<td>54</td>
<td>25</td>
<td>27</td>
</tr>
<tr>
<td>Male</td>
<td>36</td>
<td>40</td>
<td>18</td>
</tr>
</tbody>
</table>

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

432 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

<table>
<thead>
<tr>
<th>Comedy Series</th>
<th>Drama Series</th>
<th>Reality Series</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>95</td>
<td>65</td>
<td>70</td>
</tr>
<tr>
<td>Females</td>
<td>80</td>
<td>70</td>
<td>110</td>
</tr>
<tr>
<td>Total</td>
<td>175</td>
<td>135</td>
<td>180</td>
</tr>
</tbody>
</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

433 Data collected about jogging from students with two older siblings are shown in the table below.

<table>
<thead>
<tr>
<th>Neither Sibling Jogs</th>
<th>One Sibling Jogs</th>
<th>Both Siblings Jog</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Does Not Jog</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1168</td>
<td>1823</td>
<td>1380</td>
</tr>
<tr>
<td>Student Jog</td>
<td></td>
<td></td>
</tr>
<tr>
<td>188</td>
<td>416</td>
<td>400</td>
</tr>
</tbody>
</table>

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.
434 A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Favorite Type of Program</th>
<th>Sports</th>
<th>Reality Show</th>
<th>Comedy Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior</td>
<td>83</td>
<td>110</td>
<td>67</td>
</tr>
<tr>
<td>Freshmen</td>
<td>119</td>
<td>103</td>
<td>54</td>
</tr>
</tbody>
</table>

A student response is selected at random from the results. State the exact probability the student response is from a freshman, given the student prefers to watch reality shows on television.

435 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

<table>
<thead>
<tr>
<th>Practice Time</th>
<th>Juan Wins</th>
<th>Felipe Wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Practice Time</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Long Practice Time</td>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Given that the practice time was long, determine the exact probability that Filipe wins the next match. Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

436 The guidance department has reported that of the senior class, 2.3% are members of key club, $K$, 8.6% are enrolled in AP Physics, $P$, and 1.9% are in both. Determine the probability of $P$ given $K$, to the nearest tenth of a percent. The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

437 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

438 At Andrew Jackson High School, students are only allowed to enroll in AP U.S. History if they have already taken AP World History or AP European History. Out of 825 incoming seniors, 165 took AP World History, 66 took AP European History, and 33 took both. Given this information, determine the probability a randomly selected incoming senior is allowed to enroll in AP U.S. History.
Algebra II Regents Exam Questions by State Standard: Topic
Answer Section

1 ANS: 4
(1) \( \frac{B(60) - B(10)}{60 - 10} \approx 28\% \)  (2) \( \frac{B(69) - B(19)}{69 - 19} \approx 33\% \)  (3) \( \frac{B(72) - B(36)}{72 - 36} \approx 38\% \)  (4) \( \frac{B(73) - B(60)}{73 - 60} \approx 46\% \)

PTS: 2  REF: 011721aii  NAT: F.IF.B.6  TOP: Rate of Change

2 ANS: 1
\( \frac{N(10) - N(1)}{10 - 1} \approx -2.03, \quad \frac{N(20) - N(10)}{20 - 10} \approx -1.63, \quad \frac{N(25) - N(15)}{25 - 15} \approx -1.46, \quad \frac{N(30) - N(1)}{30 - 1} \approx -1.64 \)

PTS: 2  REF: 061807aii  NAT: F.IF.B.6  TOP: Rate of Change

3 ANS: 3

PTS: 2  REF: 081922aii  NAT: F.IF.B.6  TOP: Rate of Change

4 ANS: 3
\[ \log_{0.8} \left( \frac{V}{17000} \right) = t \]
\[ \frac{17,000(0.8)^3 - 17,000(0.8)^1}{3 - 1} \approx -2450 \]

\[ 0.8^t = \frac{V}{17000} \]

\[ V = 17000(0.8)^t \]

PTS: 2  REF: 081709aii  NAT: F.IF.B.6  TOP: Rate of Change

5 ANS: 3
\[ \frac{f(7) - f(-7)}{7 - (-7)} = 2^{-0.25(7)} \cdot \sin \left( \frac{\pi}{2} (7) \right) - 2^{-0.25(-7)} \cdot \sin \left( \frac{\pi}{2} (-7) \right) \]
\[ \approx -0.26 \]

PTS: 2  REF: 061721aii  NAT: F.IF.B.6  TOP: Rate of Change

6 ANS: 1
(1) \( \frac{9 - 0}{3.5 - 1} = 9 \)  (2) \( \frac{17 - 0}{3.5 - 1} = 6.8 \)  (3) \( \frac{0 - 0}{5 - 1} = 0 \)  (4) \( \frac{17 - -5}{3.5 - 1} \approx 6.3 \)

PTS: 2  REF: 011724aii  NAT: F.IF.B.6  TOP: Rate of Change
7 ANS: 
\[
\frac{306.25 - 156.25}{70 - 50} = \frac{150}{20} = 7.5 \text{ Between 50-70 mph, each additional mph in speed requires 7.5 more feet to stop.}
\]
PTS: 2 REF: 081631aii NAT: F.IF.B.6 TOP: Rate of Change

8 ANS: 
\[
\frac{13.9 - 9.4}{4 - 1} = 1.5 \text{ The average rate of change in the number of hours of daylight from January 1-April 1 is 1.5.}
\]
PTS: 2 REF: 061925aii NAT: F.IF.B.6 TOP: Rate of Change

9 ANS: 
\[
\frac{p(8) - p(4)}{8 - 4} \approx 48.78
\]
PTS: 2 REF: 081827aii NAT: F.IF.B.6 TOP: Rate of Change

10 ANS: 
\[
\frac{B(11) - B(8)}{11 - 8} \approx -10.1 \text{ The average monthly high temperature decreases 10.1º each month from August to November.}
\]
PTS: 2 REF: 011930aii NAT: F.IF.B.6 TOP: Rate of Change

11 ANS: 3
\[
-2 \left( \frac{1}{2} x^2 = -6x + 20 \right)
\]
\[
x^2 - 12x = -40
\]
\[
x^2 - 12x + 36 = -40 + 36
\]
\[
(x - 6)^2 = -4
\]
\[
x - 6 = \pm 2i
\]
\[
x = 6 \pm 2i
\]
PTS: 2 REF: fall1504aii NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: complex solutions | completing the square

12 ANS: 1
\[
x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)} = \frac{-3 \pm \sqrt{-7}}{4} = \frac{-3 \pm i\sqrt{7}}{4}
\]
PTS: 2 REF: 061612aii NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: complex solutions | quadratic formula
13 ANS: 4
\[
x = \frac{8 \pm \sqrt{(-8)^2 - 4(6)(29)}}{2(6)} = \frac{8 \pm \sqrt{-632}}{12} = \frac{8 \pm i\sqrt{4 \cdot 158}}{12} = \frac{2}{3} \pm \frac{1}{6} i\sqrt{158}
\]
PTS: 2 REF: 011711aii NAT: A.REI.B.4 TOP: Solving Quadratics KEY: complex solutions | quadratic formula

14 ANS: 4
\[
4x^2 = -98
\]
\[
x^2 = \frac{-98}{4} = -\frac{49}{2}
\]
\[
x = \pm \sqrt{-\frac{49}{2}} = \pm \frac{7i}{\sqrt{2}} = \pm \frac{7i\sqrt{2}}{2}
\]
PTS: 2 REF: 061707aii NAT: A.REI.B.4 TOP: Solving Quadratics KEY: complex solutions | taking square roots

15 ANS: 3
\[
x^2 + 2x + 1 = -5 + 1
\]
\[
(x + 1)^2 = -4
\]
\[
x + 1 = \pm 2i
\]
\[
x = -1 \pm 2i
\]
PTS: 2 REF: 081703aii NAT: A.REI.B.4 TOP: Solving Quadratics KEY: complex solutions | completing the square

16 ANS: 3
\[
x = \frac{-2 \pm \sqrt{2^2 - 4(3)(7)}}{2(3)} = \frac{-2 \pm \sqrt{-80}}{6} = \frac{-2 \pm i\sqrt{16\sqrt{5}}}{6} = \frac{1}{3} \pm \frac{2i\sqrt{5}}{3}
\]
PTS: 2 REF: 081809aii NAT: A.REI.B.4 TOP: Solving Quadratics KEY: complex solutions | quadratic formula

17 ANS: 2
\[
x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(4)}}{2(5)} = \frac{2 \pm \sqrt{-76}}{10} = \frac{2 \pm i\sqrt{4\sqrt{19}}}{10} = \frac{1}{5} \pm \frac{i\sqrt{19}}{5}
\]
PTS: 2 REF: 011905aii NAT: A.REI.B.4 TOP: Solving Quadratics KEY: complex solutions | quadratic formula
18 ANS: 4
\[wx^2 + w = 0\]
\[wx^2 = -w\]
\[x^2 = -1\]
\[x = \pm i\]

PTS: 2 REF: 061912a1i NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: complex solutions | taking square roots

19 ANS:
\[x = \frac{-5 \pm \sqrt{5^2 + 4(2)(8)}}{2(2)} = \frac{-5 \pm i\sqrt{39}}{4}\]

PTS: 2 REF: 061827a1i NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

20 ANS:
\[x^2 - 6x = -17\] The solution is imaginary because the parabola and line do not intersect.
\[x^2 - 6x + 9 = -17 + 9\]
\[(x - 3)^2 = -8\]
\[x - 3 = \pm 2i\sqrt{2}\]
\[x = 3 \pm 2i\sqrt{2}\]

PTS: 4 REF: 081936a1i NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: complex solutions | completing the square

21 ANS: 4
(1) quadratic has two roots and both are real (-2,0) and (-0.5,0), (2) \(x = \pm \sqrt{32} - 3\), (3) the real root is 3, with a multiplicity of 2, (4) \(x = \pm 4i\)

PTS: 2 REF: 011909a1i NAT: A.REI.B.4 TOP: Using the Discriminant
KEY: determine nature of roots given equation, graph, table

22 ANS: 4
If \(1 - i\) is one solution, the other is \(1 + i\). \((x - (1 - i))(x - (1 + i)) = 0\)
\[x^2 - ix - x + ix + (1 - i^2) = 0\]
\[x^2 - 2x + 2 = 0\]

PTS: 2 REF: 081601a1i NAT: A.REI.B.4 TOP: Complex Conjugate Root Theorem

23 ANS: 4
The vertex is (2,-1) and \(p = 2\). \(y = \frac{1}{4(2)} (x - 2)^2 - 1\)

PTS: 2 REF: 081619a1i NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions
A parabola with a focus of \((0,4)\) and a directrix of \(y = 2\) is sketched as follows: By inspection, it is determined that the vertex of the parabola is \((0,3)\). It is also evident that the distance, \(p\), between the vertex and the focus is 1. It is possible to use the formula \((x - h)^2 = 4p(y - k)\) to derive the equation of the parabola as follows: \((x - 0)^2 = 4(1)(y - 3)\)

\[
x^2 = 4y - 12
\]

\[
x^2 + 12 = 4y
\]

\[
\frac{x^2}{4} + 3 = y
\]

or A point \((x,y)\) on the parabola must be the same distance from the focus as it is from the directrix. For any such point \((x,y)\), the distance to the focus is \(\sqrt{(x - 0)^2 + (y - 4)^2}\) and the distance to the directrix is \(y - 2\). Setting this equal leads to:

\[
x^2 + y^2 - 8y + 16 = y^2 - 4y + 4
\]

\[
x^2 + 16 = 4y + 4
\]

\[
\frac{x^2}{4} + 3 = y
\]
28 ANS: 4
\[ \frac{5 + 9}{2} = 7, \text{ vertex: } (-2, 7); \ p = 7 - 9 = -2, \ y = \frac{1}{4(-2)} (x + 2)^2 + 7 \]

\[ y - 7 = \frac{1}{-8} (x + 2)^2 \]

\[ -8(y - 7) = (x + 2)^2 \]

PTS: 2  REF: 061821aii  NAT: G.GPE.A.2  TOP: Graphing Quadratic Functions

29 ANS: 4
The vertex is (2, 2) and \( p = 3 \). \( 3 + 2 = 5 \)

PTS: 2  REF: 081823aii  NAT: G.GPE.A.2  TOP: Graphing Quadratic Functions

30 ANS: 3
The vertex is (−3, 5) and \( p = 2 \). \( y = \frac{-1}{4(2)} (x + 3)^2 + 5 \)

PTS: 2  REF: 011914aii  NAT: G.GPE.A.2  TOP: Graphing Quadratic Functions

31 ANS:
\[ y = \frac{1}{4(2)} (x - 4)^2 - 3 \]

\[ y = \frac{-1 + -5}{2} = -3. \] The vertex is (4, −3) and \( p = 2 \).

PTS: 4  REF: 061935aii  NAT: G.GPE.A.2  TOP: Graphing Quadratic Functions

32 ANS:
The vertex of the parabola is (4, −3). The x-coordinate of the focus and the vertex is the same. Since the distance from the vertex to the directrix is 3, the distance from the vertex to the focus is 3, so the y-coordinate of the focus is 0. The coordinates of the focus are (4, 0).

PTS: 2  REF: 061930aii  NAT: G.GPE.A.2  TOP: Graphing Quadratic Functions
\[ B(t) = 750 \left( 1.16 \frac{1}{12} \right)^{12t} \approx 750(1.012)^{12t} \]

is wrong, because the growth is an annual rate that is not compounded monthly.

\[ t \approx 750(1.012)^{12} \]

\[ B(t) = 750 + 0.16 \]

\[ \frac{1}{10} \approx 0.9716 \]

\[ \left( \frac{1}{2} \right)^{\frac{1}{73.83}} \approx 0.990656 \]

\[ 1 \text{ year} = 365 \text{ days} \]

\[ 100 \left( \frac{1}{2} \right)^{\frac{d}{8}} = 100e^{\frac{k}{8}} \]

\[ \left( \frac{1}{2} \right)^{\frac{1}{8}} = e^{k} \]

\[ k \approx -0.087 \]

\[ 1.00643^{\frac{1}{12}} \approx 1.08 \]

\[ 1.04^{\frac{1}{12}} \approx 1.0032737 \]

\[ 1 + \frac{0.009}{12} = 1.00075 \]
41 ANS: 1
\[
\frac{1}{1.025^{\frac{12}{12}}} \approx 1.00206
\]
PTS: 2 REF: 081924aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions

42 ANS: 4 PTS: 2 REF: 011808aii NAT: A.SSE.B.3
TOP: Modeling Exponential Functions

43 ANS: 3
\[
1.0525^{\frac{1}{12}} \approx 1.00427
\]
PTS: 2 REF: 061621aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions

44 ANS: 4 PTS: 2 REF: 081622aii NAT: F.BF.A.1
TOP: Modeling Exponential Functions

45 ANS: 1
\[
\frac{A}{P} = e^{rt}
\]
\[
0.42 = e^{rt}
\]
\[
\ln 0.42 = \ln e^{rt}
\]
\[
-0.87 \approx rt
\]
PTS: 2 REF: 011723aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions

46 ANS: 4
\[
1.06^{\frac{1}{52}}
\]
PTS: 2 REF: 061924aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions

47 ANS: 1
\[
P(28) = 5(2)^{\frac{98}{28}} \approx 56
\]
PTS: 2 REF: 011702aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions

48 ANS: 3
\[
y = 278(0.5)^{\frac{18}{18}} \approx 0.271
\]
PTS: 2 REF: 011920aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions

49 ANS:
\[
A(t) = 100(0.5)^{\frac{7}{63}}, \text{ where } t \text{ is time in years, and } A(t) \text{ is the amount of titanium-44 left after } t \text{ years. }
\]
\[
\frac{A(10) - A(0)}{10 - 0} = \frac{89.58132 - 100}{10} = -1.041868 \text{ The estimated mass at } t = 40 \text{ is } 100 - 40(-1.041868) \approx 58.3. \text{ The actual mass is } A(40) = 100(0.5)^{\frac{40}{63}} \approx 64.3976. \text{ The estimated mass is less than the actual mass. }
\]
PTS: 6 REF: fall1517aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions
50 ANS:

\[ N(t) = 950e^{0.0475t} \]

The base is \( e \) because growth is continuous. \( N\left(\frac{36}{24}\right) \approx 1020 \)

PTS: 4 REF: 081933aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions

51 ANS: 2

The 2010 population is 110 million.

PTS: 2 REF: 061718aii NAT: F.LE.B.5 TOP: Modeling Exponential Functions

52 ANS: 1

The car lost approximately 19% of its value each year.

PTS: 2 REF: 081613aii NAT: F.LE.B.5 TOP: Modeling Exponential Functions

53 ANS: 4 PTS: 2 REF: 011805aii NAT: F.LE.B.5

TOP: Modeling Exponential Functions

54 ANS: 2 PTS: 2 REF: 061917aii NAT: F.LE.B.5

TOP: Modeling Exponential Functions

55 ANS: 3

\[ d = 10 \log \frac{6.3 \times 10^{-3}}{1.0 \times 10^{-12}} \approx 98 \]

PTS: 2 REF: 011715aii NAT: F.IF.B.4 TOP: Evaluating Logarithmic Expressions

56 ANS: 4

\[ y = 5^{-t} = \left(\frac{1}{5}\right)^t \]

PTS: 2 REF: 061615aii NAT: F.IF.C.7 TOP: Graphing Exponential Functions

57 ANS: 4

There is no \( x \)-intercept.

PTS: 2 REF: 011823aii NAT: F.IF.C.7 TOP: Graphing Exponential Functions

58 ANS: 2 PTS: 2 REF: 061802aii NAT: F.IF.C.7

TOP: Graphing Exponential Functions

59 ANS:

\[ \frac{\ln\frac{1}{2}}{1590} \]

is negative, so \( M(t) \) represents decay.

PTS: 2 REF: 011728aii NAT: F.IF.C.7 TOP: Graphing Exponential Functions
60 ANS:

61 ANS: 4
\log_2(x - 1) - 1 = 0
\log_2(x - 1) = 1
x - 1 = 2^1
x = 3

62 ANS: 1 PTS: 2 REF: 011902aii NAT: F.IF.C.7 TOP: Graphing Logarithmic Functions
63 ANS: 1

64 ANS: 2 PTS: 2 REF: 081816aii NAT: F.IF.C.7 TOP: Graphing Logarithmic Functions KEY: bimodalgraph
As $x \to -3, y \to -\infty$. As $x \to \infty, y \to \infty$.

PTS: 4  REF: 061735aii  NAT: F.IF.C.7  TOP: Graphing Logarithmic Functions

PTS: 2  REF: 011932aii  NAT: F.IF.C.7  TOP: Graphing Logarithmic Functions

PTS: 2  REF: 061927aii  NAT: F.IF.C.7  TOP: Graphing Logarithmic Functions

$5000 \left(1 + \frac{.035}{12}\right)^{12\cdot6} \approx 6166.50$

PTS: 2  REF: 081917aii  NAT: A.CED.A.1  TOP: Exponential Growth
69 ANS:

\[
720 = \frac{120000 \left( \frac{0.048}{12} \right) \left( 1 + \frac{0.048}{12} \right)^n}{\left( 1 + \frac{0.048}{12} \right)^n - 1}
\]

\[
\frac{275.2}{12} \approx 23 \text{ years}
\]

\[
720(1.004)^n - 720 = 480(1.004)^n
\]

\[
240(1.004)^n = 720
\]

\[
1.004^n = 3
\]

\[
n \log 1.004 = \log 3
\]

\[
n \approx 275.2 \text{ months}
\]

PTS: 4 REF: spr1509aii NAT: A.CED.A.1 TOP: Exponential Growth

70 ANS:

\[
A = 5000(1.045)^n
\]

\[
5000 \left( 1 + \frac{0.046}{4} \right)^{4n} - 5000(1.045)^6 \approx 6578.87 - 6511.30 \approx 67.57
\]

\[
10000 = 5000 \left( 1 + \frac{0.046}{4} \right)^{4n}
\]

\[
2 = 1.0115^{4n}
\]

\[
\log 2 = 4n \cdot \log 1.0115
\]

\[
n = \frac{\log 2}{4 \log 1.0115}
\]

\[
n \approx 15.2
\]

PTS: 6 REF: 081637aii NAT: A.CED.A.1 TOP: Exponential Growth

71 ANS:

\[
C(t) = 63000 \left( 1 + \frac{0.0255}{12} \right)^{12t}
\]

\[
63000 \left( 1 + \frac{0.0255}{12} \right)^{12t} = 100000
\]

\[
12t \log(1.002125) = \log \frac{100}{63}
\]

\[
t \approx 18.14
\]

PTS: 4 REF: 061835aii NAT: A.CED.A.1 TOP: Exponential Growth
The graph of \( A(t) \) nearly intersects the point \((17, 1000000)\).

\[
A(t) = 318000(1.07)^t \\
318000(1.07)^t = 1000000
\]

\[
1.07^t = \frac{1000}{318}
\]

\[
t \log 1.07 = \log \frac{1000}{318}
\]

\[
t = \frac{\log 1000}{\log 1.07}
\]

\[
t \approx 17
\]
75 ANS: 3
\[ e^{bt} = \frac{c}{a} \]
\[ \ln e^{bt} = \ln \frac{c}{a} \]
\[ bt \ln e = \ln \frac{c}{a} \]
\[ t = \frac{\ln \frac{c}{a}}{b} \]

PTS: 2  REF: 011813aii  NAT: F.LE.A.4  TOP: Exponential Growth

76 ANS: 1
\[ 9110 = 5000e^{30r} \]
\[ \ln \frac{911}{500} = \ln e^{30r} \]
\[ \frac{\ln 911}{500} = r \]
\[ r \approx .02 \]

PTS: 2  REF: 011810aii  NAT: F.LE.A.4  TOP: Exponential Growth

77 ANS:
\[ A = Pe^{rt} \]
\[ 135000 = 100000e^{5r} \]
\[ 1.35 = e^{5r} \]
\[ \ln 1.35 = \ln e^{5r} \]
\[ \ln 1.35 = 5r \]
\[ .06 \approx r \text{ or } 6\% \]

PTS: 2  REF: 061632aii  NAT: F.LE.A.4  TOP: Exponential Growth

78 ANS:
\[ 8.75 = 1.25x^{49} \]
\[ 7 = x^{49} \]
\[ x = \sqrt[49]{7} \approx 1.04 \]

PTS: 2  REF: 081730aii  NAT: F.LE.A.4  TOP: Exponential Growth
79 ANS:
\[ 2 = e^{0.0375t} \]
\[ t \approx 18.5 \]

PTS: 4 \hspace{1cm} REF: 081835aii \hspace{1cm} NAT: F.LE.A.4 \hspace{1cm} TOP: Exponential Growth

80 ANS:
\[ 7 = 20(0.5)^{\frac{t}{8.02}} \]
\[ \log 0.35 = \log 0.5 \]
\[ \log 0.35 = \frac{t \log 0.5}{8.02} \]
\[ \frac{8.02 \log 0.35}{\log 0.5} = t \]
\[ t \approx 12 \]

PTS: 4 \hspace{1cm} REF: 081634aii \hspace{1cm} NAT: F.LE.A.4 \hspace{1cm} TOP: Exponential Decay

81 ANS:
\[ 100 = 140 \left( \frac{1}{2} \right)^{\frac{5}{h}} \]
\[ \log \frac{100}{140} = \log \left( \frac{1}{2} \right)^{\frac{5}{h}} \]
\[ \log \frac{5}{7} = \frac{5}{h} \log \frac{1}{2} \]
\[ \log \frac{2}{7} = \log \left( \frac{1}{2} \right) \]
\[ h = \frac{5 \log \frac{1}{2}}{\log \frac{5}{7}} \approx 10.3002 \]
\[ \frac{t \log \frac{1}{2}}{10.3002} \]
\[ t = \frac{10.3002 \log \frac{2}{7}}{\log \frac{1}{2}} \approx 18.6 \]

PTS: 6 \hspace{1cm} REF: 061737aii \hspace{1cm} NAT: F.LE.A.4 \hspace{1cm} TOP: Exponential Decay
82 ANS:

\[ s(t) = 200(0.5)^{15} \]

\[ \frac{1}{10} = (0.5) \]

\[ \log \frac{1}{10} = \log(0.5) \]

\[ -1 = t \cdot \log(0.5) \]

\[ t = \frac{-15}{\log(0.5)} \approx 50 \]

PTS: 4  REF: 061934aii  NAT: F.LE.A.4  TOP: Exponential Decay

83 ANS: 4

\[ 120 = 68 + (195 - 68)e^{-0.05t} \]

\[ 52 = 127e^{-0.05t} \]

\[ \ln \frac{52}{127} = \ln e^{-0.05t} \]

\[ \ln \frac{52}{127} = -0.05t \]

\[ \ln \frac{52}{127} \]

\[ -0.05 = t \]

\[ 18 \approx t \]

PTS: 2  REF: 081918aii  NAT: F.LE.A.4  TOP: Exponential Decay

84 ANS:

\[ 100 = 325 + (68 - 325)e^{-2k} \]

\[ T = 325 - 257e^{-0.066t} \]

\[ -225 = -257e^{-2k} \]

\[ T = 325 - 257e^{-0.066(7)} \approx 163 \]

\[ k = \frac{-225}{-257} \]

\[ k \approx 0.066 \]

PTS: 4  REF: fall1513aii  NAT: F.LE.A.4  TOP: Exponential Growth
85 ANS: 4

\[ k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48 \]

\[ k^2(k^2 - 4) + 8(k^2 - 4) + 12(k^2 - 4) \]

\[(k^2 - 4)(k^2 + 8k + 12) \]

\[(k + 2)(k - 2)(k + 6)(k + 2) \]

PTS: 2 REF: fall1505aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: factoring by grouping

86 ANS: 3

\[ 2d(d^3 + 3d^2 - 9d - 27) \]

\[ 2d(d^2(d + 3) - 9(d + 3)) \]

\[ 2d(d^2 - 9)(d + 3) \]

\[ 2d(d + 3)(d - 3)(d + 3) \]

\[ 2d(d + 3)^2(d - 3) \]

PTS: 2 REF: 081615aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: factoring by grouping

87 ANS: 4

\[ m^5 + m^3 - 6m = m(m^4 + m^2 - 6) = m(m^2 + 3)(m^2 - 2) \]

PTS: 2 REF: 011703aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: higher power

88 ANS: 3

\[ 1^3 - k(1)^2 + 2(1) = 0 \]

\[ k = 3 \]

PTS: 2 REF: 061812aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: higher power

89 ANS: 2

\[ n^2\left(n^2 - 9\right) + 4n\left(n^2 - 9\right) - 12\left(n^2 - 9\right) \]

\[ \left(n^2 + 4n - 12\right)\left(n^2 - 9\right) \]

\[ (n + 6)(n - 2)(n + 3)(n - 3) \]

PTS: 2 REF: 061911aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: factoring by grouping
90. ANS: 2
\[ u = x + 2 \quad u^2 + 4u + 3 \]
\[ (u + 3)(u + 1) \]
\[ (x + 2 + 3)(x + 2 + 1) \]
\[ (x + 5)(x + 3) \]

PTS: 2  REF: 081901aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials  KEY: higher power

91. ANS: 2  PTS: 2  REF: 081904aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials  KEY: higher power

92. ANS: 3
\[ (m - 2)^2(m + 3) = (m^2 - 4m + 4)(m + 3) = m^3 + 3m^2 - 4m^2 - 12m + 4m + 12 = m^3 - m^2 - 8m + 12 \]

PTS: 2  REF: 081605aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials  KEY: factoring by grouping

93. ANS: 1
1) let \( y = x + 2 \), then \( y^2 + 2y - 8 \)
\[ (y + 4)(y - 2) \]
\[ (x + 2 + 4)(x + 2 - 2) \]
\[ (x + 6)x \]

PTS: 2  REF: 081715aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials  KEY: multivariable

94. ANS: 4
\[ x^6y^4 - 9 \]
\[ (x^3y^2 + 3)(x^3y^2 - 3)(x^2 + 4)(x^2 - 4) \]

PTS: 2  REF: 081814aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials  KEY: factoring by grouping

95. ANS:
The expression is of the form \( y^2 - 5y - 6 \) or \( (y - 6)(y + 1) \). Let \( y = 4x^2 + 5x \):
\[ \left( 4x^2 + 5x - 6 \right) \left( 4x^2 + 5x + 1 \right) \]
\[ (4x - 3)(x + 2)(4x + 1)(x + 1) \]

PTS: 2  REF: fall1512aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials  KEY: a>1

96. ANS:
\[ x^2(4x - 1) + 4(4x - 1) = (x^2 + 4)(4x - 1) \]

PTS: 2  REF: 061727aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials  KEY: factoring by grouping
97 ANS:
\[3x^3 + x^2 + 3xy + y = x^2(3x + 1) + y(3x + 1) = (x^2 + y)(3x + 1)\]

PTS: 2  REF: 011828aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials
KEY: factoring by grouping

98 ANS:
\[(x^2 - 6)(x^2 + 2)\]

PTS: 2  REF: 081825aii  NAT: A.SSE.A.2  TOP: Factoring Polynomials
KEY: higher power

99 ANS: 1

\[x^4 - 4x^3 - 9x^2 + 36x = 0\]
\[x^3(x - 4) - 9x(x - 4) = 0\]
\[(x^3 - 9x)(x - 4) = 0\]
\[x(x^2 - 9)(x - 4) = 0\]
\[x(x + 3)(x - 3)(x - 4) = 0\]
\[x = 0, \pm 3, 4\]

PTS: 2  REF: 061606aii  NAT: A.APR.B.3  TOP: Solving Polynomial Equations

100 ANS: 4  PTS: 2  REF: 081708aii  NAT: A.APR.B.3  TOP: Solving Polynomial Equations

101 ANS: 4
\[m^3 - 2m^2 + 4m - 8 = 0\]
\[m^2(m - 2) + 4(m - 2) = 0\]
\[(m^2 + 4)(m - 2) = 0\]

PTS: 2  REF: 081821aii  NAT: A.APR.B.3  TOP: Solving Polynomial Equations

102 ANS: 1
\[x^3 + 2x^2 - 9x - 18 = 0\]
\[x^3 - 9x + 2x^2 - 18 = 0\]
\[x^3 - 9x + 2x^2 - 18 = 0\]
\[x^2(x + 2) - 9(x + 2) = 0\]
\[x(x^2 - 9) + 2(x^2 - 9) = 0\]
\[x(x^2 - 9) + 2(x^2 - 9) = 0\]
\[(x + 2)(x^2 - 9) = 0\]

PTS: 2  REF: 011903aii  NAT: A.APR.B.3  TOP: Solving Polynomial Equations
103 ANS: 4
\[ f(x) = (x + 1)(x - 1)(x - 2) = (x^2 - 1)(x - 2) = x^3 - 2x^2 - x + 2 \]

PTS: 2  REF: 081921aii  NAT: A.APR.B.3  TOP: Graphing Polynomial Functions

104 ANS: 1  PTS: 2  REF: 061701aii  NAT: A.APR.B.3

TOP: Graphing Polynomial Functions

105 ANS: 4  PTS: 2  REF: 061921aii  NAT: A.APR.B.3

TOP: Graphing Polynomial Functions

106 ANS: 1
\[ x^2 + 2x + 1 = (x + 1)^2 \]

PTS: 2  REF: 011919aii  NAT: A.APR.B.3  TOP: Graphing Polynomial Functions

107 ANS: 
\[ f(x) = x^2(x + 4)(x - 3); \quad g(x) = (x + 2)^2(x + 6)(x - 1) \]

PTS: 4  REF: 011836aii  NAT: A.APR.B.3  TOP: Graphing Polynomial Functions

108 ANS: 2  PTS: 2  REF: 061620aii  NAT: F.IF.B.4

TOP: Graphing Polynomial Functions

109 ANS: 4
The maximum volume of \( p(x) = -(x + 2)(x - 10)(x - 14) \) is about 56, at \( x = 12.1 \)

PTS: 2  REF: 081712aii  NAT: F.IF.B.4  TOP: Graphing Polynomial Functions

110 ANS: 3

PTS: 2  REF: 011817aii  NAT: F.IF.B.4  TOP: Graphing Polynomial Functions

111 ANS: 1

PTS: 2  REF: 011908aii  NAT: F.IF.B.4  TOP: Graphing Polynomial Functions

112 ANS: 2  PTS: 2  REF: 081908aii  NAT: F.IF.B.4

TOP: Graphing Polynomial Functions
113 ANS:

\[16x^4 - 81 = (4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x + 3)(2x - 3).\] No, because \(\pm \frac{3i}{2}\) are roots.

PTS: 4 REF: 061933aii NAT: F.IF.B.4 TOP: Graphing Polynomial Functions

114 ANS: 1
The zeros of the polynomial are at \(-b\), and \(c\). The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as \(x\) goes to negative infinity, \(f(x)\) goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

PTS: 2 REF: spr1501aii NAT: F.IF.C.7 TOP: Graphing Polynomial Functions

115 ANS: 3
The graph shows three real zeros, and has end behavior matching the given end behavior.

PTS: 2 REF: 061604aii NAT: F.IF.C.7 TOP: Graphing Polynomial Functions

116 ANS: 2 PTS: 2 REF: 061816aii NAT: F.IF.C.7 TOP: Graphing Polynomial Functions

117 ANS:

PTS: 2 REF: 081732aii NAT: F.IF.C.7 TOP: Graphing Polynomial Functions

118 ANS:

PTS: 2 REF: 011926aii NAT: F.IF.C.7 TOP: Graphing Polynomial Functions
0 = x^2(x + 1) - 4(x + 1)
0 = (x^2 - 4)(x + 1)
0 = (x + 2)(x - 2)(x + 1)
$x = -2, -1, 2$

PTS: 4  REF: 081633aii  NAT: F.IF.C.7  TOP: Graphing Polynomial Functions

PTS: 2  REF: 011729aii  NAT: F.IF.C.7  TOP: Graphing Polynomial Functions

PTS: 2  REF: 011831aii  NAT: F.IF.C.7  TOP: Graphing Polynomial Functions
$P(x) = R(x) - C(x) = -330x^3 + 9000x^2 - 67000x + 167000$  
Least profitable at year 5 because there is a minimum in $P(x)$. Most profitable at year 13 because there is a maximum in $P(x)$.

\[ P(5) = 2(5)^3 - 3(5) + 5 = 240 \]

Since $x + 4$ is a factor of $p(x)$, there is no remainder.

Since there is no remainder when the quartic is divided by $x - 2$, this binomial is a factor.
Since there is a remainder when the cubic is divided by \(x + 4\), this binomial is not a factor.

\[
\begin{array}{c|cccc}
-4 & 1 & -11 & 16 & 84 \\
 & -4 & 60 & -304 \\
1 & -15 & 76 \\
\end{array}
\]

Any method that demonstrates 4 is a zero of \(f(x)\) confirms that \(x - 4\) is a factor, as suggested by the Remainder Theorem.

\[f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 128 - 80 - 44 - 4 = 0\]

\[0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 \quad z(x) = 6x^3 + 19x^2 - 52x + 15\]

\[0 = -750 + 25b + 260 + 15 \quad 475 = 25b\]

\[19 = b\]

\[
\begin{array}{c|cccc}
-5 & 6 & 19 & -52 & 15 \\
 & 6 & 30 & 55 & 15 \\
6 & -11 & 3 & 0 \\
\end{array}
\]

\[6x^2 - 11x + 3 = 0\]

\[(2x - 3)(3x - 1) = 0\]

\[x = \frac{3}{2}, \frac{1}{3}, -5\]
132 ANS:

\[ j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48 = 2 + 1 - 35 - 16 + 48 = 0; \ x + 1 \ is \ a \ factor \ of \ j(x); \]

\[ 2x^3 - 3x^2 - 32x + 48 = 0 \]

\[ x^2(2x - 3) - 16(2x - 3) = 0 \]

\[ \left( x^2 - 16 \right) (2x - 3) = 0 \]

\[ x = \pm 4, \frac{3}{2} \]

PTS: 4 REF: 081834aii NAT: A.APR.B.2 TOP: Remainder Theorem

133 ANS:

\[
\begin{array}{r}
2x^2 + 6x + 23 \\
\underline{x - 5} \\
2x^3 - 4x^2 - 7x - 10
\end{array}
\]

Since there is a remainder, \( x - 5 \) is not a factor.

\[ 2x^3 - 10x^2 \]

\[ 6x^2 - 7x \]

\[ 6x^2 - 30x \]

\[ 23x - 10 \]

\[ 23x - 115 \]

\[ 105 \]

PTS: 2 REF: 061627aii NAT: A.APR.B.2 TOP: Remainder Theorem

134 ANS:

\[ r(2) = -6. \ Since \ there \ is \ a \ remainder \ when \ the \ cubic \ is \ divided \ by \ x - 2, \ this \ binomial \ is \ not \ a \ factor. \]

\[
\begin{array}{c|cccc}
 & 2 & 1 & -4 & 4 & 6 \\
\hline
1 & 2 & -4 & 0 \\
\hline
 & 1 & -2 & 0 & -6
\end{array}
\]

PTS: 2 REF: 061725aii NAT: A.APR.B.2 TOP: Remainder Theorem

135 ANS:

\[ P(-2) = 60 \ Q(-2) = 0 \ (x + 2) \ is \ a \ factor \ of \ Q(x) \ since \ Q(-2) = 0. \]

PTS: 2 REF: 081929aii NAT: A.APR.B.2 TOP: Remainder Theorem

136 ANS:

\[ (x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3 \]

PTS: 2 REF: 081620aii NAT: A.APR.C.4 TOP: Polynomial Identities
\[(x - y)^2 = x^2 - 2xy + y^2 \quad (x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3\]

**PTS:** 2  
**REF:** 061902a i  
**NAT:** A.APR.C.4  
**TOP:** Polynomial Identities

**ANS:** 4

\[(x + 7)(x - 1) = x^2 + 6x - 7 = x^2 + 6x + 9 - 7 = (x + 3)^2 - 16\]

**PTS:** 2  
**REF:** 061808aiii  
**NAT:** A.APR.C.4  
**TOP:** Polynomial Identities

**ANS:** 1

\[(a + b + c)^2 = a^2 + ab + ac + ab + b^2 + bc + ac + ab + c^2\]

\[x = a^2 + b^2 + c^2 + 2(ab + bc + ac)\]

\[x = y + 2z\]

**PTS:** 2  
**REF:** 061822aiii  
**NAT:** A.APR.C.4  
**TOP:** Polynomial Identities

**ANS:**

Let \(x\) equal the first integer and \(x + 1\) equal the next. \((x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1\). \(2x + 1\) is an odd integer.

**PTS:** 2  
**REF:** fall1511aii  
**NAT:** A.APR.C.4  
**TOP:** Polynomial Identities

**ANS:**

\[\frac{x^3 + 9}{x^3 + 8} = \frac{x^3 + 8 + 1}{x^3 + 8}\]

\[\frac{x^3 + 9}{x^3 + 8} = \frac{x^3 + 9}{x^3 + 8}\]

**PTS:** 2  
**REF:** 061631aiii  
**NAT:** A.APR.C.4  
**TOP:** Polynomial Identities

**ANS:**

\[2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k \quad h = -2\]

\[-2x^2 + 8x + 5 = hx^2 - 4hx + k\]

\[k = 5\]

**PTS:** 4  
**REF:** 011733aii  
**NAT:** A.APR.C.4  
**TOP:** Polynomial Identities

**ANS:**

\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\]

\[x^4 + 2x^2y^2 + y^4 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2\]

\[x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4\]

**PTS:** 2  
**REF:** 081727aiii  
**NAT:** A.APR.C.4  
**TOP:** Polynomial Identities
145 ANS:

\[(a + b)^3 = a^3 + b^3\]  

No. Erin’s shortcut only works if \(a = 0, b = 0\) or \(a = -b.\)

\[a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3\]

\[3ab^2 + 3a^2b = 0\]

\[3ab(b + a) = 0\]

\[a = 0, b = 0, a = -b\]

PTS: 2  REF: 011927aii  NAT: A.APR.C.4  TOP: Polynomial Identities

146 ANS: 2

\[4x \cdot x^2 + 2x^3 = 4x \cdot x^2 + 6x^3 = 6\sqrt[3]{x^5}\]


KEY: with variables, index > 2

147 ANS: 3

\[x^{\frac{2}{3}} \cdot x^{\frac{5}{2}} = x^{\frac{4}{6}} \cdot x^{\frac{15}{6}} = x^{\frac{18}{6}} = x^3\]


KEY: with variables, index > 2

148 ANS: 4

\[\sqrt[3]{3x^2y} \cdot \sqrt[3]{27x^3y^2} = \frac{1}{3} xy \cdot \frac{1}{3} x^2y^3 = 3^2 x^2y^6\]

PTS: 2  REF: 081914aii  NAT: N.RN.A.2  TOP: Operations with Radicals

KEY: with variables, index > 2

149 ANS:

\[\sqrt[3]{x} \cdot \sqrt[3]{x} = x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{2}{3}} \cdot x^{\frac{2}{3}} = x^2\]


KEY: with variables, index > 2
\[
\sqrt{56 - x} = x \quad \text{--8 is extraneous.}
\]
\[
56 - x = x^2
\]
\[
0 = x^2 + x - 56
\]
\[
0 = (x + 8)(x - 7)
\]
\[
x = 7
\]

PTS: 2  REF: 061605aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: extraneous solutions

\[
\sqrt{x + 14} = \sqrt{2x + 5} + 1 \quad \sqrt{22 + 14} - \sqrt{2(22) + 5} = 1
\]
\[
x + 14 = 2x + 5 + 2\sqrt{2x + 5} + 1
\]
\[
-x + 8 = 2\sqrt{2x + 5}
\]
\[
x^2 - 16x + 64 = 8x + 20
\]
\[
x^2 - 24x + 44 = 0
\]
\[
(x - 22)(x - 2) = 0
\]
\[
x = 2, 22
\]

PTS: 2  REF: 081704aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: advanced

\[
\sqrt{x + 1} = x + 1
\]
\[
x + 1 = x^2 + 2x + 1
\]
\[
0 = x^2 + x
\]
\[
0 = x(x + 1)
\]
\[
x = -1, 0
\]

PTS: 2  REF: 011802aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: extraneous solutions
153 ANS: 3
\[ x^2 - 4x - 5 = 4x^2 - 40x + 100 \]
\[ 3x^2 - 36x + 105 = 0 \]
\[ x^2 - 12x + 35 = 0 \]
\[ (x - 7)(x - 5) = 0 \]
\[ x = 5, 7 \]

PTS: 2 REF: 081807aii NAT: A.REI.A.2 TOP: Solving Radicals
KEY: extraneous solutions

154 ANS: 2
\[ b^2 = 2b^2 - 64 \text{ is extraneous.} \]
\[ -b^2 = -64 \]
\[ b = \pm 8 \]

PTS: 2 REF: 061919aii NAT: A.REI.A.2 TOP: Solving Radicals
KEY: extraneous solutions
ANS: 

\[ B = 1.69\sqrt{30 + 4.45} - 3.49 \approx 6, \text{ which is a steady breeze.} \]

\[ 15 = 1.69\sqrt{s + 4.45} - 3.49 \]

\[ 18.49 = 1.69\sqrt{s + 4.45} \]

\[ \frac{18.49}{1.69} = \sqrt{s + 4.45} \]

\[ \left( \frac{18.49}{1.69} \right)^2 = s + 4.45 \]

\[ s = \left( \frac{18.49}{1.69} \right)^2 - 4.45 \]

\[ s \approx 115 \]

\[ 9.5 = 1.69\sqrt{s + 4.45} - 3.49 \]

\[ 12.99 = 1.69\sqrt{s + 4.45} \]

\[ \frac{12.99}{1.69} = \sqrt{s + 4.45} \]

\[ \left( \frac{12.99}{1.69} \right)^2 = s + 4.45 \]

\[ s = \left( \frac{12.99}{1.69} \right)^2 - 4.45 \]

\[ s \approx 55 \]

\[ 10.49 = 1.69\sqrt{s + 4.45} - 3.49 \]

\[ 13.98 = 1.69\sqrt{s + 4.45} \]

\[ \frac{13.98}{1.69} = \sqrt{s + 4.45} \]

\[ \left( \frac{13.98}{1.69} \right)^2 = s + 4.45 \]

\[ s = \left( \frac{13.98}{1.69} \right)^2 - 4.45 \]

\[ s \approx 64 \]
\[
\sqrt{x - 5} = -x + 7 \quad \sqrt{x - 5} = -9 + 7 = -2 \text{ is extraneous.}
\]

\[
x - 5 = x^2 - 14x + 49
\]

\[
0 = x^2 - 15x + 54
\]

\[
0 = (x - 6)(x - 9)
\]

\[
x = 6, 9
\]


\[
\left( \sqrt{2x - 7} \right)^2 = (5 - x)^2 \quad \sqrt{2(4) - 7} + 4 = 5 \quad \sqrt{2(8) - 7} + 8 = 5
\]

\[
2x - 7 = 25 - 10x + x^2
\]

\[
0 = x^2 - 12x + 32
\]

\[
0 = (x - 8)(x - 4)
\]

\[
x = 4, 8
\]


\[
\sqrt{x - 4} = -x + 6 \quad \sqrt{x - 4} = -8 + 6 = -2 \text{ is extraneous.}
\]

\[
x - 4 = x^2 - 12x + 36
\]

\[
0 = x^2 - 13x + 40
\]

\[
0 = (x - 8)(x - 5)
\]

\[
x = 5, 8
\]

ANS: 
\[\sqrt{6-2x} + x = 2x + 30 - 9 \quad \sqrt{6-2(-29)} \neq -29 + 21, \text{ so } -29 \text{ is extraneous.}\]
\[\sqrt{6-2x} = x + 21 \quad \sqrt{64} \neq -8\]
\[6 - 2x = x^2 + 42x + 441\]
\[x^2 + 44x + 435 = 0\]
\[(x + 29)(x + 15) = 0\]
\[x = -29, -15\]

PTS: 4  REF: 061833aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: extraneous solutions

ANS: 
\[3\sqrt{x} - 2x = -5 \quad 1 \text{ is extraneous.}\]
\[3\sqrt{x} = 2x - 5\]
\[9x = 4x^2 - 20x + 25\]
\[4x^2 - 29x + 25 = 0\]
\[(4x - 25)(x - 1) = 0\]
\[x = \frac{25}{4}, 1\]

PTS: 4  REF: 011936aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: extraneous solutions

ANS: 
\[0 = \sqrt{t} - 2t + 6 \quad 2\left(\frac{9}{4}\right) - 6 < 0, \text{ so } \frac{9}{4} \text{ is extraneous.}\]
\[2t - 6 = \sqrt{t}\]
\[4t^2 - 24t + 36 = t\]
\[4t^2 - 25t + 36 = 0\]
\[(4t - 9)(t - 4) = 0\]
\[t = \frac{9}{4}, 4\]
\[\left(\sqrt{1} - 2(1) + 6\right) - \left(\sqrt{3} - 2(3) + 6\right) = 5 - \sqrt{3} \approx 3.268 \quad 327 \text{ mph}\]

PTS: 6  REF: 011737aii  NAT: A.REI.A.2  TOP: Solving Radicals
KEY: context
Applying the commutative property, \( \left( \frac{1}{3^2} \right)^{\frac{1}{5}} \) can be rewritten as \( \left( 3^2 \right)^{\frac{1}{5}} \) or \( 9^{\frac{1}{5}} \). A fractional exponent can be rewritten as a radical with the denominator as the index, or \( 9^{\frac{1}{5}} = \sqrt[5]{9} \).

Rewrite \( \frac{4}{3} \) as \( \frac{1 \cdot 4}{3} \), using the power of a power rule.

The denominator of the rational exponent represents the index of a root, and the 4th root of 81 is 3 and \( 3^3 \) is 27.

The denominator of the rational exponent represents the index of a root, and the numerator of the rational exponent represents the power of the base. \( \left( \sqrt[5]{9} \right)^5 = 243 \)

\( \left( \frac{5}{3} \right)^{-\frac{1}{2}} = m^{-\frac{5}{6}} = \frac{1}{\sqrt[6]{m^5}} \)

\( \left( -\frac{54x^9}{y^4} \right)^{\frac{2}{3}} = \left( \frac{2 \cdot -27}{8} \right)^{\frac{2}{3}} \cdot \left( \frac{x^3}{y^8} \right)^{\frac{18}{3}} = \frac{2^\frac{2}{3} \cdot 9x^6}{y^2 \cdot y^2 \cdot \sqrt[3]{y^2}} = \frac{9x^6 \sqrt[3]{4}}{y^2 \cdot y^2 \sqrt[3]{y^2}} \)
\[
\frac{n}{m} = \sqrt[n]{a^5} = \frac{\frac{5}{2}}{a^{\frac{1}{2}}} = a^{\frac{3}{2}} = \sqrt{a^3}
\]

170 ANS: 4

\[
\begin{align*}
\frac{n}{m} &= \sqrt[n]{a^5} \\
&= \frac{\frac{5}{2}}{a^{\frac{1}{2}}} \\
&= a^{\frac{3}{2}} \\
&= \sqrt{a^3}
\end{align*}
\]

PTS: 2 KEY: variables

REF: 011811aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

171 ANS: 1

\[
\left(x^{\frac{3}{2}}\right)^2 = x^3
\]

PTS: 2 KEY: variables

REF: 061908aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

172 ANS:

\[
\begin{align*}
\frac{\frac{8}{3}}{x^{\frac{4}{3}}} &= x^y \\
\frac{\frac{4}{3}}{x^{\frac{1}{3}}} &= x^y \\
\frac{4}{3} &= y
\end{align*}
\]

PTS: 2 KEY: numbers

REF: spr1505aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

173 ANS:

\[
\left(\left(\frac{\frac{5}{3}}{\frac{3}{x}}\right)^{\frac{6}{5}}\right) = \left(\left(\frac{\frac{5}{6}}{\frac{5}{y}}\right)^{\frac{6}{5}}\right)
\]

\[
x^2 = y
\]

PTS: 2 KEY: variables

REF: 011730aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

174 ANS:

\[
\frac{\frac{1}{2}}{2x^{\frac{1}{2}}} = x^{\frac{1}{2}} = \sqrt{x}
\]

PTS: 2 KEY: variables

REF: 081826aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents
175 ANS: 
\[
\frac{3}{4} \sqrt[3]{x^2 y^5} = \frac{2}{x} \cdot \frac{5}{y^3} = \frac{8}{x} \cdot \frac{20}{y^{12}} = x \cdot \frac{1}{12} \cdot \frac{2}{y^3}
\]
PTS: 2
REF: 011925aii
NAT: N.RN.A.2
TOP: Radicals and Rational Exponents
KEY: variables

176 ANS: 
No. \( \sqrt[3]{x^2} \left( \sqrt[5]{x^3} \right) = x^{\frac{2}{7}} \cdot x^{\frac{3}{5}} = x^{\frac{31}{35}} = \sqrt[35]{x^{31}} \)
PTS: 2
REF: 061929aii
NAT: N.RN.A.2
TOP: Radicals and Rational Exponents
KEY: variables

177 ANS: 2
\((2 - yi)(2 - yi) = 4 - 4yi + y^2 i^2 = -y^2 - 4yi + 4\)
PTS: 2
REF: 061603aii
NAT: N.CN.A.2
TOP: Operations with Complex Numbers

178 ANS: 2
\(6xi(-4xi + 5) = -24x^2 i^4 + 30xi^3 = -24x^2(1) + 30x(-1) = -24x^2 - 30xi\)
PTS: 2
REF: 061704aii
NAT: N.CN.A.2
TOP: Operations with Complex Numbers

179 ANS: 3
\((3k - 2i)^2 = 9k^2 - 12ki + 4i^2 = 9k^2 - 12ki - 4\)
PTS: 2
REF: 081702aii
NAT: N.CN.A.2
TOP: Operations with Complex Numbers

180 ANS: 3
\((x + 3i)^2 - (2x - 3i)^2 = x^2 + 6xi + 9i^2 - \left(4x^2 - 12xi + 9i^2\right) = -3x^2 + 18xi\)
PTS: 2
REF: 061805aii
NAT: N.CN.A.2
TOP: Operations with Complex Numbers

181 ANS: 3
\(-3 + 5i - \left(4 + 24i - 2i - 12i^2\right) = -3 + 5i - (16 + 22i) = -19 - 17i\)
PTS: 2
REF: 081815aii
NAT: N.CN.A.2
TOP: Operations with Complex Numbers

182 ANS: 1
\((2x - i)^2 - (2x - i)(2x + 3i)\)
\((2x - i)[(2x - i) - (2x + 3i)]\)
\((2x - i)(-4i)\)
\(-8xi + 4i^2\)
\(-8xi - 4\)
PTS: 2
REF: 011911aii
NAT: N.CN.A.2
TOP: Operations with Complex Numbers
183 ANS: 1
\[ 6 - (3x - 2i)(3x - 2i) = 6 - \left( 9x^2 - 12xi + 4i^2 \right) = 6 - 9x^2 + 12xi + 4 = -9x^2 + 12xi + 10 \]

PTS: 2  REF: 061915aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

184 ANS:
\[
\begin{align*}
(4 - 3i)(5 + 2yi - 5 + 2yi) \\
(4 - 3i)(4yi) \\
16yi - 12yi^2 \\
12y + 16yi
\end{align*}
\]

PTS: 2  REF: spr1506aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

185 ANS:
\[
xi(-6i)^2 = xi(36i^2) = 36xi^3 = -36xi
\]

PTS: 2  REF: 081627aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

186 ANS:
\[
(1 - i)(1 - i) = (1 - 2i + i^2)(1 - i) = -2i(1 - i) = -2i + 2i^2 = -2 - 2i
\]

PTS: 2  REF: 011725aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

187 ANS:
\[
\frac{-1}{2}i^3 (3i - 4) - 3i^2 = -\frac{3}{2}i + 2i^3 - 3i^2 = -\frac{3}{2}i - 2i + 3 = \frac{3}{2} - 2i
\]

PTS: 2  REF: 081927aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

188 ANS:
\[
i^2 = -1, \text{ and not 1}; \quad 10 + 10i
\]

PTS: 2  REF: 011825aii  NAT: N.CN.A.2  TOP: Operations with Complex Numbers

189 ANS: 1
\[
x^2 + 2x - 8 = 0 \\
(x + 4)(x - 2) = 0 \\
x = -4, 2
\]

PTS: 2  REF: 081701aii  NAT: A.APR.D.6  TOP: Undefined Rationals

190 ANS: 4
\[
\frac{-3x^2 - 5x + 2}{x^3 + 2x^2} = \frac{(-3x + 1)(x + 2)}{x^2(x + 2)} = \frac{-3x}{x^2} + \frac{1}{x^2} = -3x^{-1} + x^{-2}
\]

PTS: 2  REF: 061723aii  NAT: A.APR.D.6  TOP: Expressions with Negative Exponents

KEY: variables
\[
\frac{3x^5 + 4x - 1}{2x + 3} \div \frac{6x^5 + 9x^2}{8x^2 + 16x + 8x^2 + 12x} - \frac{2x + 2}{2x - 3}
\]

ANS: 1

PTS: 2  REF: fall1503aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: division

\[
\frac{2x^2 - 3x + 7}{2x + 3} \div \frac{4x^3 + 6x^2}{4x^3 + 0x^2 + 5x + 10} - \frac{6x^2 + 5x}{-6x^2 - 9x}
\]

ANS: 2

PTS: 2  REF: 061614aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: division

\[
\frac{x^2 + 0x + 1}{x + 2} \div \frac{x^3 + 2x^2 + x + 6}{x^3 + 2x^2} - \frac{0x^2 + x}{0x^2 + 0x} - \frac{x + 6}{x + 2}
\]

ANS: 2

PTS: 2  REF: 081611aii  NAT: A.APR.D.6  TOP: Rational Expressions
KEY: division
194 ANS: 1

\[
\frac{2x^2 + x + 5}{2x - 1} = \frac{4x^3 + 0x^2 + 9x - 5}{4x^3 - 2x^2}
\]

\[
\frac{2x^2 + 9x}{2x^2 - x}
\]

\[
\frac{10x - 5}{10x - 5}
\]

PTS: 2 REF: 081713aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division

195 ANS: 4

\[
\frac{5x^2 + x - 3}{2x - 1} = \frac{10x^3 - 3x^2 - 7x + 3}{10x^3 - 5x^2}
\]

\[
\frac{2x^2 - 7x}{2x^2 - x}
\]

\[
-6x + 3
\]

PTS: 2 REF: 011809aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division

196 ANS: 3

\[
\frac{c^2 - d^2}{d^2 + cd - 2c^2} = \frac{(c + d)(c - d)}{(d + 2c)(d - c)} = \frac{-(c + d)}{d + 2c} = -\frac{c - d}{d + 2c}
\]

PTS: 2 REF: 011818aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: factoring

197 ANS: 3

\[
\frac{x^2(x + 2) - 9(x + 2)}{x(x^2 - x - 6)} = \frac{(x^2 - 9)(x + 2)}{x(x - 3)(x + 2)} = \frac{(x + 3)(x - 3)}{x(x - 3)} = \frac{x + 3}{x}
\]

PTS: 2 REF: 061803aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: factoring
198 ANS: 3

\[
\begin{align*}
2x^3 - 4x^2 - x + \frac{14}{x + 6} \\
2x^4 + 12x^3 \\
\quad - 4x^3 - 25x^2 \\
\quad - 4x^3 - 24x^2 \\
\quad - x^2 - 6x \\
\quad - x^2 - 6x
\end{align*}
\]

PTS: 2 REF: 081805aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division

199 ANS: 4

\[
\frac{x^2 - 4x}{2x} = \frac{x(x - 4)}{2x} = \frac{x - 4}{2} = \frac{x}{2} - 2 \quad \frac{x - 1}{2} - \frac{3}{2} = \frac{x - 1 - 3}{2} = \frac{x - 4}{2}
\]

PTS: 2 REF: 011921aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: factoring

200 ANS: 1

\[
\begin{align*}
\frac{3x - 1}{3x + 1} \\
9x^2 + 0x - 2 \\
\quad - 3x - 2 \\
\quad - 3x - 1 \\
\quad - 1
\end{align*}
\]

PTS: 2 REF: 081910aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division

201 ANS:

\[
\begin{align*}
\frac{3x + 13}{x - 2} \\
\quad - \frac{3x^2 + 7x - 20}{x^2 - 2} \\
\quad - 13x - 20 \\
\quad - 13x - 26 \\
\quad 6
\end{align*}
\]

PTS: 2 REF: 011732aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division
202 ANS:

\[
\begin{array}{c}
2a^2 + 5a + 2 \\
3a - 2 \quad 6a^3 + 11a^2 - 4a - 9 \\
6a^3 - 4a^2 \\
15a^2 - 4a \\
15a^2 - 10a \\
6a - 9 \\
6a - 4 \\
- 5 \\
\end{array}
\]

PTS: 2 REF: 061829aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division

203 ANS:

\[
\begin{array}{c}
x^3 + 4 \\
x + 2 \quad x^4 + 2x^3 + 4x - 10 \\
x^3 + 4 - \frac{18}{x + 2} \\
x^4 + 2x^3 \\
4x - 10 \\
4x + 8 \\
- 18 \\
\end{array}
\]

No, because there is a remainder.

PTS: 4 REF: 011934aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division

204 ANS:

\[
\frac{p(x)}{x - 1} = x^2 + 7 + \frac{5}{x - 1}
\]

\[
p(x) = x^3 - x^2 + 7x - 7 + 5 \\
p(x) = x^3 - x^2 + 7x - 2
\]

PTS: 2 REF: 061930aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division
205 ANS: 2
\[
2 - \frac{x - 1}{x + 2}
\]
\[
1 + \frac{x + 2}{x + 2} - \frac{x - 1}{x + 2}
\]
\[
1 + \frac{x + 2 - (x - 1)}{x + 2}
\]
\[
1 + \frac{3}{x + 2}
\]

PTS: 2 REF: 081907aii NAT: A.APR.D.7 TOP: Addition and Subtraction of Rationals

206 ANS: 1
\[
\frac{x}{x} - \frac{20}{x} = 8
\]
\[
x^2 - 8x - 20 = 0
\]
\[
(x - 10)(x + 2) = 0
\]
\[
x = 10, -2
\]

PTS: 2 REF: 061916aii NAT: A.CED.A.1 TOP: Modeling Rationals

207 ANS: 3 PTS: 2 REF: 061602aii NAT: A.CED.A.1
TOP: Modeling Rationals

208 ANS: 3 PTS: 2 REF: 061722aii NAT: A.CED.A.1
TOP: Modeling Rationals

209 ANS: 3 PTS: 2 REF: 061824aii NAT: A.CED.A.1
TOP: Modeling Rationals

210 ANS:
\[
\frac{1}{8} + \frac{1}{6} = \frac{1}{t_b}, \quad \frac{24t_b}{8} + \frac{24t_b}{6} = \frac{24t_b}{t_b}
\]
\[
3t_b + 4t_b = 24
\]
\[
t_b = \frac{24}{7} \approx 3.4
\]

PTS: 2 REF: 011827aii NAT: A.CED.A.1 TOP: Modeling Rationals
211 ANS: 4

\[ x(x + 7) \left[ \frac{3x + 25}{x + 7} - 5 = \frac{3}{x} \right] \]

\[ x(3x + 25) - 5x(x + 7) = 3(x + 7) \]
\[ 3x^2 + 25x - 5x^2 - 35x = 3x + 21 \]
\[ 2x^2 + 13x + 21 = 0 \]
\[ (2x + 7)(x + 3) = 0 \]
\[ x = -\frac{7}{2}, -3 \]

PTS: 2  REF: fall1501aiai  NAT: A.REI.A.2  TOP: Solving Rationals  KEY: rational solutions

212 ANS: 3

\[ \frac{1}{J} = \frac{1}{F} - \frac{1}{W} \]
\[ \frac{1}{J} = \frac{W - F}{FW} \]
\[ J = \frac{FW}{W - F} \]


213 ANS: 1

\[ \frac{2(x - 4)}{(x + 3)(x - 4)} + \frac{3(x + 3)}{(x - 4)(x + 3)} = \frac{2x - 2}{x^2 - x - 12} \]
\[ 2x - 8 + 3x + 9 = 2x - 2 \]
\[ 3x = -3 \]
\[ x = -1 \]


214 ANS: 1

\[ x - \frac{4}{x - 1} = 2 \]
\[ x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)} = \frac{3 \pm \sqrt{17}}{2} \]
\[ x(x - 1) - 4 = 2(x - 1) \]
\[ x^2 - x - 4 = 2x - 2 \]
\[ x^2 - 3x - 2 = 0 \]

\[
\frac{2}{x} = \frac{4x}{x + 3}
\]

\[
2x + 6 = 4x^2
\]

\[
4x^2 - 2x - 6 = 0
\]

\[
2\left(2x^2 - x - 3\right) = 0
\]

\[
(2x - 3)(x + 1) = 0
\]

\[
x = \frac{3}{2}, -1
\]

PTS: 2  REF: 061809aii  NAT: A.REI.A.2  TOP: Solving Rationals

\[
\frac{2}{3x + 1} = \frac{1}{x} - \frac{6x}{3x + 1} - \frac{1}{3}
\]

is extraneous.

\[
\frac{6x + 2}{3x + 1} = \frac{1}{x}
\]

\[
6x^2 + 2x = 3x + 1
\]

\[
6x^2 - x - 1 = 0
\]

\[
(2x - 1)(3x + 1) = 0
\]

\[
x = \frac{1}{2}, -\frac{1}{3}
\]

PTS: 2  REF: 011915aii  NAT: A.REI.A.2  TOP: Solving Rationals

\[
x(x - 2)\left(\frac{10}{x^2 - 2x} + \frac{4}{x} = \frac{5}{x - 2}\right)
\]

is extraneous.

\[
10 + 4(x - 2) = 5x
\]

\[
10 + 4x - 8 = 5x
\]

\[
2 = x
\]

PTS: 2  REF: 081915aii  NAT: A.REI.A.2  TOP: Solving Rationals

KEY: rational solutions
\[
\frac{2x}{x-2} \left( \frac{x}{x} \right) - \frac{11}{x} \left( \frac{x-2}{x-2} \right) = \frac{8}{x^2 - 2x}
\]

\[
2x^2 - 11x + 22 = 8
\]

\[
2x^2 - 11x + 14 = 0
\]

\[
(2x - 7)(x - 2) = 0
\]

\[
x = \frac{7}{2}, 2
\]

PTS: 2  REF: 061719aii  NAT: A.REI.A.2  TOP: Solving Rationals

\[
\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}
\]

\[
\frac{3-x}{3x} = -\frac{1}{3x}
\]

\[
3 - x = -1
\]

\[
x = 4
\]

PTS: 2  REF: 061625aii  NAT: A.REI.A.2  TOP: Solving Rationals

KEY: rational solutions

\[
\frac{3p}{p-5} = \frac{p+2}{p+3}
\]

\[
3p^2 + 9p = p^2 - 3p - 10
\]

\[
2p^2 + 12p + 10 = 0
\]

\[
p^2 + 6p + 5 = 0
\]

\[
(p + 5)(p + 1) = 0
\]

\[
p = -5, -1
\]

PTS: 4  REF: 081733aii  NAT: A.REI.A.2  TOP: Solving Rationals

KEY: rational solutions
221 ANS: 
\[-6(x + 3)\left(\frac{-3}{x + 3} - \frac{x}{6} + 1 = 0\right)\]

18 + x(x + 3) – 6(x + 3) = 0
18 + x^2 + 3x - 6x - 18 = 0
\[x^2 - 3x = 0\]
\[x(x - 3) = 0\]
\[x = 0, 3\]

PTS: 2 REF: 081829aii NAT: A.REI.A.2 TOP: Solving Rationals KEY: rational solutions

222 ANS: 
\[\frac{7}{2x} - \frac{2}{x + 1} = \frac{1}{4}\]
\[\frac{7x + 7 - 4x}{2x^2 + 2x} = \frac{1}{4}\]
\[2x^2 + 2x = 12x + 28\]
\[x^2 - 5x - 14 = 0\]
\[(x - 7)(x + 2) = 0\]
\[x = 7, -2\]


223 ANS: 4
3x – (–2x + 14) = 16 3(6) – 4z = 2
\[5x = 30\]
\[-4z = -16\]
\[x = 6\]
\[z = 4\]

PTS: 2 REF: 011803aii NAT: A.REI.C.6 TOP: Solving Linear Systems KEY: three variables

224 ANS: 2
Combining (1) and (3): –6c = –18  Combining (1) and (2): 5a + 3c = –1  Using (3): –(–2) – 5b – 5(3) = 2
\[c = 3\]
\[5a + 3(3) = -1\]
\[2 - 5b - 15 = 2\]
\[5a = -10\]
\[b = -3\]
\[a = -2\]

PTS: 2 REF: 081623aii NAT: A.REI.C.6 TOP: Solving Linear Systems KEY: three variables
225 ANS: 2
\[\begin{align*}
2x + 2y - 2z &= 12 \\
5y - 4z &= 31 \\
x + 3 - (-4) &= 6 \\
-x + 4y - z &= 17 \\
2x - 3y + 2z &= -19 \\
5y - 2z &= 23
\end{align*}\]
\[\begin{align*}
x &= -1 \\
5y - 2z &= 23 \\
5y - 4z &= 31 \\
-2z &= 8 \\
y &= 3 \\
z &= -4
\end{align*}\]

PTS: 2 REF: 061923aii NAT: A.REI.C.6 TOP: Solving Linear Systems
KEY: three variables

226 ANS:
\[\begin{align*}
6x - 3y + 2z &= -10 \\
x + 3y + 5z &= 45 \\
4x + 10z &= 62 \\
4x + 4(7) &= 20 \\
6(-2) - 3y + 2(7) &= -10 \\
-2x + 3y + 8z &= 72 \\
6x - 3y + 2z &= -10 \\
4x + 4z &= 20 \\
4x &= -8 \\
-3y &= -12 \\
4x + 10z &= 62 \\
7x + 7z &= 35 \\
6z &= 42 \\
x &= -2 \\
y &= 4 \\
4x + 4z &= 20 \\
z &= 7
\end{align*}\]

PTS: 4 REF: spr1510aii NAT: A.REI.C.6 TOP: Solving Linear Systems
KEY: three variables

227 ANS:
\[\begin{align*}
x + y + z &= 1 \\
x + y + z &= 1 \\
x + y + z &= 1 \\
-2z - z &= 3 \\
y - (-1) &= 3 \\
x + 2 = 1 &= 1
\end{align*}\]
\[\begin{align*}
x + 2y + 3z &= 1 \\
x + 2y + 3z &= 1 \\
x + 3y - 5z &= 11 \\
-3z &= 3 \\
y &= 2 \\
x &= 0
\end{align*}\]
\[\begin{align*}
-x + 3y - 5z &= 11 \\
y + 2z &= 0 \\
4y - 4z &= 12 \\
z &= -1 \\
y &= -2z \\
y - z &= 3
\end{align*}\]

PTS: 4 REF: 061733aii NAT: A.REI.C.6 TOP: Solving Linear Systems
KEY: three variables

228 ANS:
\[\begin{align*}
4x + 6y - 8z &= -2 \\
4x + 6y - 8z &= -2 \\
4x - 8y + 20z &= 12 \\
z + 2 &= 3z - 4 \\
y &= 3 + 2 \\
-4x + 5 + 3 &= 16
\end{align*}\]
\[\begin{align*}
4x - 8y + 20z &= 12 \\
-4x + y + z &= 16 \\
-4x + y + z &= 16 \\
6 &= 2z \\
-4x &= 8
\end{align*}\]
\[\begin{align*}
-4x + y + z &= 16 \\
7y - 7z &= 14 \\
-7y + 21z &= 28 \\
z &= 3 \\
x &= -2
\end{align*}\]
\[\begin{align*}
y - z &= 2 \\
y - 3z &= -4 \\
y &= z + 2 \\
y &= 3z - 4
\end{align*}\]

PTS: 4 REF: 081833aii NAT: A.REI.C.6 TOP: Solving Linear Systems
KEY: three variables
229 ANS:
\[
\begin{align*}
    a + 4b + 6c &= 23 & a + 2b + c &= 2 & 8b + 3c &= 16 & 2b + 5(4) &= 21 & a + 4\left( \frac{1}{2} \right) + 6(4) &= 23 \\
    a + 2b + c &= 2 & -a + 6b + 2c &= 14 & 8b + 20c &= 84 & 2b &= 1 & a + 2 + 4 &= 23 \\
    2b + 5c &= 21 & 8b + 3c &= 16 & 17c &= 68 & b &= \frac{1}{2} & a &= -3 \\
    c &= 4
\end{align*}
\]

PTS: 4  REF: 011933aii  NAT: A.REI.C.6  TOP: Solving Linear Systems

KEY: three variables

230 ANS: 4
\[
y = g(x) = (x - 2)^2 \\
(x - 2)^2 = 3x - 2 & \quad y = 3(6) - 2 = 16 \\
x^2 - 4x + 4 = 3x - 2 & \quad y = 3(1) - 2 = 1 \\
x^2 - 7x + 6 = 0 \\
(x - 6)(x - 1) = 0 \\
x = 6, 1
\]

PTS: 2  REF: 011705aii  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems

231 ANS: 3
\[
(x + 4)^2 - 10 = 3x + 6 & \quad y = 3(-5) + 6 = -9 \\
x^2 + 8x + 16 - 10 = 3x + 6 & \quad y = 3(0) + 6 = 6 \\
x^2 + 5x = 0 \\
x(x + 5) = 0 \\
x = -5, 0
\]

PTS: 2  REF: 061903aii  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems

232 ANS: 3
\[
x^2 + (2x)^2 = 5 & \quad y = 2x = \pm 2 \\
x^2 + 4x^2 = 5 \\
5x^2 = 5 \\
x = \pm 1
\]

PTS: 2  REF: 081916aii  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems

233 ANS: 1
\[
(x + 3)^2 + (2x - 4)^2 = 8 & \quad b^2 - 4ac \\
x^2 + 6x + 9 + 4x^2 - 16x + 16 = 8 & \quad 100 - 4(5)(17) < 0 \\
5x^2 - 10x + 17 = 0
\]

PTS: 2  REF: 081719aii  NAT: A.REI.C.7  TOP: Quadratic-Linear Systems
\[ x^2 + 4x - 1 = x - 3 \quad y + 3 = -1 \]
\[ x^2 + 3x + 2 = 0 \quad y = -4 \]
\[(x + 2)(x + 1) = 0 \]
\[ x = -2, -1 \]

\[ -2x + 1 = -2x^2 + 3x + 1 \]
\[ 2x^2 - 5x = 0 \]
\[ x(2x - 5) = 0 \]
\[ x = 0, \frac{5}{2} \]

\[ y = -x + 5 \quad y = -7 + 5 = -2 \]
\[ (x - 3)^2 + (-x + 5 + 2)^2 = 16 \quad y = -3 + 5 = 2 \]
\[ x^2 - 6x + 9 + x^2 - 14x + 49 = 16 \]
\[ 2x^2 - 20x + 42 = 0 \]
\[ x^2 - 10x + 21 = 0 \]
\[ (x - 7)(x - 3) = 0 \]
\[ x = 7, 3 \]
237 ANS:

\[ x^2 + (x - 28)^2 = 400 \]
\[ y = 12 - 28 = -16 \quad y = 16 - 28 = -12 \]
\[ x^2 + x^2 - 56x + 784 = 400 \]
\[ 2x^2 - 56x + 384 = 0 \]
\[ x^2 - 28x + 192 = 0 \]
\[ (x - 16)(x - 12) = 0 \]
\[ x = 12, 16 \]

PTS: 2 | REF: 081831a | NAT: A.REI.C.7 | TOP: Quadratic-Linear Systems

238 ANS: 3

\[ -33t^2 + 360t = 700 + 5t \]
\[ -33t^2 + 355t - 700 = 0 \]
\[ t = \frac{-355 \pm \sqrt{355^2 - 4(-33)(-700)}}{2(-33)} \approx 3, 8 \]

PTS: 2 | REF: 081606a | NAT: A.REI.D.11 | TOP: Quadratic-Linear Systems

239 ANS: 2
Algebra II Regents Exam Questions by State Standard: Topic
Answer Section

240 ANS: 2

PTS: 2  REF: 011712aii  NAT: A.REI.D.11  TOP: Other Systems

241 ANS: 1  PTS: 2  REF: 011814aii  NAT: A.REI.D.11
TOP: Other Systems

242 ANS: 3  PTS: 2  REF: 081819aii  NAT: A.REI.D.11
TOP: Other Systems

243 ANS: 4

PTS: 2  REF: 011924aii  NAT: A.REI.D.11  TOP: Other Systems

244 ANS: 2

PTS: 2  REF: 081920aii  NAT: A.REI.D.11  TOP: Other Systems
245 ANS: 4

246 ANS: 2

247 ANS: 2

248 ANS: 1

\[ 1240(1.06)^x = 890(1.11)^x \]

\[ x \approx 7 \]

249 ANS: 4

PTS: 2 REF: 061814aii NAT: A.REI.D.11 TOP: Other Systems
250 ANS:
\[20e^{0.05t} = 30e^{0.03t}\]
\[\frac{2}{3}e^{0.05t} = \frac{e^{0.03t}}{e^{0.05t}}\]
\[\ln\frac{2}{3} = \ln e^{-0.02t}\]
\[\ln\frac{2}{3} = -0.02t \ln e\]
\[\frac{\ln\frac{2}{3}}{-0.02} = t\]
\[20.3 \approx t\]

PTS: 2  REF: 011829aii  NAT: A.REI.D.11  TOP: Other Systems

251 ANS:

PTS: 2  REF: fall1510aii  NAT: A.REI.D.11  TOP: Other Systems

252 ANS:
\[P(16) = \log(16 - 4) \approx 1.1, \quad 14000\]

PTS: 6  REF: 061837aii  NAT: A.REI.D.11  TOP: Other Systems
253 ANS: 

\[ A(t) = 800e^{-0.347t} \]

\[ B(t) = 400e^{-0.231t} \]

\[ 800e^{-0.347t} = 400e^{-0.231t} \]

\[ \ln 2 + \ln e^{-0.347t} = \ln e^{-0.231t} \]

\[ \ln 2 - 0.347t = -0.231t \]

\[ \ln 2 \approx 0.116t \]

\[ 6 \approx t \]

PTS: 2 REF: 081932aii NAT: A.REI.D.11 TOP: Other Systems

254 ANS: 

\[ A(t) = 800e^{-0.347t} \]

\[ B(t) = 400e^{-0.231t} \]

\[ 800e^{-0.347t} = 400e^{-0.231t} \]

\[ \ln 2 + \ln e^{-0.347t} = \ln e^{-0.231t} \]

\[ \ln 2 - 0.347t = -0.231t \]

\[ \ln 2 \approx 0.116t \]

\[ 6 \approx t \]

PTS: 6 REF: 061637aii NAT: A.REI.D.11 TOP: Other Systems
At 1.95 years, the value of the car equals the loan balance. Zach can cancel the policy after 6 years.

\[ x(30 - 0.01x) - (0.15x^3 + 0.01x^2 + 2x + 120) = 30x - 0.01x^2 - 0.15x^3 - 0.01x^2 - 2x - 120 \]

\[ = -0.15x^3 - 0.02x^2 + 28x - 120 \]

\[ p(x) = r(x) - c(x) \]

\[ -0.5x^2 + 250x - 300 = -0.3x^2 + 150x - c(x) \]

\[ c(x) = 0.2x^2 - 100x + 300 \]

\[ \frac{m(c)}{g(c)} = \frac{c + 1}{1 - c^2} = \frac{c + 1}{(1 + c)(1 - c)} = \frac{1}{1 - c} \]

\[ (2x^2 + x - 3) \cdot (x - 1) - \left[ (2x^2 + x - 3) + (x - 1) \right] \]

\[ (2x^3 - 2x^2 + x^2 - x - 3x + 3) - (2x^2 + 2x - 4) \]

\[ 2x^3 - 3x^2 - 6x + 7 \]
ANS: 3  PTS: 2  REF: 061906aii  NAT: F.LE.A.2  TOP: Families of Functions

ANS: 1  PTS: 2  REF: 081903aii  NAT: F.LE.A.2  TOP: Families of Functions

ANS: 
\[
\frac{f(4) - f(-2)}{4 - -2} = \frac{80 - 1.25}{6} = 13.125
\]
g(x) has a greater rate of change

\[
\frac{g(4) - g(-2)}{4 - -2} = \frac{179 - -49}{6} = 38
\]

PTS: 4  REF: 061636aii  NAT: F.IF.C.9  TOP: Comparing Functions

ANS:
0 = \log_{10} (x - 4)  The x-intercept of h is (2,0).  f has the larger value.

\[
10^0 = x - 4
\]

\[
1 = x - 4
\]

\[
x = 5
\]

PTS: 2  REF: 081630aii  NAT: F.IF.C.9  TOP: Comparing Functions

ANS:
q has the smaller minimum value for the domain [-2,2].  h's minimum is \(-1 (2(-1) + 1)\) and q's minimum is -8.

PTS: 2  REF: 011830aii  NAT: F.IF.C.9  TOP: Comparing Functions

ANS: 4
The maximum of p is 5.  The minimum of f is \(-\frac{21}{4} (x = \frac{-6}{2(4)} = -\frac{3}{4}

\[
f \left( \frac{3}{4} \right) = 4 \left( \frac{3}{4} \right)^2 + 6 \left( \frac{3}{4} \right) - 3 = 4 \left( \frac{9}{16} \right) - \frac{18}{4} - \frac{12}{4} = \frac{21}{4} \right) - \frac{20}{4} \left( - \frac{21}{4} \right) = \frac{41}{4} = 10.25
\]

PTS: 2  REF: 011922aii  NAT: F.IF.C.9  TOP: Comparing Functions

ANS: 1  PTS: 2  REF: 081804aii  NAT: F.IF.C.9  TOP: Comparing Functions

TOP: Relating Graphs to Events

TOP: Transformations with Functions

ANS: 3
f(x) = -f(x), so f(x) is odd.  g(-x) \neq g(x), so g(x) is not even.  g(-x) \neq -g(x), so g(x) is not odd.  h(-x) = h(x), so h(x) is even.

PTS: 2  REF: fall1502aii  NAT: F.BF.B.3  TOP: Even and Odd Functions
The graph of $y = \sin x$ is unchanged when rotated $180^\circ$ about the origin.

$$f(x) = f(-x)$$

$$x^2 - 4 = (-x)^2 - 4$$

$$x^2 - 4 = x^2 - 4$$

Since $f(x) = f(-x)$, the function is even.

$$j(-x) = (-x)^4 - 3(-x)^2 - 4 = x^2 - 3x^2 - 4$$

$$x = \frac{3}{4}y + 2$$

$$-4x = 3y - 8$$

$$-4x + 8 = 3y$$

$$\frac{4}{3}x + \frac{8}{3} = y$$

$$x = \frac{y + 1}{y - 2}$$

$$xy - 2x = y + 1$$

$$xy - y = 2x + 1$$

$$y(x - 1) = 2x + 1$$

$$y = \frac{2x + 1}{x - 1}$$
281 ANS: 2
\[ x = -6(y - 2) \]
\[ \frac{x}{6} = y - 2 \]
\[ \frac{x}{6} + 2 = y \]

PTS: 2  REF: 011821aii  NAT: F.BF.B.4  TOP: Inverse of Functions  KEY: linear

282 ANS: 3
\[ y = x^3 - 2 \]
\[ x = y^3 - 2 \]
\[ x + 2 = y^3 \]
\[ \sqrt[3]{x + 2} = y \]

PTS: 2  REF: 061815aii  NAT: F.BF.B.4  TOP: Inverse of Functions  KEY: other

283 ANS: 2
\[ y = \frac{1}{2}x + 8 \]
\[ x = \frac{1}{2}y + 8 \]
\[ 2x = y + 16 \]
\[ y = 2x - 16 \]

PTS: 2  REF: 081806aii  NAT: F.BF.B.4  TOP: Inverse of Functions  KEY: linear

284 ANS: 3  PTS: 2  REF: 011917aii  NAT: F.BF.B.4  TOP: Inverse of Functions  KEY: other

285 ANS: 2
\[ x = 4y + 5 \]
\[ x - 5 = 4y \]
\[ \frac{1}{4} x - \frac{5}{4} = y \]

PTS: 2  REF: 061909aii  NAT: F.BF.B.4  TOP: Inverse of Functions  KEY: linear
\[ x = \frac{y}{y+2} \]

\[ xy + 2x = y \]
\[ xy - y = -2x \]
\[ y(x - 1) = -2x \]
\[ y = \frac{-2x}{x - 1} \]

\[ x = \left(y - 3\right)^3 + 1 \]
\[ x - 1 = \left(y - 3\right)^3 \]
\[ \sqrt[3]{x - 1} = y - 3 \]
\[ \sqrt[3]{x - 1} + 3 = y \]
\[ f^{-1}(x) = \sqrt[3]{x - 1} + 3 \]

\[ a_1 = 4 \]
\[ a_n = 3a_{n-1} \]
293 ANS:
\[ a_1 = 4 \quad a_8 = 639 \]
\[ a_n = 2a_{n-1} + 1 \]

PTS: 2   REF: 081729aii   NAT: F.LE.A.2   TOP: Sequences
KEY: recursive

294 ANS:
\[ a_n = x^{n-1}(x + 1) \quad x^{n-1} = 0 \quad x + 1 = 0 \]
\[ x = 0 \quad x = -1 \]

PTS: 4   REF: spr1511aii   NAT: F.LE.A.2   TOP: Sequences
KEY: recursive

295 ANS: 2
\[ 121(b)^2 = 64 \quad 64 \left(\frac{8}{11}\right)^2 \approx 34 \]
\[ b = \frac{8}{11} \]

PTS: 2   REF: 011904aii   NAT: F.IF.A.3   TOP: Sequences
KEY: explicit

296 ANS: 1
\[ d = 18; \quad r = \pm \frac{5}{4} \]

PTS: 2   REF: 011714aii   NAT: F.IF.A.3   TOP: Sequences
KEY: explicit

297 ANS:
\[ a_1 = 3 \quad a_2 = 7 \quad a_3 = 15 \quad a_4 = 31; \] No, because there is no common ratio: \( \frac{7}{3} \neq \frac{15}{7} \)

PTS: 2   REF: 061830aii   NAT: F.IF.A.3   TOP: Sequences
KEY: recursive

298 ANS: 3   PTS: 2   REF: 061623aii   NAT: F.BF.A.2   TOP: Sequences

299 ANS: 4   PTS: 2   REF: 081624aii   NAT: F.BF.A.2   TOP: Sequences

300 ANS: 3   PTS: 2   REF: 011824aii   NAT: F.BF.A.2   TOP: Sequences

301 ANS: 3   PTS: 2   REF: 081724aii   NAT: F.BF.A.2   TOP: Sequences


303 ANS: 4
The scenario represents a decreasing geometric sequence with a common ratio of 0.80.

PTS: 2   REF: 061610aii   NAT: F.BF.A.2   TOP: Sequences
\begin{align*}
a_1 &= 2.5 + 0.5(1) = 3 \\
\text{PTS: 2} & \quad \text{REF: 011916aii} \quad \text{NAT: F.BF.A.2} \quad \text{TOP: Sequences} \\
305 & \quad \text{ANS: 3} \\
\text{PTS: 2} & \quad \text{REF: 061910aii} \quad \text{NAT: F.BF.A.2} \\
\text{TOP: Sequences} \\
306 & \quad \text{ANS:} \\
\frac{9}{6} &= 1.5 \quad a_1 = 6 \\
\frac{a_n - a_{n-1}}{a_n} &= 1.5 \\
\text{PTS: 2} & \quad \text{REF: 061931aii} \quad \text{NAT: F.BF.A.2} \quad \text{TOP: Sequences} \\
307 & \quad \text{ANS:} \\
\frac{6.25 - 2.25}{21 - 5} &= \frac{4}{16} = 0.25 \text{ fine per day. } 2.25 - 5(0.25) = \$1 \text{ replacement fee. } a_n = 1.25 + (n - 1)(0.25). \quad a_{60} = \$16 \\
\text{PTS: 4} & \quad \text{REF: 081734aii} \quad \text{NAT: F.BF.A.2} \quad \text{TOP: Sequences} \\
308 & \quad \text{ANS:} \\
\text{Jillian’s plan, because distance increases by one mile each week. } a_1 = 10 \quad a_n = n + 12 \\
a_n &= a_{n-1} + 1 \\
\text{PTS: 4} & \quad \text{REF: 011734aii} \quad \text{NAT: F.BF.A.2} \quad \text{TOP: Sequences} \\
309 & \quad \text{ANS: 1} \\
\text{PTS: 2} & \quad \text{REF: 081609aii} \quad \text{NAT: F.BF.B.6} \\
\text{TOP: Sigma Notation} \quad \text{KEY: represent} \\
310 & \quad \text{ANS: 1} \\
\text{PTS: 2} & \quad \text{REF: 081813aii} \quad \text{NAT: A.SSE.B.4} \\
\text{TOP: Series} \\
311 & \quad \text{ANS: 4} \\
d &= 32(0.8)^{b-1} \quad S_n = \frac{32 - 32(0.8)^{12}}{1 - 0.8} \approx 149 \\
\text{PTS: 2} & \quad \text{REF: 081721aii} \quad \text{NAT: A.SSE.B.4} \quad \text{TOP: Series} \\
312 & \quad \text{ANS: 4} \\
S_7 = \frac{85000 - 85000(1.06)^7}{1 - 1.06} \approx 713476.20 \\
\text{PTS: 2} & \quad \text{REF: 061905aii} \quad \text{NAT: A.SSE.B.4} \quad \text{TOP: Series} \\
313 & \quad \text{ANS: 2} \\
S_{20} = \frac{0.01 - 0.01(3)^{20}}{1 - 3} = 17,433,922 \\
\text{PTS: 2} & \quad \text{REF: 011822aii} \quad \text{NAT: A.SSE.B.4} \quad \text{TOP: Series} \\
\end{align*}
314 ANS: 3
\[ 8r^3 = 216 \quad S_{12} = \frac{8 - 8(3)^{12}}{1 - 3} = 2125760 \]
\[ r^3 = 27 \]
\[ r = 3 \]

PTS: 2    REF: 081902aii    NAT: A.SSE.B.4    TOP: Series

315 ANS: 2    PTS: 2    REF: 061724aii    NAT: A.SSE.B.4

TOP: Series

316 ANS:
\[ S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04} \quad S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04} \approx 660778.39 \]

PTS: 4    REF: 061634aii    NAT: A.SSE.B.4    TOP: Series

317 ANS:
\[ 20000 = PMT \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right) \quad 21000 - x = 300 \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right) \]
\[ PMT \approx 400.76 \quad x \approx 6028 \]

PTS: 4    REF: 011736aii    NAT: A.SSE.B.4    TOP: Series

318 ANS:
\[ M = 172600 \cdot \frac{0.00305(1 + 0.00305)^{12 \cdot 15}}{(1 + 0.00305)^{12 \cdot 15} - 1} \approx 1247 \quad 1100 = (172600 - x) \cdot \frac{0.00305(1 + 0.00305)^{12 \cdot 15}}{(1 + 0.00305)^{12 \cdot 15} - 1} \]
\[ 1100 \approx (172600 - x) \cdot (0.007228) \]
\[ 152193 \approx 172600 - x \]
\[ 20407 \approx x \]

PTS: 4    REF: 061734aii    NAT: A.SSE.B.4    TOP: Series

319 ANS:
\[ M = \frac{(152500 - 15250) \left( \frac{0.036}{12} \right) \left( 1 + \frac{0.036}{12} \right)^{360}}{\left( 1 + \frac{0.036}{12} \right)^{360} - 1} \approx 624 \]

PTS: 2    REF: 061831aii    NAT: A.SSE.B.4    TOP: Series

320 ANS:
\[ S_{10} = \frac{15 - 15(1.03)^{10}}{1 - 1.03} \approx 171.958 \]

PTS: 2    REF: 011929aii    NAT: A.SSE.B.4    TOP: Series
321  ANS: 1  PTS: 2  REF: 081616aii  NAT: F.TF.A.1  TOP: Unit Circle  KEY: bimodalgraph

322  ANS: 1  PTS: 2  REF: 011815aii  NAT: F.TF.A.2  TOP: Unit Circle

323  ANS:

$$t^2 + \left( \frac{4}{7} \right)^2 = 1$$

$$t^2 + \frac{16}{49} = \frac{49}{49}$$

$$t^2 = \frac{33}{49}$$

$$t = \pm \frac{\sqrt{33}}{7}$$

324  ANS:

csc $\theta = \frac{1}{\sin \theta}$, and $\sin \theta$ on a unit circle represents the $y$ value of a point on the unit circle. Since $y = \sin \theta$, 

csc $\theta = \frac{1}{y}$.


326  ANS: 1

A reference triangle can be sketched using the coordinates $(-4,3)$ in the second quadrant to find the value of $\sin \theta$.

327  ANS: 1  PTS: 2  REF: 011931aii  NAT: F.TF.A.2  TOP: Unit Circle

328  ANS: 4  PTS: 2  REF: 011727aii  NAT: F.TF.A.2  TOP: Reciprocal Trigonometric Relationships

329  ANS: 1  PTS: 2  REF: spr1503aii  NAT: F.TF.A.2  TOP: Determining Trigonometric Functions  KEY: extension to reals
327 ANS: 1

\[ \frac{\cos \theta}{\frac{6}{10}} = \frac{3}{5} \]

PTS: 2 REF: 061617aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions KEY: extension to reals

328 ANS: 2 PTS: 2 REF: 011804aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions KEY: radians

329 ANS: 3

\[ T(19) = 8 \sin(0.3(19) - 3) + 74 \approx 77 \]

PTS: 2 REF: 061922aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions KEY: radians

330 ANS:

\[ \frac{-1}{\sqrt{2^2 + (-1)^2}} = -\frac{1}{\sqrt{5}} \]

PTS: 2 REF: 061832aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions KEY: extension to reals

331 ANS: 2

\[ \cos \theta = \pm \sqrt{1 - \left(\frac{-\sqrt{2}}{5}\right)^2} = \pm \sqrt{\frac{25}{25} - \frac{2}{25}} = \pm \frac{\sqrt{23}}{5} \]

PTS: 2 REF: 061712aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions

332 ANS: 2

If \( \cos \theta = \frac{7}{25} \), \( \sin \theta = \pm \frac{24}{25} \), and \( \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{24}{7}}{\frac{25}{7}} = -\frac{24}{7} \)

PTS: 2 REF: 081811aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions

333 ANS: 1

\[ -\sqrt{1 - \left(\frac{3}{4}\right)^2} = -\sqrt{\frac{16}{16} - \frac{9}{16}} = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4} \]

PTS: 2 REF: 081905aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions
334 ANS:
\[ \sin^2 \theta + (-0.7)^2 = 1 \]
Since \( \theta \) is in Quadrant II, \( \sin \theta = \sqrt{0.51} \) and \( \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{0.51}}{-0.7} \approx -1.02 \)
\[ \sin^2 \theta = 0.51 \]
\[ \sin \theta = \pm \sqrt{0.51} \]

PTS: 2 REF: 081628aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions

335 ANS:
\[ \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-7}{25} \]
\[ \cos \theta = \frac{-24}{25} \]

PTS: 2 REF: 061928aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions

336 ANS: 1 PTS: 2 REF: 011704aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions

337 ANS: 1 PTS: 2 REF: 061708aii NAT: F.TF.B.5 TOP: Determining Trigonometric Functions

338 ANS: 4
\[ a = \frac{14 - 4}{2} = 5, \quad d = \frac{14 + 4}{2} = 9 \]

PTS: 2 REF: 061810aii NAT: F.TF.B.5 TOP: Determining Trigonometric Functions

339 ANS: 1
The cosine function has been translated +3. Since the maximum is 5 and the minimum is 1, the amplitude is 2.
\[ \frac{\pi}{3} = \frac{2\pi}{b} \]
\[ b = 6 \]

PTS: 2 REF: 011913aii NAT: F.TF.B.5 TOP: Determining Trigonometric Functions

340 ANS: 4
period = \( \frac{2\pi}{B} \)
\[ \frac{1}{60} = \frac{2\pi}{B} \]
\[ B = 120\pi \]

PTS: 2 REF: 061624aii NAT: F.TF.B.5 TOP: Determining Trigonometric Functions
$H(t)$ is at a minimum at $70(-1) + 80 = 10$

PTS: 2  REF: 061613aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions  KEY: maximum/minimum

ANS: 3

342 ANS: 2  PTS: 2  REF: 011701aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions

343 ANS: 4  PTS: 2  REF: 061706aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions

344 ANS: 2  PTS: 2  REF: 081610aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions  KEY: increasing/decreasing

345 ANS: 4

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Bar Harbor</th>
<th>31.386</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midline</td>
<td>55.3</td>
<td>86.729</td>
</tr>
<tr>
<td>Maximum</td>
<td>79.214</td>
<td>106.967</td>
</tr>
<tr>
<td>Range</td>
<td>47.828</td>
<td>40.476</td>
</tr>
</tbody>
</table>

PTS: 2  REF: 061715aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions  KEY: maximum/minimum

346 ANS: 3  PTS: 2  REF: 081705aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions  KEY: increasing/decreasing

347 ANS: 1

\[-4(-1) - 3 = 1  \quad 8 = \frac{2\pi}{b}\]

\[b = \frac{\pi}{4}\]

PTS: 2  REF: 081820aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions  KEY: maximum/minimum

348 ANS:

\[\frac{h(2) - h(1)}{2 - 1} = -12, \quad h(t) = 0 \text{ at } t \approx 2.2, 3.8, \text{ using a graphing calculator to find where } h(t) = 0.\]

PTS: 4  REF: 061836aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions

349 ANS:

\[250(1) + 2450 = 2700 \quad \text{The maximum lung capacity of a person is 2700 mL.}\]

PTS: 2  REF: 081928aii  NAT: F.IF.B.4  TOP: Graphing Trigonometric Functions
350 ANS: 3
(3) repeats 3 times over \(2\pi\).

351 ANS: 4 PTS: 2 REF: 081718aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions KEY: amplitude

352 ANS: 2
\[ P = \frac{2\pi}{\frac{\pi}{45}} = 90 \]

353 ANS: 4
As the range is [4,10], the midline is \(y = \frac{4 + 10}{2} = 7\).

354 ANS: 1
The time of the next high tide will be the midpoint of consecutive low tides.

355 ANS: 4 PTS: 2 REF: 081912aaii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions KEY: mixed

356 ANS: Amplitude, because the height of the graph shows the volume of the air.

357 ANS: \[ \frac{10.1 - \left(-\frac{2}{2}\right)}{2} - \frac{2.5 - \left(-\frac{0.1}{2}\right)}{2} = 6.05 - 1.3 = 4.75 \]

358 ANS: 2 REF: 081930aaii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions KEY: amplitude
358 ANS:
period is $\frac{2}{3}$. The wheel rotates once every $\frac{2}{3}$ second.

PTS: 2 REF: 061728aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions
KEY: period

359 ANS:

PTS: 2 REF: 081830aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions
KEY: graph

360 ANS:

PTS: 2 REF: 061628aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions
KEY: graph

361 ANS:

$y = 2 \sin 4x$

PTS: 4 REF: 081934aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions
KEY: graph
Part a sketch is shifted $\frac{\pi}{3}$ units right.

The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of $-12$ and a maximum of 12. The value of $A$ is $-12$ since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter $B$. By experimentation with technology or using the relation $P = \frac{2\pi}{B}$ (where $P$ is the period), it is determined that $B = \frac{2\pi}{13}$.

$$f(t) = -12 \cos \left( \frac{2\pi}{13} t \right)$$

In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from $t = 13$ to $t = 19.5$ (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.
The period of $P$ is $\frac{2}{3}$, which means the patient’s blood pressure reaches a high every $\frac{2}{3}$ second and a low every $\frac{2}{3}$ second. The patient’s blood pressure is high because $\frac{144}{96}$ is greater than $\frac{120}{80}$.

365 ANS: 

$$\text{period} = \frac{2\pi}{0.8\pi} = 2.5.$$ The wheel rotates once every 2.5 seconds.

No, because the maximum of $f(t) = 26$.

366 ANS: 4

$$4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324$$

$$4(x - 3)^2 + 4(y + 9)^2 = 436$$

367 ANS: 3

368 ANS: 3

369 ANS: 2

370 ANS: 

No. $0.499 \pm 2(0.049) \rightarrow 0.401 - 0.597$. Since 0.43 falls within this interval, Robin’s coin is likely not unfair.
138.905 ± 2 · 7.95 = 123 − 155. No, since 125 (50% of 250) falls within the 95% interval.

ANS: 4  REF: 011835aiai  NAT: S.I.C.A.2  TOP: Analysis of Data

29.101 ± 2 · 0.934 = 27.23 − 30.97. Yes, since 30 falls within the 95% interval.

ANS: 4  REF: 011935aiai  NAT: S.I.C.A.2  TOP: Analysis of Data

Sample: pails of oranges; population: truckload of oranges. It is likely that about 5% of all the oranges are unsatisfactory.

ANS: 2  REF: 011726aiai  NAT: S.I.C.A.2  TOP: Analysis of Data

Since there are six flavors, each flavor can be assigned a number, 1-6. Use the simulation to see the number of times the same number is rolled 4 times in a row.

ANS: 1  REF: 081728aiai  NAT: S.I.C.A.2  TOP: Analysis of Data

II. Ninth graders drive to school less often; III. Students know little about adults; IV. Calculus students love math!

ANS: 3  REF: 081602aiai  NAT: S.I.C.B.3  TOP: Analysis of Data

Self selection causes bias.

ANS: 2  REF: 061703aiai  NAT: S.I.C.B.3  TOP: Analysis of Data

Self selection is a cause of bias because people with more free time are more likely to respond.
Randomly assign participants to two groups. One group uses the toothpaste with ingredient $X$ and the other group uses the toothpaste without ingredient $X$.

\[ ME = z \sqrt{\frac{p(1-p)}{n}} = 1.96 \sqrt{\frac{(0.55)(0.45)}{900}} \approx 0.03 \]

\[ ME = z \sqrt{\frac{p(1-p)}{n}} = 1.96 \sqrt{\frac{(0.16)(0.84)}{1334}} \approx 0.02 \]

Yes. The margin of error from this simulation indicates that 95% of the observations fall within $\pm 0.12$ of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to be 0.06 in the dotplot, by 2, or applying the estimated standard error formula, \( \sqrt{\frac{p(1-p)}{n}} \) and multiplying by 2. The interval 0.25 $\pm$ 0.12 includes plausible values for the true proportion of people who prefer Stephen’s new product. The company has evidence that the population proportion could be at least 25%. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 (9 out of 50) or less several times, even when the population proportion is 0.25, due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

\[ 2(0.042) = 0.084 \approx 0.08 \] The percent of users making in-app purchases will be within 8% of 35%.

\[ 0.254 \pm 2(0.060) \rightarrow (0.134,0.374) \]

\[ 0.301 \pm 2(0.058) \rightarrow 0.185 \pm 0.417 \frac{14}{60} \approx 0.23. \] It is not unusual because 0.23 falls within this interval.
The mean difference between the students’ final grades in group 1 and group 2 is –3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an online subscription. One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students’ final grades many (500) times. If the observed difference –3.64 is the result of the assignment of students to groups alone, then a difference of –3.64 or less should be observed fairly regularly in the simulation output. However, a difference of –3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups’ mean final grades.

\[0.602 \pm 2 \cdot 0.066 = 0.47 – 0.73.\] Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

Some of the students who did not drink energy drinks read faster than those who did drink energy drinks. \[17.7 – 19.1 = –1.4\] Differences of -1.4 and less occur \(\frac{25}{232}\) or about 10% of the time, so the difference is not unusual.

\[0.506 \pm 2 \cdot 0.078 = 0.35 – 0.66.\] The 32.5% value falls below the 95% confidence level.

\[23 – 18 = 5, \bar{x} \pm 2\sigma = –3.07 – 3.13,\] Yes, a difference of 5 or more occurred three times out of a thousand, which is statistically significant.

John found the means of the scores of the two rooms and subtracted the means. The mean score for the classical room was 7 higher than the rap room (82-75). Yes, there is less than a 5% chance this difference occurring due to random chance. It is likely the difference was due to the music.

Using a 95% level of confidence, \(x \pm 2\) standard deviations sets the usual wait time as 150-302 seconds. 360 seconds is unusual.
The pattern suggests an exponential pattern, not linear or sinusoidal. A 4% growth rate is accurate, while a 43% growth rate is not.

\[
y = 4.168(3.981)^x.
\]

\[
\log \frac{100}{4.168} = \log(3.981)^x
\]

\[
\log \frac{100}{4.168} = x \log(3.981)
\]

\[
\log \frac{100}{4.168} = x
\]

\[
x \approx 2.25
\]

\[
\bar{x} + 2\sigma \text{ represents approximately 48% of the data.}
\]
406 ANS: 3

PTS: 2 REF: 081604aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: probability

407 ANS: 4
496 ± 2(115)

PTS: 2 REF: 011718aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: interval

408 ANS: 1

PTS: 2 REF: 081711aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: percent

409 ANS: 3
440 × 2.3% ≈ 10

PTS: 2 REF: 011807aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: predict

410 ANS: 2

PTS: 2 REF: 061817aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: probability

411 ANS: 1
84.1% × 750 ≈ 631

PTS: 2 REF: 011923aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: predict
412 ANS: 4
400 \cdot 0.954 \approx 380

PTS: 2 REF: 061918aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: predict

413 ANS: 1

PTS: 2 REF: 081919aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: percent

414 ANS: 2 PTS: 2 REF: 011901aii NAT: S.ID.A.4
TOP: Normal Distributions KEY: mean and standard deviation

415 ANS:

PTS: 2 REF: 061726aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: percent

416 ANS: 1200 \cdot 0.784 \approx 941

PTS: 2 REF: 081828aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: predict

417 ANS:

\[
\text{normcdf}(510, 540, 480, 24) = 0.0994 \quad z = \frac{510 - 480}{24} = 1.25 \quad 1.25 = \frac{x - 510}{20} \quad 2.5 = \frac{x - 510}{20} \quad 535-560
\]
\[
z = \frac{540 - 480}{24} = 2.5 \quad x = 535 \quad x = 560
\]

PTS: 4 REF: fall1516aii NAT: S.ID.A.4 TOP: Normal Distributions
KEY: probability
The events are independent because \( P(A \text{ and } B) = P(A) \cdot P(B) \).

\[
0.125 = 0.5 \cdot 0.25
\]

If \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.25 + 0.5 - 0.125 = 0.625 \), then the events are not mutually exclusive because \( P(A \text{ or } B) = P(A) + P(B) \)

\[
0.625 \neq 0.5 + 0.25
\]

\[ P(S \cap M) = P(S) + P(M) - P(S \cup M) = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376} \]

\[ P(S \cup I) = 0.8 \]

\[ P(S \cap I) = 0.4 \]

If \( S \) and \( I \) are independent, then the Product Rule must be satisfied. However, \((0.5)(0.7) \neq 0.4\). Therefore, salary and insurance have not been treated independently.

This scenario can be modeled with a Venn Diagram:

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad A \text{ and } B \text{ are independent since } P(A \cap B) = P(A) \cdot P(B)
\]

\[
0.8 = 0.6 + 0.5 - P(A \cap B) \quad 0.3 = 0.6 \cdot 0.5
\]

\[ P(A \cap B) = 0.3 \quad 0.3 = 0.3 \]

\[
\frac{85}{210 + 85}
\]

\[ \frac{85}{210 + 85} \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.25 \]

\[ 0.125 \]

\[ 0.625 \]

\[ 0.5 \]

\[ 0.25 \]
ANS: 2

\[ P(B) \cdot P(A|B) = P(A \text{ and } B) \]
\[ P(B) \cdot 0.8 = 0.2 \]
\[ P(B) = 0.25 \]

ANS: 1

The probability of rain equals the probability of rain, given that Sean pitches.

ANS: 2

(1) 0.4 \cdot 0.3 \neq 0.2, (2) 0.8 \cdot 0.25 = 0.2, (3) P(A|B) = P(A) = 0.2, (4) 0.2 \neq 0.15 \cdot 0.05

0.2 \neq 0.2 \cdot 0.2

ANS: 2

\[ \frac{47}{108} = \frac{1}{4} + \frac{116}{459} \]

\[ P(M \text{ and } J) = \frac{31}{459} \]

ANS: 1

\[ \frac{157}{25 + 47 + 157} \]

ANS:

Based on these data, the two events do not appear to be independent. \( P(F) = \frac{106}{200} = 0.53 \), while

\( P(F|T) = \frac{54}{90} = 0.6 \), \( P(F|R) = \frac{25}{65} = 0.39 \), and \( P(F|C) = \frac{27}{45} = 0.6 \). The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.
ANS: No, because $P(M/R) \neq P(M)$
\[
\frac{70}{180} \neq \frac{230}{490}
\]
\[0.38 \neq 0.47\]


ANS:
A student is more likely to jog if both siblings jog. 1 jogs: $\frac{416}{2239} \approx 0.19$. Both jog: $\frac{400}{1780} \approx 0.22$


ANS: \[
\frac{103}{110 + 103} = \frac{103}{213}
\]


ANS: $P(F/L) = \frac{12}{27}$ $P(F) = \frac{22}{45}$ Since $P(F/L) \neq P(F)$, the events are not independent.


ANS: \[
P(P/K) = \frac{P(P^K)}{P(K)} = \frac{1.9}{2.3} \approx 82.6\% \text{ A key club member has an 82.6\% probability of being enrolled in AP Physics.}
\]


ANS: \[
P(W/D) = \frac{P(W^D)}{P(D)} = \frac{.4}{.5} = .8
\]

PTS: 2 REF: 081726aii NAT: S.C.P.B.6 TOP: Conditional Probability

ANS: \[
\frac{165 + 66 - 33}{825} = \frac{198}{825}
\]

PTS: 2 REF: 081925aii NAT: S.C.P.B.6 TOP: Conditional Probability